COMP. 233.

ASSIGN. 1.

1. How many bit strings of length 10 containing at least 5 consecutive 0's or at least 5 consecutive 1's are there?

2. A Palindrome is a string whose reversal is identical to the original string.
   How many bit stings of length N are Palindromes?

3. How many ways are there for a horse race to finish if ties are allowed? There are four horses in the race.

4. A Peruvian human sacrificial temple, near Lima, Peru, has three levels, levels J, M, and H, where tourists have the only access to the temple.
   The probabilities that a tourist visiting the temple will visit the different levels are:

   Visit level J: .74
   Visit level M: .70
   Visit level H: .62
   Visit levels J and M: .52
   Visit levels J and H: .46
   Visit levels M and H: .44
   Visit levels J and M and H: .34.

   Find the probabilities that a person visiting the temple will:

   A. Visit level M given that he will go to level J.
   B. Visit level H given that he will go to level J and level M.
   C. Not visit level J given that he will visit level M and/or visit level H.
   D. Visit level H and visit level J given that he will not visit level M.

5. In a major city, 4% of the population has Klingon Influenza. In a certain hospital, a test for Klingon Influenza is administered. The test is 97% effective in detecting the disease in those people having the disease. People not having the disease test positive 2% of the time. What is the probability that:

   A. A person testing positive with this disease actually has the disease?
   B. A person testing positive with this disease, in fact, does not have the disease?
   C. A person testing negative for this disease actually has the disease?
   D. A person testing negative for this disease, in fact, does not have the disease?

6. A company receives a shipment of 24 computers, of which 5 are defective. The Quality Control Department (Q.C.) samples 7 of these computers.
   What is the probability that:

   A. Q.C. finds 0 defectives in its sample of 7?
   B. Q.C. finds exactly 2 defectives in its sample of 7?
   C. Q.C. finds at least 1 defective computer in its sample of 7?

7. Experiment. Two fair die are rolled.
   Let F: event that the sum of the dice is 5.
   T: event that the first die equals 3.
   W: event that the second die equals 2.

   Are the events F and TW independent?

8. A group of 5 boys and 10 girls is lined up in random order, that is, each of the 15! permutations is assumed to be equally likely.
   (a) What is the probability that the person in the 4th position is a boy?
   (b) What is the probability that the person in the 12th position is a boy?
   (c) What is the probability that a particular boy is in the 3rd position?

9. In how many total distinct ways could six identical objects be partitioned?

10. Ten parallel lines are perpendicular to ten other parallel lines. What is the total number of rectangles formed?

11. To transfer into a particular technical department, a company requires an employee to pass a screening test. A maximum of three attempts are allowed, at six month intervals between trials. From past records, it is found that 40% pass on the first trial; of those that fail the first trial and take the test a second time, 60% pass; and of those that fail on the second trial and take the test a third time, 20% pass.
For an employee desiring to transfer:

   A. What is the probability of passing the test on the first or second try?

   B. What is the probability of failing on all three attempts?

   C. What is the probability of failing on the first two trials, and passing on the third?

12. A company needs to hire a new director of advertising. It has decided to try to hire either person A or person B, both of whom are assistant advertising directors for major competitors.
To decide between A and B, the company does research on the campaigns managed by A or B, and finds that A is in charge of twice as many advertising campaigns as B. Also, A's campaigns have yielded satisfactory results three out of four times, while B's campaigns have yielded satisfactory results two out of five times.
Suppose one of the competitors advertising campaigns is selected randomly.

   A. What is the probability that the selected campaign is a satisfactory campaign belonging to B?

   B. What is the probability that the selected campaign is unsatisfactory?

13. What is the probability that a number chosen at random from the first 1000 positive integers is exactly divisible by 24 or 36?

assign. 1 Solutions.

1. Set A: set containing all bit strings of length 11 with at least 5 consecutive 0's.

   B: set containing all bit strings of length 11 with at least 5 consecutive 1's.

Set A (Set B is similar).
The number of bit strings in set A is a function of where the 5 consecutive 0's start.
If the 5 consecutive 0's start:

at bit 1 : 2^5 bit strings exist.
" " 2 : 2^4 " " " .
" " 3 : 2^4 " " " .
" " 4 : 2^4 " " " .
" " 5 : 2^4 " " " .
" " 6 : 2^4 " " " .

