

**ADM 2304  
Fall 2012  
Solutions and Marking Guide to Assignment 1.**

**Problem 1. [ 3 marks ]**

*3 marks: done by MyStatLab*

**Problem 2. [ 10 marks ]**

- (a)  $H_0: p = .306$ ,  $H_a: p > .306$   
 $n = 1836$  with  $\hat{p} = .36$   
 $z = (.36 - .306) / \sqrt{.306 * .694 / 1836} = 5.0$   
 At the .05 significance level, we reject  $H_0$  since  $5.45 > 1.645$   
 Conclude there has been an increase in NDP support.

Test of  $p = 0.306$  vs  $p > 0.306$

| Sample | X   | N    | Sample p | 95%         |         |         |
|--------|-----|------|----------|-------------|---------|---------|
|        |     |      |          | Lower Bound | Z-Value | P-Value |
| 1      | 661 | 1836 | 0.360022 | 0.341595    | 5.02    | 0.000   |

*4 marks:*

*1 for hypotheses (deduct .5 for 2-sided alternative)*

*1 for z-statistic or CI or p-value (deduct .5 if z is based on  $SE = \sqrt{.36 * .64 / 1836}$  instead of  $SD$  based on  $p = .306$ .)*

*1 for decision to reject the null  $H_0$ .*

*1 for conclusion*

(b)

$$n = z^2 pq / M^2 = 1.96^2 .36 * .64 / .01^2 = 8851$$

*2 marks: 1 mark for using  $\hat{p}$  of .36, 1 mark for answer.*

(c)

$H_0: p = .306$ ,  $H_a: p > .306$

$n = 17$  is not sufficient to assume a normal distribution for  $\hat{p}$  since  $np = 17 * .306 = 5.2$  is not greater or equal to 10 or since  $X = 8$  is not greater or equal to 10.

Use the binomial ( $n = 17$ ,  $p = .306$ ) to calculate

the p-value =  $P(X \geq 8) = 1 - P(X \leq 7) = 1 - P(0) - P(1) - \dots - P(7) = 1 - 0.885 = .115$

(see calculations below)

Since p-value not  $< .05$ , we do not reject the null  $H_0$ . To use the critical value approach, we reject the null hypothesis if  $X \geq 9$  since  $P(X \geq 9) = 1 - P(X \leq 8) = 0.05$ . Since  $X = 8$  is not in the rejection region, we do not reject the null hypothesis.

Conclude that there is insufficient evidence to suggest that NDP support among students exceeds .306

**Cumulative Distribution Function**

Binomial with  $n = 17$  and  $p = 0.306$

```
x  P( X <= x )
7  0.884737
```

```
x    binomial prob'y
0    0.002010
1    0.015063
2    0.053134
3    0.117140
4    0.180774
5    0.207239
6    0.182752
7    0.126625
Total 0.884737
```

Note that Minitab handles this easily if you do not assume a normal distribution for  $\hat{p}$ .

### Test and CI for One Proportion

Test of  $p = 0.306$  vs  $p > 0.306$

|        |   |    |          | 95%         |               |
|--------|---|----|----------|-------------|---------------|
| Sample | X | N  | Sample p | Lower Bound | Exact P-Value |
| 1      | 8 | 17 | 0.470588 | 0.260114    | <b>0.115</b>  |

If normal approximation is used, then  $z = (.47 - .306)/\sqrt{.306*.694/17}$   
 $= .164/.112=1.46$ , which has a p-value of  $P(z>1.46) = .07$ . Clearly this is not close to correct value of 0.115.

Another possibility is to use the adjustment  $\hat{p} = (x+2)/(n+4) = 10/21 = .476$   
and to calculate the CI based on this value:

at least  $.476 - 1.645 * \sqrt{.476*.524/21} = .476 - 1.645 * 0.109 = .476 - .179 = .297$ .

Since this CI covers the value .306, we do not reject the null hypothesis.

Note that, using .476,  $z$  is  $(.476 - .306)/.109 = 1.56$  with a p-value of  $P(z > 1.56) = 0.06$ .  
this is still not a good approximation.

