# Module: Calculus 1

## Rates of Change Using Equations

If we have the equation of a function, $y = f(x)$, we can make more accurate estimates of the instantaneous rate of change when $x = a$ by finding the average rate of change over a small interval $a \leq x \leq a + h$.

### Example

The position (in meters) of a moving object is given by $s(t) = 2t^2 + 3t + 2$, where $t$ is measured in seconds. What is the instantaneous velocity of the object at time $t = 2$s?

The slope of the secant line (which we are using as an approximation of the tangent line) passing through $P = (2, s(2))$ and $Q = (2 + h, s(2 + h))$ is

- $\dfrac{\Delta s}{\Delta t} = \dfrac{s(2+h) - s(2)}{(2+h) - 2}$
- $= \dfrac{2(2+h)^2 + 3(2+h) + 2 - (2(2)^2 + 3(2) + 2))}{h}$
- $= \dfrac{2(4 + 4h + h^2) + 6 + 3h + 2 - 16}{h}$
- $= \dfrac{8 + 8h + 2h^2 + 6 + 3h + 2 - 16}{h}$
- $= \dfrac{11h + 2h^2}{h}$

so if $h = 1$ s, $\dfrac{\Delta s}{\Delta t} = 13$ m/s

so if $h = 0.1$ s, $\dfrac{\Delta s}{\Delta t} = 11.2$ m/s

so if $h = 0.01$ s, $\dfrac{\Delta s}{\Delta t} = 11.02$ m/s

and so it appears that the instantaneous velocity of the object at time $t = 2$ s is 11 m/s.

What are we actually doing here? We're using secant lines to approximate the tangent line.

a graph of a function with 3 secant lines through points P=(a,f(a)) and Q=(a+h+,f(a+h)) for 3 different values of h and the tangent line at P

The smaller $h$ is, the closer $Q$ is to $P$ and the better the secant line approximates the tangent line. And as we shrink $h$ to zero, the difference quotient $\frac{f(a+h)-f(a)}{h}$, which is the slope of the secant line or the average rate of change on the interval $a \le x \le a+h$, becomes closer and closer to the slope of the tangent line or the instantaneous rate of change at $x = a$.

But what do we mean by shrinking $h$ to 0 and something becoming closer and closer to something else?

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