

**CVG2171/CVG2571**  
**Surveying and Measurements/Mesures et Arpentage**

Final Examination/Examen Final  
Friday, 24 April, 2015/ **vendredi 24 avril, 2015**  
Duration/**Durée**: 3 hours/**heures**

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Closed Book Examination/ **Examen à livre fermé**  
Non-programmable calculators allowed/**Les calculatrices non-programmables sont permises**  
All problems are of equal value/ **Tous les problèmes sont de poids égal**

### Question 1

The coordinates of the traverse ABCDEFA with respect to the two axes  $X$  and  $Y$  are as follows:/ **Les coordonnées d'un terrain ABCDEFA sont comme suit, en termes des axes  $X$  et  $Y$  :**

Point	X(m)	Y(m)
A	33.07	0.00
B	0.00	91.90
C	18.50	279.53
D	216.68	257.19
E	222.47	196.32
F	216.65	60.05

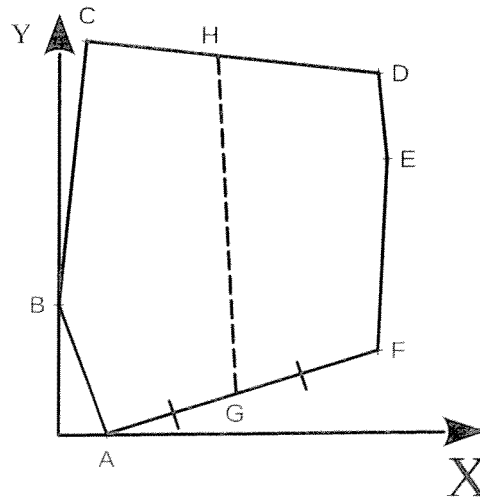


Figure 1: Terrain ABCDEFA

This field is to be divided into two equal parts by a line  $GH$ , where  $G$  is the mid-point of line  $AF$ , and  $H$  will be on line  $CD$  as is apparent from this diagram./ **Ce terrain doit être divisé en deux parties égales par la ligne  $GH$ , où  $G$  est le milieu du segment  $AF$ , et  $H$  se trouve le long du segment  $CD$  tel qu'illustré.**

- a) Find the area of this field by the coordinate method./ **Trouvez l'aire de ce terrain par la méthode des coordonnées.**
- b) Compute the coordinates of points  $G$  and  $H$ ./ **Calculez les coordonnées des points  $G$  et  $H$ .**

## Question 2

An equal-tangent vertical curve, 800.00 ft long, is to be used to connect a -4% grade to a +3% grade. The vertex or point of intersection of these grade lines is at station 105+00, elevation 320.10 ft. Calculate./ **Une courbe verticale à tangentes égales d'une longueur de 800,00 ft doit être utilisée pour raccorder une pente de -4% à une pente de +3%. Le point d'intersection de ces pentes se trouve à la station 105+00, à une altitude de 320,10 ft. Calculez :**

- a) The rate of change in grade per station./ **Le taux de changement de pente par station.**
- b) Elevations at 100 ft stations along the curve. Tabulate the result and do the usual check./ **Les élévations à des stations de 100 ft le long de la courbe.**
- c) The station and elevation of the lowest point of the curve./ **La station et l'altitude du point le plus bas le long de la courbe.**

## Question 3

The back and forward tangents  $AV$ , and  $VB$  of a proposed highway intersect at point  $V$ , station 75+00. The angle of intersection,  $I$ , is  $20^{\circ}07'36''$ . It is desired to connect these two tangents by a circular curve with a radius ( $R$ ) of 800.0 ft./ **Les tangentes avant,  $AV$ , et arrière,  $VB$  d'une autoroute proposée, s'intersectent au point  $V$ , à la station 75+00. L'angle d'intersection,  $I$ , est de  $20^{\circ}07'36''$ . Il est souhaité que les deux tangentes soient rattachées par une courbe circulaire ayant un rayon ( $R$ ) de 800,0 ft.**

