**Exercise Problems for the Final**

**MATH 265 / Mast 219, Winter 2009**

This collection contains only problems related to the second part of the course starting with line integrals. For the problems related to the previous material see Exercise Problems for the Midterm.

**Problem -1:** Evaluate the integral

$$\int_C x \exp(yz) ds \ ,$$

where $C$ is the line segment from $(0, 0, 0)$ to $(1, 2, 3)$.

**Problem 0:** Find the work done by the force $F = [x, y]$ on the particle which moves counterclockwise on the upper half ($y \geq 0$) of the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \ .$$

**Problem 1:** Evaluate the integral

$$\oint_C (1 + 10xy + y^2)dx + (6xy + 5x^2)dy \ ,$$

where $C$ is the square with the vertices $(0, 0)$, $(a, 0)$, $(a, a)$ and $(0, a)$.

**Problem 2:** Show that if $F = \text{ grad } f$, then

$$\oint_C F dr = 0 \ ,$$

for any simple, piecewise smooth, closed curve $C$.

**Problem 3:** Find the area of the surface $3z^2 = (x + y)^3$, with $x + y \leq 2$, $x \geq 0$, $y \geq 0$, $z \geq 0$.

**Problem 4:** Find the area of the surface $r(u, v) = [u \cos v, u \sin v, u^2]$, with $0 \leq u \leq 1$, $0 \leq v \leq 3\pi$.

**Problem 5:** Evaluate

$$\iint_S x^2 z \, dS \ ,$$

where $S$ is the part of the cylinder $x^2 + z^2 = 1$ between the planes $y = 0$ and $y = 2$ and above the $xy$-plane.

**Problem 6:** Determine the flux of $F = [-xy^2, z, 0]$ across the surface $z = xy$, with $0 \leq x \leq 1$, $0 \leq y \leq 2$, in the direction of the upper unit normal.

**Problem 7:** Determine the flux of $F = [x, y, 0]$ across the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the direction of the outward unit normal.

**Problem 8:** Prove:

(a) curl (grad $f$)$= 0$ , for any $f \in C^2$;

(b) div (curl $F$) $=0$ , for any vector field $F$ with continuous second partial derivatives.

**Problem 9:** Find the flux of $F = [x, 2y^2, 3z^2]$ out of the region bounded by the surfaces $x^2 + y^2 = 9$, $z = 0$ and $z = 1$.

**Problem 10:** What is the flux of $F = [Ax, By, Cz]$ out of the region of volume $V$ ?

**Problem 11:** Let $F = [\frac{1}{2}y, 2xz, -3x]$ and $S$ be the surface $y = 1 - x^2 - z^2$ from $y = -8$ to $y = 1$. Calculate the flux of curl$(F)$ in the direction of the unit normal with positive $y$-component.

**Problem 12:** The cylinder $x^2 + y^2 = b^2$ intersects the plane $y + z = a^2$ in a curve $C$. Assume that $a^2 \geq b > 0$. Calculate the circulation of $F = [xy, yz, xz]$ about $C$.