ENGR311/1/AA **Quiz Six** **June 13, 2017**

$Solve\ the\ differential\ equation$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad where\ \ u(0,t) = 0, u(1,t) = 0,$$

$$u(x,0) = 1, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad homogeneous$$

$$0 < x < 1, \ \ t \geq 0$$

$$u(0,t) = 0, \ \ u(1,t) = 0, \qquad homogeneous$$

$$u(x,0) = 1, \qquad nonhomogeneous \ \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, homogeneous$$

$$u(x,t) = g(t)h(x)$$

$$h_{xx} + \lambda h = 0, \ \ h(0) = 0, \ \ h(1) = 0$$

$Eigenvalues\ \ \lambda > 0,\ \ \lambda_n = (n\pi)^2 \qquad h_n(x) = c_n sin(n\pi x)$

$g_{tt} + \lambda g = 0 \qquad g(t) = c_3 cosn\pi t + c_4 sinn\pi t \quad g_t'(0) = 0,\ \ u(x,0) = 1$

$g(t) = c_3 cosn\pi t$

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} A_n sin(n\pi x) cos(n\pi t)$$

$$u(x,0) = \sum_{n=1}^{\infty} A_n sin(n\pi x) = 1$$

$$A_n = \frac{2}{p}\int_0^p f(x)\, sin(n\pi x) dx$$

$$A_n = 2\int_0^1 1.\ sin n\pi x dx = -\frac{2}{n\pi}\{cos n\pi - 1\}$$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\{1 - cos n\pi\} sin(n\pi x) cos(n\pi t)$$