- a) Find,  $D_a$ , the degree of the curve,  $E$ , the external distance, and the stations of the beginning of the curve ( $A$ ) and the end of it ( $B$ ) using the arc definition./ **Trouvez,  $D_a$ , le degré de la courbe,  $E$ , la contre-flèche, ainsi que les stations du début ( $A$ ) et de la fin de la courbe ( $B$ ) par la définition de l'arc.**
- b) Calculate and tabulate all data required to lay out this curve in the field by the deflection angles and chords method. Stations are 100 ft apart./ **Calculez et dressez un tableau contenant toutes les données requises pour implanter la courbe sur le terrain par la méthode des angles de déflexion et des cordes. Les stations sont aux 100 ft.**

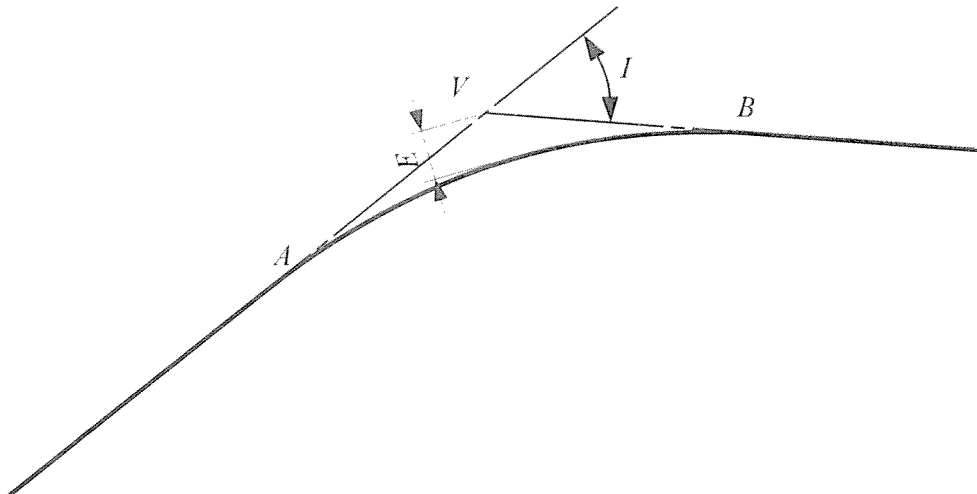


Figure 2: Horizontal Curve/ *Courbe horizontale*

### Question 4

The following notes are for cross sections at stations 95+00 and 96+00 of a proposed highway: / **Les notes de terrain suivantes ont été faites pour deux sections consécutives aux stations 95+00 et 96+00 d'une autoroute proposée :**

Station 95+00	C3.0 21.0	C3.4	C11.4 37.8
Station 96+00	C4.7 24.4	C4.8	C7.0 29.1

- i. Find the side slopes of these sections if the roadbed width is 30 ft./ **Trouvez les pentes latérales de ces sections si la largeur de la route est de 30 ft.**
- ii. Compute the volume of CUT between these two stations by the Average-End-Area formula/ **Calculez le volume de remblais entre ces deux stations par la méthode moyenne des aires.**
- iii. Compute the volume by the prismoïdal formula by applying a correction to the volume obtained in the previous part (ii)./ **Calculez le volume du prismoïde en appliquant la correction au volume calculé en (ii).**

## Question 5

All parts are INDEPENDENT/ **Toutes les parties sont INDÉPENDENTES**

- a) A bridge, 110 m long, measures 36.7 mm on a vertical photograph. Find the approximate dimensions (in meters) of a large rectangular building that also appears on this photograph and whose sides measures 22.0 mm by 18.3 mm. / **Un pont de 110 m de long mesure 36,7 mm sur une photographie aérienne. Trouvez les dimensions approximatives, en mètres, d'un édifice apparaissant également sur cette photo, et dont les côtés mesurent 22,0 mm par 18,3 mm.**
- b) Determine the height of the smoke-stack (tower) in front of Colonel By Hall at the University of Ottawa which appears on a vertical photograph taken from an aircraft flying at a height,  $H$ , of 7000 ft above its base. The distance from the principal point of this photo to the tower base is 86.5 mm, and to the tower top is 88.9 mm. / **Déterminez la hauteur de la cheminée devant le pavillon Colonel By de l'université d'Ottawa qui apparaît sur une photographie aérienne prise par un appareil volant à une altitude,  $H$ , de 7000 ft au-dessus de sa base. La distance du point principal de la photo à la base de la cheminée est de 86,5 mm, et au haut de la cheminée elle est de 88,9 mm.**
- c) A 20-mile strip of terrain is to be photographed for highway mapping. The aerial camera to be used has a lens of focal length 12 inches and takes 9×9 inches photographs. The ground elevation varies from a low of 900 ft to a high of 1100 ft above mean sea level. The plane has to make one flight line and the average width of photographic coverage is to be 1800 ft. / **Une**