$|A| = 112$ " " " in set A.
Because of symmetry, $|B| = 112$.
$\therefore$ Total Number $= |A| + |B| - |A \cap B|$

$= 112 + 112 - 2$

Total number $= 222$.

2. If n is even: $2^{n/2}$

   n is odd: $2^{(n+1)/2}$

See next Page.

n even:

$$\underbrace{\overbrace{(2)\ldots\ldots(2)(2)(2)(2)(2)}^{n/2}\;\overbrace{(1)(1)(1)(1)(1)\ldots\ldots(1)}^{n/2}}_{n\text{ even}}$$

n odd:

$$\underbrace{(2)\ldots\ldots(2)(2)(2)(2)(2)(2)}_{\frac{n+1}{2}}\;\underbrace{(1)(1)(1)(1)(1)\ldots\ldots(1)}_{\frac{n-1}{2}}.$$

3. There are 5 cases.

Case 1. No ties. ${}_4P_4 = 24$ ways.

Case 2. All 4 horses tie. 1 way.

Case 3. 3 horses tie. $({}_4C_3)(2) = (4)(2) = 8$ ways.

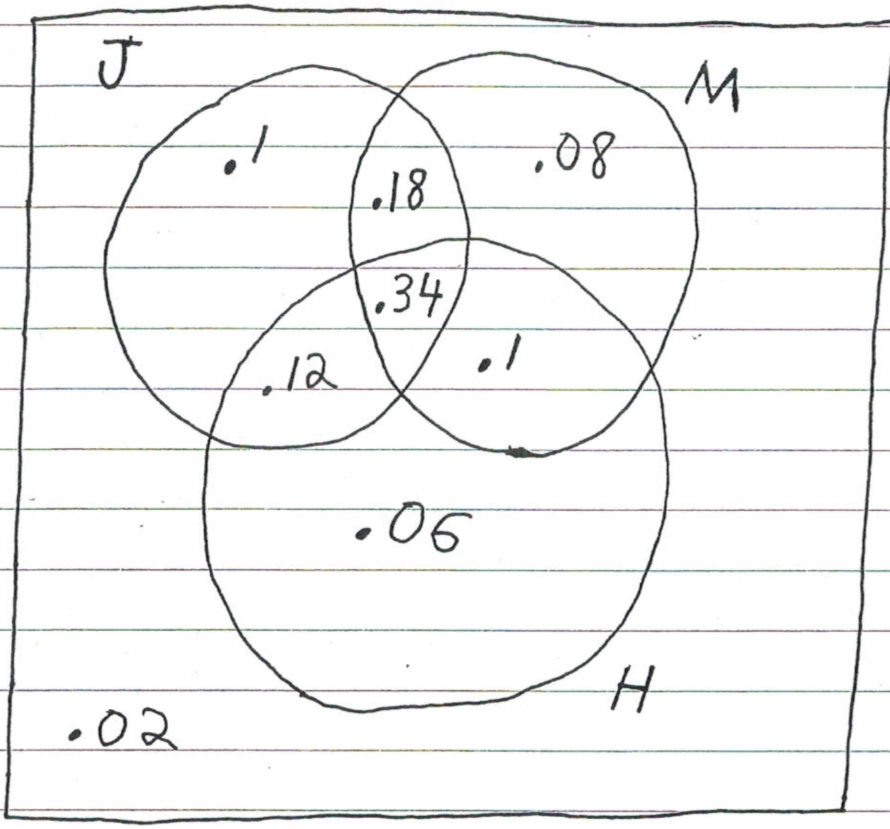
Case 4. Only 2 horses tie. $({}_4C_2)({}_3P_3) = (6)(6) = 36$ way

Case 5. Two pairs of horses tie. ${}_4C_2 = 6$ ways.

Total ways: $24 + 1 + 8 + 36 + 6 = 75$ ways.