*4 marks:*

*1 for hypotheses*

*1 for some recognition that the normal approximation is not appropriate*

*1 for showing p-value of .115 or rejection region of  $X \geq 9$  by calculating the binomial prob'y or using Minitab.*

*1 for decision and conclusion (if the answer uses the  $z$  of 1.46, then the decision would be to not reject the null  $H$  since  $z$  not  $> 1.645$ , and we cannot conclude that NDP support among students is  $> .306$ )*

If students calculate the z-statistic manually based on the assumption that the sample proportion is normally distributed, then give them 2 marks out of 4.

If they use the adjustment  $(x+2)/(n+4)$ , then give them 3 marks out of 4.

We cannot infer anything about the national level of support for the NDP since the sample of U of Ottawa students from Quebec is not a sample from that population.  
(no mark)

**Problem 3. [ 10 marks ]**

(a) Using Minitab, we can calculate the population mean of the male BMIs as 26.244

| Variable | N   | N* | Mean          | SE Mean | StDev | Minimum | Q1     | Median | Q3     |
|----------|-----|----|---------------|---------|-------|---------|--------|--------|--------|
| bmi_m    | 639 | 0  | <b>26.244</b> | 0.153   | 3.873 | 17.100  | 23.600 | 25.600 | 28.500 |

*1 mark for mean of **26.244***

(b) *3 marks for evidence of 20 CIs.*

(c) *1 mark for any graph of data*

*2 marks for any reasonable comment re data distribution— that the small sample can or cannot be assumed to come from a normal distribution or that the large sample came from a population that is not extremely skewed and how this means the CI is or is not valid.*

