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Midterm 2 - July 12

Exercise 1. Find a basis for the following subspace of $\mathcal{M}_{2\times 2}(\mathbb{R})$:

$$V = \left\{ M \in \operatorname{span}\left\{ \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}, \text{ such that } \operatorname{tr}(M) = 0 \right\}$$

Reminder: The trace of M, denoted by $tr(M)$, is the sum of the elements in the main diagonal

Exercise 2. Consider the linear transformation[1] $T : \mathbb{P}_3 \to \mathbb{P}_3$ such that $T(p(x)) = p(x) + p(1)$.

i) Find the kernel of T.

ii) Prove that T is an isomorphism.

Exercise 3. Consider the following bases for $\mathbb{P}_2$: $B = \{1, x, x^2\}$ and $C = \{x^2, x+1, 1\}$ (you can assume that these sets are bases). Let $T : \mathbb{P}_2 \to \mathbb{P}_2$ be the linear transformation such that $_B[T]_C = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$. Let $p(x) = 1 + x + x^2$. Find $[p]_C$ and $T(p)$.

Exercise 4. Find the change-of-coordinate matrix from the standard basis $C = \{e_1, e_2, e_3\}$ for $\mathbb{R}^3$ to the basis $B = \{e_1 - 2e_2 + e_3, 3e_1 - 5e_2 + 4e_3, 2e_2 + 3e_3\}$ (you can assume that both are bases for $\mathbb{R}^3$). Then find the B-coordinate vector for $-e_1 + 2e_2$.

Exercise 5. Let $T : V \to W$ be a linear transformation and $v_1, \ldots, v_n$ be vectors of V. Prove that $T(\operatorname{span}\{v_1, \ldots, v_n\}) = \operatorname{span}\{T(v_1), \ldots, T(v_n)\}$.

[1] You can assume it is a linear transformation.