4. A. $P(M|J) = \dfrac{P(MJ)}{P(J)} = \dfrac{.52}{.74} = \dfrac{26}{37}.$

B. $P(H|JM) = \dfrac{P(HJM)}{P(JM)} = \dfrac{.34}{.52} = \dfrac{17}{26}.$



C. $P(\bar{J} \mid M \cup H) = \dfrac{P(\bar{J}(M \cup H))}{P(M \cup H)} = \dfrac{.24}{.88} = \dfrac{3}{11}.$

D. $P(HJ \mid \bar{M}) = \dfrac{P(HJ\bar{M})}{P(\bar{M})} = \dfrac{.12}{.3} = \dfrac{2}{5}.$

5. Let A: event that a randomly chosen person in hospital is infected with this disease.

P: event that a randomly chosen person tests positive for this disease.

$P(A) = .04$; $P(P \mid A) = .97$; $P(\bar{P} \mid A) = .03$ (false negative);

$P(P \mid \bar{A}) = .02$ (false positive); $P(\bar{P} \mid \bar{A}) = .98.$

A. $P(A|P) = \dfrac{P(AP)}{P(P)}$

$$= \frac{P(A)\,P(P|A)}{P(P)}$$

$$= \frac{P(A)\,P(P|A)}{P(PA \cup P\bar{A})}$$

$$= \frac{P(A)\,P(P|A)}{P(PA) + P(P\bar{A})}$$

$$= \frac{P(A)\,P(P|A)}{P(A)P(P|A) + P(\bar{A})P(P|\bar{A})}$$

$$= \frac{(.04)(.97)}{(.04)(.97) + (.96)(.02)}$$

$P(A|P) = .669.$

B. $P(\bar{A}|P) = 1 - P(A|P) = 1 - .669 = .331.$

C. $P(A|\bar{P}) = \dfrac{P(A\bar{P})}{P(\bar{P})}$

$$= \frac{P(A)\,P(\bar{P}|A)}{P(\bar{P})}$$

$$= \frac{P(A)\,P(\bar{P}|A)}{P(\bar{P}A \cup \bar{P}\bar{A})}$$

$$= \frac{P(A)\,P(\bar{P}|A)}{P(\bar{P}A) + P(\bar{P}\bar{A})}$$

$$= \frac{P(A)\,P(\bar{P}|A)}{P(A)\,P(\bar{P}|A) + P(\bar{A})\,P(\bar{P}|\bar{A})}$$

$$= \frac{(.04)(.03)}{(.04)(.03) + (.96)(.98)}$$

$P(A|\bar{P}) = .001.$

D. $P(\bar{A}|\bar{P}) = 1 - P(A|\bar{P}) = 1 - .001 = .999.$

6. A. $\frac{{}_{19}C_7}{{}_{24}C_7} = .145 = P(0 \text{ defectives}).$

B. $P(2 \text{ defectives}) = \frac{({}_5C_2)({}_{19}C_5)}{{}_{24}C_7} = .336.$

C. $P(\text{at least 1 defective}) = 1 - P(0 \text{ defectives})$

$= 1 - .145 = .855.$

7. No F and TW are not independent.

$P(F|TW) = 1.$

If F and TW were independent then
$P(F|TW) = P(F).$

SOLUTION TO ASSIGN. 1, #8.

8. A group of 5 boys and 10 girls is lined up in random order, that is, each of the 15! permutations is assumed to be equally likely.

   (a) What is the probability that the person in the 4th position is a boy?
   (b) What is the probability that the person in the 12th position is a boy?
   (c) What is the probability that a particular boy is in the 3rd position?

Solution.
   (a) Since 5 out of the 15 children are boys, the probability is 5/15 =1/3.
   (b) 1/3, since the position is not important.
   (c) Similar to (a), the probability is 1/15.

Extra Problem. (9).

Problem:

In how many total distinct ways could six identical objects be partitioned?

Solution:

Let n: number of seperators used.

$N(n)$: number of distinct ways to partition the six objects using n seperators.

$T = \sum_{n=0}^{5} N(n)$: Total number of distinct ways to partition the six objects.

000000: The six objects (identical).

$n=0$: 000000 $\Rightarrow N(0) = 1 = \binom{5}{0}$.

$n=1$: 0/00000
00/0000
000/000
0000/00
00000/0 $\Rightarrow N(1) = 5 = \binom{5}{1}$.

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$n=2$: 0|0|0 0 0 0
0|0 0|0 0 0
0|0 0 0|0 0
0|0 0 0 0|0
0 0|0|0 0 0
0 0|0 0|0 0
0 0|0 0 0|0

0 0 0|0|0 0
0 0 0|0 0|0
0 0 0 0|0|0 $\Rightarrow N(2) = 10 = \binom{5}{2}$.