*1 mark for manual calculation of CI*

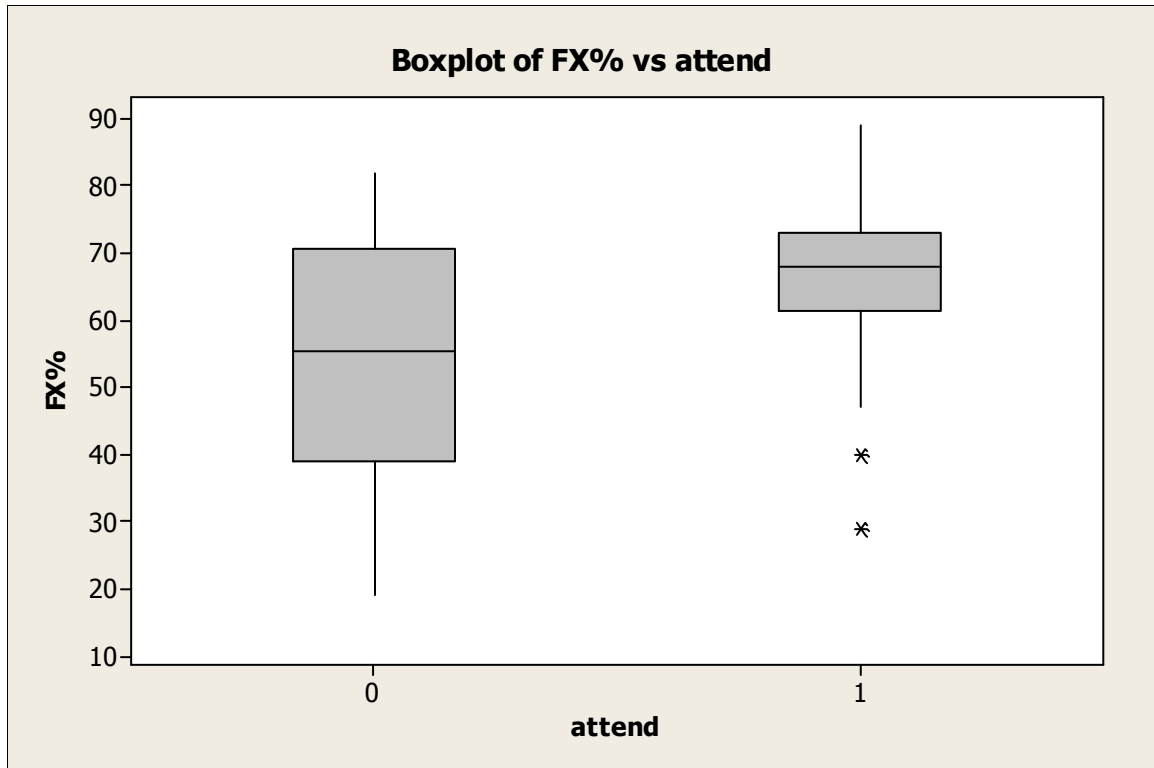
(d) *1 mark for count*

(e) *1 mark for one of the binomial probabilities below.*

| <i>x</i> | <i>prob</i> |
|----------|-------------|
| 20       | 0.358486    |
| 19       | 0.377354    |
| 18       | 0.188677    |
| 17       | 0.059582    |
| 16       | 0.013328    |
| 15       | 0.002245    |
| 14       | 0.000295    |
| 13       | 0.000031    |

**Problem 4. [ 12 marks ]**

(a)



Since first sample is small, we need to assume the population of marks is normally distributed. This is a reasonable assumption since the boxplot is symmetric and there are no outliers.

Since the second sample is large, we only need to assume the population is not extremely skewed. This is a valid assumption given the boxplot distribution.

The 2-sample t test is valid.

*2 marks: 1 for a relevant comment on the assumptions for each population based on the relevant sample.*

(b)

### Two-Sample T-Test and CI: FX%, attend

Two-sample T for FX%

| attend | N  | Mean | StDev | SE Mean |
|--------|----|------|-------|---------|
| 0      | 28 | 54.4 | 17.8  | 3.4     |
| 1      | 48 | 66.4 | 10.8  | 1.6     |

Difference = mu (0) - mu (1)

Estimate for difference: -11.9256

99% CI for difference: (-21.9768, -1.8744)

T-Test of difference = 0 (vs not =): T-Value = -3.22 P-Value = 0.003 DF = 38

Ho:  $\mu(0) = \mu(1)$ ; Ha:  $\mu(0) \neq \mu(1)$

$$t = (66.4 - 54.4) / \sqrt{17.8^2/28 + 10.8^2/48} = 3.24 \text{ or } -3.24$$

We reject the null hypothesis at the .01 level of significance since  $3.22 > 2.57$  or  $2.58$  which is the approximate critical value based on z approximation. The exact t critical value based on 38 d.f. is 2.71.

Conclude there is a difference in average marks based on attendance.

### Inverse Cumulative Distribution Function

Student's t distribution with 38 DF

|             |          |
|-------------|----------|
| P( X <= x ) | x        |
| 0.005       | -2.71156 |

4 marks:

1 for hypotheses

1 for t-statistic (must show manual calculation)

1 for decision based on critical value

1 for conclusion

(c) p-value is  $2 * \text{Prob}(t > 3.22)$  or  $2 * \text{Prob}(t < -3.22)$

Using the normal approximation, this is  $2 * 0.0006 = 0.0012$

To get the exact p-value, use Minitab to find

$$2 * P(t < -3.22) = 2 * 0.0013 = 0.0026 = 0.003.$$

Student's t distribution with 38 DF

|       |             |
|-------|-------------|
| x     | P( X <= x ) |
| -3.22 | 0.0013131   |

2 marks: 1 for showing p-value is double the tail probability of  $P(t < -3.24)$  or  $P(t > 3.24)$

1 for a reasonable attempt at showing how the tail prob of .0013 can be found.

(d) 2-sided CI is:

$$12 \pm 2.576 * \sqrt{17.8^2/28 + 10.8^2/48} = 12 \pm 2.576 * 3.71$$

$$= 12 \pm 9.56 = (2.44, 21.56) \text{ if we use the z critical value}$$

Using the t-distribution, we have:

$$12 \pm 2.71 * 3.71 = 12 \pm 10 = (2, 22) \text{ or } (-22, -2)$$

2 marks for showing manual calculation of CI (do not worry about accuracy)

(e) We reject the null H again since the p-value of 0.003 is less than .01 or because the CI does not cover zero.

1 mark for noting the same decision and conclusion as before

(f) We could use the Mann-Whitney test a.k.a. Wilcoxon Rank Sum test.

1 mark for identification of the alternative non-parametric test