bande de terrain de 20 miles de long doit être photographiée pour fin de cartographie. La caméra utilisée a une distance focale de 12 pouces et prend des photos de 9×9 pouces. L'altitude du terrain varie entre 900 et 1100ft au-dessus du niveau de la mer. L'avion doit faire une seule ligne de vol, et la largeur moyenne de la couverture photographique est de 1800 ft.

- i. What will be the average scale of the photographs? / **Quelle sera l'échelle moyenne des photographies?**
- ii. What flying height, above mean sea level, should be used for this flight?/ **Quelle altitude moyenne, au-dessus du niveau de la mer, devra être utilisée pour ce vol?**
- iii. If it is specified that the overlap (end lap) between photos be 60% and that 2 extra exposures be added at each end of the flight line, find how many photographs will be required./ **Si la superposition (longitudinale) entre photos doit être de 60%, et que 2 photos additionnelles doivent être prises à chaque extrémité de chaque ligne de vol, combien de photos seront nécessaires?**

Frequently Used Equations/Équations Fréquemment Utilisées

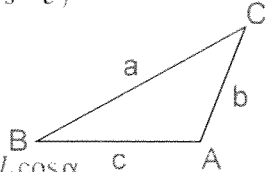
Area/Aires

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cos A$$

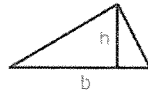
$$Dep. = L \sin \alpha \quad Lat. = L \cos \alpha$$



$$A_{triangle} = \frac{b \cdot h}{2}$$

$$m_{comp} = -1/m$$

(perpendicular slopes)



Volume (note: 1 yd<sup>3</sup>=27 ft<sup>3</sup>)

average end area:

$$V_e = \frac{A_1 + A_2}{2} \times L \quad (\text{vol. in units of length}^3)$$

$$C_p = \frac{L}{12} \cdot (c_1 - c_2)(w_1 - w_2) \quad (\text{vol. in units of length}^3)$$

borrow pit method:

$$V = \sum (h_{i,j} \cdot n) \left( \frac{A}{4} \right) \quad (\text{vol. in units of length}^3)$$

Solving a quadratic/Solution quadratique

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Photogrammetry/Photogrammétrie

$$S = \frac{f}{H} \quad S = \frac{f}{H-h}$$

$$X_A = \frac{(H-h_A)x_a}{f} \quad ; \quad Y_i = \frac{(H-h_i)y_i}{f}$$

$$d = \frac{r_{mp} \cdot h_{cover}}{H-h_{cover}} \quad ; \quad d \text{ is relief displacement}$$

$$\text{photo scale} = \frac{\text{photo distance}}{\text{map distance}} \times \text{map scale}$$

12.34 \cdot \cos(2\theta/2) / (1 - \cos(2\theta/2)) Determination of the Meridian/Détermination du Méridien

$$L.C.T. = G.C.T \pm \Delta\lambda$$

$$360^\circ \text{ de longitude} = 24 \text{ hours}$$

$$15^\circ \text{ de longitude} = 1 \text{ hour}$$

$$1^\circ \text{ de longitude} = 4 \text{ min (time)}$$

Circular Curves/Courbes Circulaires

$$R = \frac{50 \text{ ft}}{\sin\left(\frac{D}{2}\right)} \quad , \text{ Chord Definition}$$