$n=3$: 0|0|0|0 0 0

0|0|0 0|0 0

0|0|0 0 0|0

0|0 0|0|0 0

0|0 0|0 0|0

0|0 0 0|0|0

0 0|0|0|0 0

0 0|0|0 0|0
0 0|0 0|0|0
0 0 0|0|0|0 $\Rightarrow N(3) = 10 = N(2) = (5$

$n=4$: 0|0|0|0|0 0

0|0|0|0 0|0

0|0|0 0|0|0

0|0 0|0|0|0

0 0|0|0|0|0 $\Rightarrow N(4) = 5 = N(1) = \binom{5}{4}$.

$n=5$: 0|0|0|0|0|0 $\Rightarrow N(5) = 1 = N(0) = \binom{5}{5}$.

$\therefore, \; T = \sum_{n=0}^{5} N(n) = 1+5+10+10+5+1 = 32 = 2^5.$

Total number of distinct ways to partition m identical objects:

$$T = \sum_{n=0}^{(m-1)} N(n) = 2^{(m-1)}.$$

Extra Problem 10.

$\binom{10}{2}\binom{10}{2} = 2{,}025.$

R: N ∥ lines ⊥ M ∥ lines: $\binom{N}{2}\binom{M}{2}$.

11.

Let F: event of passing test on first try.

S: " " " " " second try.

T: " " " " " third " .

$P(F) = 0.4.$

$P(S|\bar{F}) = 0.6.$

$P(T|\bar{F}\bar{S}) = 0.2.$

A. $$P(F \cup \bar{F}S) = P(F) + P(\bar{F}S) - P(F \cap \bar{F}S)$$
$$= P(F) + P(S|\bar{F})P(\bar{F}) - P(F \cap \bar{F}S)$$
$$= 0.4 + (0.6)(0.6) - 0$$
$$P(F \cup \bar{F}S) = 0.76.$$

B. $$P(\bar{T}\bar{S}\bar{F}) = P(\bar{S}\bar{F})P(\bar{T}|\bar{S}\bar{F})$$
$$= P(\bar{F})P(\bar{S}|\bar{F})P(\bar{T}|\bar{S}\bar{F})$$
$$= (0.6)(0.4)(0.8)$$
$$P(\bar{T}\bar{S}\bar{F}) = 0.192.$$

C.
$$P(T\bar{S}\bar{F}) = P(\bar{S}\bar{F})P(T|\bar{S}\bar{F})$$
$$= P(\bar{F})P(\bar{S}|\bar{F})P(T|\bar{S}\bar{F})$$
$$= (0.6)(0.4)(0.2)$$
$$P(T\bar{S}\bar{F}) = 0.048.$$

~~10~~ 12.

Let S_B: event of selecting a satisfactory campaign of B.

S: " " " " " " .

B: " " " " campaign of B.

U: " " " an unsatisfactory campaign.

U_B: " " " " " " of B.

U_A: " " " " " " " A.

A: " " " a campaign of A.

A. $P(S_B) = P(B \cap S) = P(B)P(S|B)$

$$= \left(\frac{1}{3}\right)\left(\frac{2}{5}\right)$$

$$= \frac{2}{15}.$$

B. $P(U) = P(U_B \cup U_A)$

$$= P(U_B) + P(U_A)$$

$$= P(B \cap U) + P(A \cap U)$$

$$= P(B)P(U|B) + P(A)P(U|A)$$

$$= \left(\frac{1}{3}\right)\left(\frac{3}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{4}\right)$$

$$P(U) = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}.$$

13.

Let A : Set of numbers which are divisible by 24.

B : " " " " " " " 36.

$P(A \cup B)$?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$|A| = \left\lfloor \frac{1000}{24} \right\rfloor = 41.$$

$$|B| = \left\lfloor \frac{1000}{36} \right\rfloor = 27.$$

$$|A \cap B| = \left\lfloor \frac{1000}{L.C.M.(24, 36)} \right\rfloor = 13.$$

$\therefore$,

$$P(A \cup B) = \frac{41}{1000} + \frac{27}{1000} - \frac{13}{1000} = \frac{55}{1000}.$$

NOTA BENE: L.C.M. (24, 36) = 72.