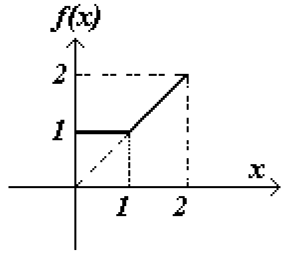
ENGR311/1/AA ***Quiz Four*** ***June 1/ 2017***

Find the Fourier expansion of the function which is



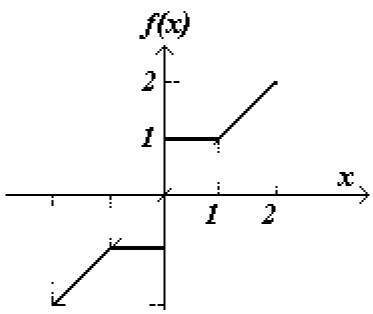
graphically described as: [7]

. . . .

. . *The Function* $f(x) = \begin{cases} 1 & 0 < \ x \leq 1 \\ x & 1 \leq \ x \leq 2 \end{cases}$ . . . .

Student is expected to derive only one expansion

The sine expansion

. . . 

[2]

the period $2p = 4$ [2]

$$b_n = \frac{2}{p}\int_0^p f(x) sin\frac{n\pi x}{p} dx$$

$$b_n = \frac{2}{2}\int_0^2 f(x) sin\frac{n\pi x}{2} dx$$

$$= \int_0^1 1. sin\frac{n\pi x}{2} dx + \int_1^2 x. sin\frac{n\pi x}{2} dx$$ [1]

$$= -\frac{2}{n\pi}\int_0^1 dcos\frac{n\pi x}{2} - \frac{2}{n\pi}\int_1^2 xdcos\frac{n\pi x}{2}$$

$$= -\frac{2}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right) - 1\right] - \frac{2}{n\pi}\left[xcos\frac{n\pi x}{2}\Big|_1^2 - \int_1^2 cos\frac{n\pi x}{2} dx\right]$$

$$= \frac{2}{n\pi}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right] - \frac{2}{n\pi}\left[2cosn\pi - \cos\left(\frac{n\pi}{2}\right) - \int_1^2 cos\frac{n\pi x}{2} dx\right]$$

$$= \frac{2}{n\pi}\left[1-\cos\left(\frac{n\pi}{2}\right)\right] - \frac{4}{n\pi}cosn\pi + \frac{2}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{4}{n^2\pi^2}sin\frac{n\pi x}{2}\Big|_1^2$$

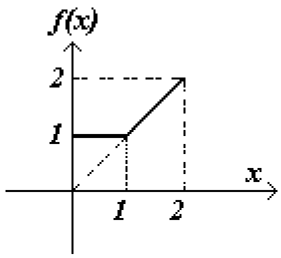
$$= \frac{2}{n\pi}\left[1-\cos\left(\frac{n\pi}{2}\right)\right] - \frac{4}{n\pi}cosn\pi + \frac{2}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{4}{n^2\pi^2}\left(sinn\pi - sin\frac{n\pi}{2}\right)$$

$$= \frac{2}{n\pi}\left[1-\cos\left(\frac{n\pi}{2}\right)\right] - \frac{4}{n\pi}cosn\pi + \frac{2}{n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{4}{n^2\pi^2}sin\frac{n\pi}{2}$$

$$= \frac{2}{n\pi} - \frac{4}{n\pi}cosn\pi - \frac{4}{n^2\pi^2}sin\frac{n\pi}{2} \qquad [2]$$

$$f(x) = \sum_{n=1}^{n=\infty}\left(\frac{2}{n\pi} - \frac{4}{n\pi}cosn\pi - \frac{4}{n^2\pi^2}sin\frac{n\pi}{2}\right)\sin\left(\frac{n\pi x}{2}\right) \quad 0 \le x \le 2$$

General expansion



$$2p = 2, p = 1$$

$$a_0 = \frac{1}{1}\int_0^2 f(x)\,dx = \int_0^1 1.\,dx + \int_1^2 x\,dx = 1 + \frac{x^2}{2}\Big|_1^2 = \frac{5}{2}$$

$$a_n = \frac{1}{1}\int_0^2 f(x)\cos(n\pi x)\,dx = \int_0^1 \cos(n\pi x)\,dx + \int_1^2 x.\cos(n\pi x)\,dx$$

$$= \frac{1}{n\pi}sinn\pi x|_0^1 + \frac{1}{n\pi}\int_1^2 x.\,dsin(n\pi x)$$

$$= 0 + \frac{1}{n\pi}x.\sin(n\pi x)|_1^2 - \frac{1}{n\pi}\int_1^2 \sin(n\pi x)\,dx$$

$$a_n = 0 - \frac{1}{n\pi}\left(-\frac{1}{n\pi}\cos(n\pi x)|_1^2\right) = \frac{1}{n^2\pi^2}(\cos(2n\pi) - \cos(n\pi)) = \frac{1}{n^2\pi^2}(1-(-1)^n)$$

$$b_n = \frac{1}{1}\int_0^2 f(x)\sin(n\pi x)\,dx = \int_0^1 \sin(n\pi x)\,dx + \int_1^2 x.\sin(n\pi x)\,dx$$

$$= \frac{-1}{n\pi}\cos(n\pi x)|_0^1 - \frac{1}{n\pi}\int_1^2 x.\,dcos(n\pi x)$$

$$= \frac{1}{n\pi}(1-(-1)^n) - \frac{1}{n\pi}\left[x.\cos(n\pi x)|_1^2 - \int_1^2 \cos(n\pi x)\,dx\right]$$

$$= \frac{1}{n\pi}(1-(-1)^n) - \frac{1}{n\pi}(2-(-1)^n) + \frac{1}{n\pi}.\frac{1}{n\pi}\sin(n\pi x)|_1^2$$

$$b_n = \frac{-1}{n\pi}$$

$$f(x) = \frac{5}{2} + \sum_{n=1}^{\infty} \frac{1}{n^2\pi^2}(1-(-1)^n)\cos(n\pi x) - \frac{1}{n\pi}\sin(n\pi x) \quad 0 \le x \le 2$$