$$R = \frac{5729.58}{D} (\text{ft}) = \frac{1746.37}{D} (\text{m}) \quad , \text{ Arc Definition}$$

$$T = R \tan \frac{I}{2} \quad ; \quad L = R \theta^{\text{rad}} = R \theta \cdot \frac{\pi}{180} \quad ; \quad L = 100 \frac{I}{D}$$

$$\frac{R}{R+E} = \cos \frac{I}{2} \quad ; \quad E = R \left( \sec \frac{I}{2} - 1 \right)$$

$$L.C. = 2R \sin \frac{I}{2} \quad ; \quad \text{chord} = 2R \sin \left( \frac{\theta}{2} \right)$$

$$\frac{R-M}{R} = \cos \frac{I}{2} \quad ; \quad M = R \left( 1 - \cos \frac{I}{2} \right)$$

$$c_s = 2R \sin \delta_s \quad ; \quad \text{or} \quad \delta_s = 0.3 c_s D \quad (\delta_s \text{ in minutes})$$

$\delta_s$  = def. angle. incr.;  $c_s$  = subchord

Vertical (Parabolic) Curves/ Courbes Verticales

$$\frac{\text{offset at } a}{\text{offset at } V} = \left[ \frac{x_a}{L/2} \right]^2$$

$$Y = Y_{BVC} + g_1 x + \left[ \frac{r}{2} \right] x^2 \quad ; \quad r = \frac{g_2 - g_1}{L}$$

$$L = \frac{g_2 - g_1}{\text{max. allowable change in grade per station}}$$

$$x = \left[ \frac{g_1 L}{g_1 - g_2} \right] \quad ; \quad x = \text{distance of min/max from BVC}$$

# CVG 2171/2571

## Examen / Exam 2015 Solution

Q1.

Point	X	Y	$X_1 Y_2$	$Y_2 Y_1$
A	33,07	0,00	3039,13	0,00
B	0,00	91,90	0,00	1700,15
C	18,50	279,53	4758,01	60568,56
D	216,68	257,19	42538,62	57217,06
E	222,47	196,32	13359,32	42532,73
F	216,65	60,05	0,00	1985,85
A	33,07	0,00		
			63695,08	164004,35
				<u>- 63695,08</u>

$$2A = 100309,27 \text{ m}^2$$

$$A = 50154,64 \text{ m}^2$$

a)

$$b) A_{ABCHG} = \frac{1}{2} A = 25104,82 \text{ m}^2$$

$$X_C = X_A + \frac{1}{2} (X_F - X_A) = 33,07 + \frac{1}{2} (216,65 - 33,07) = 124,86$$

$$Y_G = Y_A + \frac{1}{2} (Y_F - Y_A) = 0,00 + \frac{1}{2} (60,05 - 0,00) = 30,02$$

$$X_H = X_C + h (X_D - X_C) = 124,86 + h (216,68 - 124,86) = 124,86 + 91,82h$$

$$Y_H = Y_G + h (Y_D - Y_G) = 30,02 + h (257,19 - 30,02) = 30,02 + 227,17h$$

Point	X	Y	X <sub>1</sub> Y <sub>2</sub>	X <sub>2</sub> Y <sub>1</sub>
A	33,07	0,00	3039,13	0
B	0,00	91,90	0,00	1700,15
C	18,50	279,53	5171,3 - 413,29h	5171,3 + 55397,3h
H	18,5 + 198,18h	279,53 - 22,34h	5171,3 - 413,29h	5171,3 + 55397,3h
G	124,86	30,02	555,37 + 594936h	34902,1 - 2789,4h
A	33,07	0,00	0	99276

$$8765,8 + 5536,07h \quad 42766,3 + 52607,9h$$

$$2A = \frac{34000,5 + 47071,8h}{-8765,8 - 5536,11}$$

$$A = \frac{17000 + 23536h}{23536} = 25077,3$$

$$h = \frac{(25077,3 - 17000)}{23536} = \frac{8077,3}{23536}$$

$$h = 0,343$$

$$X_H = 18,50 + 198,18(0,343) = \boxed{86,48}$$

$$Y_H = 279,53 - 22,34(0,343) = \boxed{271,86}$$

Q2

$$g_1 = -4\% , g_2 = +3\%$$

$$\text{PVI: } 105+00, 320.10 \text{ ft}$$

$$L = 800.00 \text{ ft}$$

$$a) r = \frac{g_2 - g_1}{L} = \frac{0.03 - (-0.04)}{800.00} = 8.75 \times 10^{-5} \text{ ft}^{-1} = 0.875 \text{ ft}^{-1} \text{ station}^{-1}$$

