midTerm Soln's

COURSE TITLE Probability and Statistics for Computer Science

COURSE NUMBER COMP 233

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NUMBER OF ORIGINALS 8

COST 40/45

APPROX. NO. STUDENTS

START DATE June 9/10

EXPIRATION E.O.T

39-C

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EXPIRED ORIGINALS

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40

COMP. 233.

Midterm Solutions.

1. $$\frac{\binom{15}{7}}{\binom{20}{7}} = ({}_{15}C_7)({}_{20}C_7)^{-1} = .083.$$

2. Range: $(9999-1000)+1 = 9000$ numbers.

A. Every ninth number is divisible by 9.

$\therefore$ $\frac{9000}{9} = 1000$ numbers are divisible by 9.

B. Every second number is even.

$\frac{9000}{2} = 4{,}500$ numbers are even.

C. $(9)(9)(8)(7) = 4536$ numbers have distinct digits.

D. $\frac{9000}{3} = 3000$ numbers are divisible by 3.

$\therefore$ $9000 - 3000 = 6000$ numbers are not divisible by 3.

3. Let A: Set of Bit Strings starting with 00.

B: " " " " ending with 111.

C: " " " " starting with 00 or ending with 111.

$|A| = 2^5 = 32.$

$|B| = 2^4 = 16.$

$|C| = |A \cup B| = |A| + |B| - |A \cap B|$

$= 32 + 16 - 2^2$

$C = 44.$

4. A. $\binom{12}{3} = {}_{12}C_3 = 220$ strings.

B. $\binom{12}{3} + \binom{12}{2} + \binom{12}{1} + \binom{12}{0} = 220 + 66 + 12 + 1$

$= 299$ strings.

C.

$2^{12} - \left[\binom{12}{2} + \binom{12}{1} + \binom{12}{0}\right] = 4017$ strings.

D. ${}_{12}C_6 = 924$ strings.

5. Bit string length: 18.

Number of 01 pairs: 8.

(8)(2) = 16 bit positions occupied ⇒ the 2 remaining positions must be occupied by 1's.

Number of grouped bit string positions

8 pairs + 2 singles = 10 bit string positions

(grouped bit string length is 10).

$\therefore \binom{10}{2} = 45$ strings.

6. One way of choosing winning numbers.

$\therefore \dfrac{1}{_{36}C_6} = 5.134 \times 10^{-7} = .0000005134.$

7. Let H: event person has the disease.

T: event person tests positive for the disease

$P(H|T)$?

$P(H) = \frac{1}{100{,}000} = .00001.$

$P(\bar{H}) = .99999.$

$P(T|H) = .99.$

$P(\bar{T}|H) = .01.$

$P(\bar{T}|\bar{H}) = .995.$

$P(T|\bar{H}) = .005.$

$\therefore$ $P(H|T) = \frac{P(HT)}{P(T)}$

$= \frac{P(H)\,P(T|H)}{P(T)}$

$= \frac{P(H)\,P(T|H)}{P(TH \cup T\bar{H})}$

$= \frac{P(H)\,P(T|H)}{P(TH) + P(T\bar{H})}$

$$= \frac{P(H)\,P(T|H)}{P(H)\,P(T|H) + P(\bar{H})\,P(T|\bar{H})}$$

$$= \frac{(.00001)(.99)}{(.00001)(.99) + (.99999)(.005)}$$

$P(H|T) \doteq .002.$

8. A.

$$P(10 < X < 19) = \sum_{i=11}^{19} \frac{e^{-9.1}(9.1)^i}{i!}$$

$= .3047$ (or use table).

B.

$$P(X \geq 20) = \sum_{i=20}^{\infty} \frac{e^{-9.1}(9.1)^i}{i!}$$

$= .0011$ (or use table).

C.

$P(X \leq 10) = 1 - (.3047 + .0011)$

$= .6942.$

9. $P(|X| \le 15) = P(-15 < X < 15)$

$$= P\left(\frac{-15-20}{2} < \frac{X-20}{2} < \frac{15-20}{2}\right)$$

$$= P\left(\frac{-35}{2} < Z < \frac{-5}{2}\right)$$

$$= P(-17.5 < Z < -2.5)$$

$$= P(Z < 17.5) - P(Z < 2.5)$$

$$\doteq 1 - .9938$$

$P(|X| \le 15) \approx .0062.$

10. A.

$$P(X=0) = \frac{{}_5C_3}{{}_{12}C_3} + \frac{({}_3C_1)({}_4C_1)({}_5C_1)}{{}_{12}C_3}$$

$$= .31\overline{8}.$$

$P(X=0) = P(3 \text{ black balls}) + P(1 \text{ white}, 1 \text{ red}, 1 \text{ black})$

B. $P(X=1) = P(1W, 2B) + P(2W, 1R)$

$$= \frac{({}_3C_1)({}_5C_2)}{{}_{12}C_3} + \frac{({}_3C_2)({}_4C_1)}{{}_{12}C_3}$$

$P(X=1) = .19.$

11. A. $P(0,1) = P(R=0, W=1, B=2)$

$$= \frac{({}_3C_1)\,({}_4C_0)\,({}_5C_2)}{{}_{12}C_3}$$

$P(0,1) = .1\overline{36}.$

B. $P(1,1) = P(R=1, W=1, B=1)$

$$= \frac{({}_4C_1)({}_3C_1)({}_5C_1)}{{}_{12}C_3}$$

$P(1,1) = .\overline{27}.$

12.

$$P(X<a) = \int_0^a \int_0^\infty 2e^{-x} e^{-2y}\, dy\, dx$$

$$P(X<a) = \int_0^a e^{-x}\, dx = 1 - e^{-a}.$$

13. A. X has a Geometric Dist., with parameter: $p = \frac{1}{6}$.

B. $E[X] = \frac{1}{p} = 6$ (mean of a Geometric Dist. is $\frac{1}{p}$).

C. $Var(X) = \frac{1-p}{p^2} = \frac{\left(1-\frac{1}{6}\right)}{\left(\frac{1}{36}\right)} = 30.$

Variance of a Geometric Dist. is $\frac{1-p}{p^2}$.

14.

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{elsewhere.} \end{cases}$$

$$\phi_X(t) = E[e^{tX}] = \int_a^b e^{tx}\left(\frac{1}{b-a}\right)dx$$

$$= \frac{1}{b-a}\int_a^b e^{tx}\,dx$$

$$\phi_X(t) = \left(\frac{1}{b-a}\right)\frac{e^{tx}}{t}\Big|_a^b = \frac{e^{tb}-e^{ta}}{t(b-a)}.$$