MATH 2107 Linear Algebra II Summer 2017

Name: Student IDs:

Midterm 1 - May 29

In the following exercises we consider the vector space $\mathcal{M}_{2\times 2}(\mathbb{R})$ (2×2 matrices with real entries) with the addition and scalar multiplication given by:

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{pmatrix} \text{ and } \alpha \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{pmatrix}.$$

1) (4 pts) Given two matrices $A, B \in \mathcal{M}_{2\times 2}(\mathbb{R})$, prove that the map $T : \mathcal{M}_{2\times 2}(\mathbb{R}) \to \mathcal{M}_{2\times 2}(\mathbb{R})$ given by $T(M) = AMB$ is a linear transformation.

2) (4 pts) Given two matrices $A, B \in \mathcal{M}_{2\times 2}(\mathbb{R})$, prove that the set $V = \{M \in \mathcal{M}_{2\times 2}(\mathbb{R}) : AMB = 0\}$ is a subspace of $\mathcal{M}_{2\times 2}(\mathbb{R})$.

3) (4 pts) Let $A = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $V = \{M \in \mathcal{M}_{2\times 2}(\mathbb{R}) : AMB = 0\}$. Find a spanning set for V with 2 elements.

4) (4 pts) Let $C = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, D = \begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix}, E = \begin{pmatrix} 1 & 4 \\ 1 & 4 \end{pmatrix}$ and $W = \text{span}\{C, D, E\}$. Show that one of the matrices can be written as a linear combination of the others (indicate what are the scalars of this linear combination) and then find a basis for W.

5) (4 pts) Find a basis for the vector space $V + W$ (where V and W are the subspaces defined as above). For each step of the spanning set reduction algorithm, show explicitly what is the vector that can be written as a linear combination of the others.

JUSTIFY ALL THE ANSWERS.

Obs. To do part 5 you need to do part 3 first. The other parts can be done independently of the others (however sometimes using the previous part can help).