$$b) \text{BVC} = \text{DCV} = 105+00 - 4+00 = 101+00$$

$$Z_A = Z_{\text{PVI}} - g_1 \times L/2 = (320.10 - (-0.04) \times 400) \text{ ft} = 336.10 \text{ ft}$$

$$y = Z_A + g_1 X + \frac{1}{2} r X^2 = 336.10 \text{ ft} - 4 X + 0.4375 X^2$$

Station	X (stations)	X <sup>2</sup> (stations <sup>2</sup> )	Y	Δ	Δ <sup>2</sup>
101+00	0	0	336.10	-3.56	
102+00	1	1	332.54	-2.69	0.87
103+00	2	4	329.85	-1.81	0.88
104+00	3	9	328.04	-0.94	0.87
105+00	4	16	327.10	-0.06	0.88
106+00	5	25	327.04	0.81	0.87
107+00	6	36	327.85	1.69	0.88
108+00	7	49	329.54	2.56	0.87
109+00	8	64	332.10		

$$\overline{\Delta^2} = r = 0.875$$

$$c) x_{\min} = \frac{g_1}{g_1 - g_2} = \frac{-4 \times 8+00}{-4 - 3} = \frac{-32+00}{-7} = 4+57,14$$

$$\text{Station}_{\min} = 101+00 + 4+57,14 = \boxed{105+57,14}$$

$$y_{\min} = 336,10 - 4(4,5714) + 0,4375(20,8977) = \boxed{326,96 \text{ ft}}$$

$$Q3. \text{PI} = 75+00, \quad I = 20^\circ 07' 36'' = 20 + \frac{7}{60} + \frac{36}{3600} = 20,1267^\circ$$

$$R = 800,00 \text{ ft}$$

$$a) 100 \text{ ft} = \theta R = D_a \cdot \frac{\pi}{180} \cdot R$$

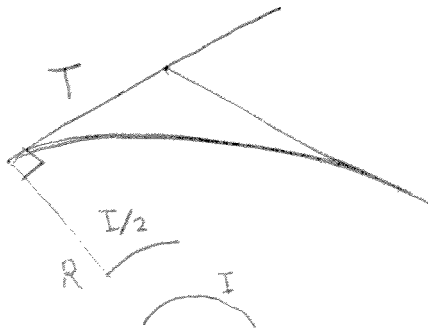
$$D_a = \frac{100 \text{ ft} \cdot \frac{180^\circ}{\pi}}{R} = \frac{100 \text{ ft} \times 180^\circ}{800 \text{ ft} \times \pi} = 7,162^\circ = \boxed{7^\circ 09' 43,1''}$$

$$E = R \left[ \sec\left(\frac{I}{2}\right) - 1 \right] = 800 \left[ \frac{1}{\cos(10,06335^\circ)} - 1 \right] = \boxed{12,50 \text{ ft}}$$

$$L = \theta R = I \cdot \frac{\pi}{180^\circ} \cdot R = 20,1267^\circ \times \frac{\pi}{180^\circ} \cdot 800 \text{ ft} = 281,02 \text{ ft}$$

$$\tan\left(\frac{I}{2}\right) = \frac{T}{R}$$

$$\begin{aligned} T &= R \tan\left(\frac{I}{2}\right) \\ &= 800 \text{ ft} \tan(10,06335^\circ) \\ &= 141,97 \text{ ft} \end{aligned}$$



$$\begin{aligned} \text{Station}_{TC} &= 75+00 - 1+41,97 \\ &= \boxed{73+58,03} \end{aligned}$$

$$\begin{aligned} \text{Station}_{CT} &= \text{Station}_{TC} + L = 73+58,13 + 2+81,02 \\ &= \boxed{76+39,05} \end{aligned}$$

b) Station	S (ft)	d (°)	δ (°)	C (ft)	Σ δ (°)
73+58.03	41.97	3.0059°	1.50295°	41.97	0
74+00	100.00	7.162°	3.581°	99.94	1.503°
75+00	100.00	7.162°	3.581°	99.94	5.084
76+00	39.05	2.7968°	1.3984	39.05	8.665
76+39.05					<u>10.063</u> = I/2 ✓

$$S_a = 74+00 - 73+58.03 = 41.97 \text{ ft}$$

$$S_b = 100 \text{ ft}$$

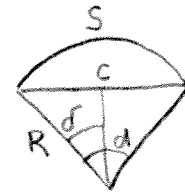
$$S_c = 76+39.05 - 76+00 = 39.05 \text{ ft}$$

$$S_a = \theta_a R = d_a \cdot \frac{\pi}{180} \cdot R \Rightarrow d_a = \frac{S_a}{R} \cdot \frac{180^\circ}{\pi} = \frac{41.97 \text{ ft}}{800 \text{ ft}} \cdot \frac{180^\circ}{\pi} = 3.0059^\circ$$

$$d_b = \frac{100 \text{ ft}}{800 \text{ ft}} \cdot \frac{180^\circ}{\pi} = 7.162^\circ \quad (\text{degree of curve})$$

$$d_c = \frac{39.05 \text{ ft}}{800 \text{ ft}} \cdot \frac{180^\circ}{\pi} = 2.7968^\circ$$

$$\delta = d/2$$



$$\sin(d/2) = \frac{C/2}{R} \Rightarrow C = 2R \sin(d/2) = 2R \sin(\delta)$$

$$C_a = 2(800 \text{ ft}) \sin(1.50295^\circ) = 41.97 \text{ ft}$$

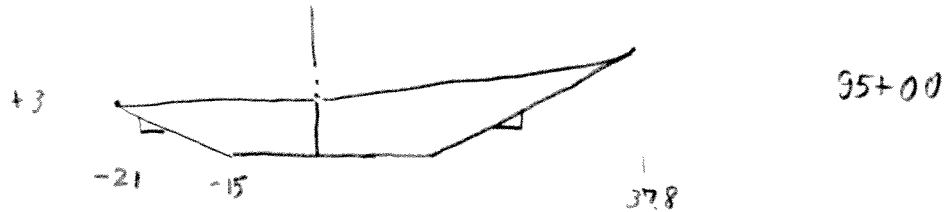
$$C_b = 2(800 \text{ ft}) \sin(3.581^\circ) = 99.94 \text{ ft}$$

$$C_c = 2(800 \text{ ft}) \sin(1.3984^\circ) = 39.05 \text{ ft}$$

Q4. Station 95+00       $\frac{C3.0}{21.0}$       C3.4       $\frac{C11.4}{37.8}$

Station 96+00       $\frac{C4.7}{24.4}$       C4.8       $\frac{C7.0}{29.1}$

i)

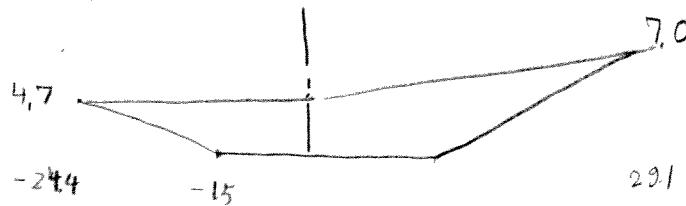


$b = 30 \text{ ft}$

$$\text{slope left: } 3 / (21 - 15) = 3 / 6 = \boxed{1:2}$$

$$\text{slope right: } 11.4 / (37.8 - 15) = \frac{11.4}{22.8} = \boxed{1:2}$$

Station 96+00



$$\text{slope left: } 4.7 / (24.4 - 15) = 4.7 / 9.4 = \boxed{1:2}$$

$$\text{slope right: } 7.0 / (29.1 - 15) = 7.0 / 14.1 = \boxed{1:2.01 \approx 1:2}$$

ii) X	Y	$X_1 Y_2$	$X_2 Y_1$
-15	0	-45	0
-21	3.0	-71,4	0
0	3.4	0	120,52
37.8	11.4	0	0
15	0	0	171,0
-15	0	0	0
		-116,4	299,52

$$2A = 415,92$$

$$A = 207,96 \text{ ft}^2$$

X	Y	$X_1 Y_2$	$X_2 Y_1$
-15	0	-70,50	0
-24.4	4.7	-117,12	0
0	4,8	0	139,68
29.1	7.0	0	105,00
15	0	0	0
-15	0	0	0
		-187,62	244,68
			+187,62

$$2A = 432,30 \text{ ft}^2$$

$$A = 216,15 \text{ ft}^2$$

$$V = \frac{1}{2} (207,96 \text{ ft}^2 + 216,15 \text{ ft}^2)$$

$$V = 21205,5 \text{ ft}^3$$

$$V = 785,39 \text{ yd}^3$$

$$\text{iii) } C_p = \frac{L}{12} (c_1 - c_2) (W_1 - W_2)$$

$$= \frac{100 \text{ ft}}{12} (3.4 - 4.8) \text{ ft} ([37.8 + 21.0] - [29.1 + 24.4]) \text{ ft}$$

$$= \frac{100}{12} (-1.4) (58.8 - 53.5) = \frac{100}{12} (-1.4) (5.3) = -61.83 \text{ ft}^3$$

$$V_p = V - C_p = 21205.50 - (-61.83) = \boxed{21267.33 \text{ ft}^3}$$

$$V_p = 787.68 \text{ yd}^3$$

Q5

$$\text{a) } S = \frac{36.7 \text{ mm}}{110 \text{ m}} = \frac{22.0 \text{ mm}}{X} = \frac{18.3 \text{ mm}}{Y}$$

$$X = 110 \text{ m} \times \frac{22.0 \text{ mm}}{36.7 \text{ mm}} = 65.94 \text{ m}$$

$$Y = 110 \text{ m} \times \frac{18.3 \text{ mm}}{36.7 \text{ mm}} = 54.85 \text{ m}$$

$$\text{b) } d = \frac{r_{\text{top}} h_t}{H - h_{\text{base}}} \Rightarrow h_t = d \frac{H - h_{\text{base}}}{r_{\text{top}}} = \frac{(r_{\text{top}} - r_{\text{base}})}{r_{\text{top}}} (H - h_{\text{base}})$$

$$h_t = \frac{88.9 - 86.5}{88.9} \times 7000 \text{ ft} = \boxed{188.97 \text{ ft}}$$

$$\text{c) } f = 12'' \quad l = 9'' \quad \bar{h} = \frac{1}{2} (900 \text{ ft} + 1100 \text{ ft}) = 1000 \text{ ft}$$

$$L = 1800 \text{ ft}$$

$$\text{i) } S = \frac{9''}{1800 \text{ ft}} = \frac{9/12}{1800} = 4.167 \times 10^{-4} = \boxed{1:2400}$$

$$ii) \quad S = \frac{f}{H-h}$$

$$H-h = \frac{f}{S} = 2400 \times 1 \text{ ft} = 2400 \text{ ft}$$

$$H = 2400 \text{ ft} + \bar{h} = 2400 \text{ ft} + 1000 \text{ ft}$$

$$\boxed{H = 3400 \text{ ft}}$$



$$iii) \quad \text{Cover.} = (100\% - 60\%) \times 1800 \text{ ft} = 720 \text{ ft}$$

$$L_{\text{Terrain}} = 20 \text{ mil} \times 80 \text{ GC/mil} \times 66 \text{ ft/GC} = 105600 \text{ ft}$$

$$n_{\text{photos}} = 1 + \frac{105600 \text{ ft}}{720 \text{ ft}} = 148 \text{ photos}$$

$$n_{\text{total}} = n_{\text{photos}} + 2 + 2 = 148 + 2 + 2 = 152 \text{ photos}$$