

Family Name, First Name ID[3]

Solve the following equation using Laplace Transforms:

$$y' + y = f(t) \quad y(0) = 0$$

$$f(t) = \begin{cases} 1 & 0 \leq t < 1/2 \\ -1 & t \geq 1/2 \end{cases} \quad [7]$$

$$f(t) = 1 - 2U\left(t - \frac{1}{2}\right) \quad [2]$$

$$y' + y = 1 - 2U\left(t - \frac{1}{2}\right) \quad y(0) = 0$$

$$sF + F = \frac{1}{s} - 2\frac{e^{-\frac{s}{2}}}{s}$$

$$F(s+1) = \frac{1}{s}\left(1 - 2e^{-\frac{s}{2}}\right) \quad F = \frac{1}{s(s+1)}\left(1 - 2e^{-\frac{s}{2}}\right) \quad [3]$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+1}\right\} = 1 - e^{-t}$$

$$\mathcal{L}^{-1}\left\{\frac{-2e^{-\frac{s}{2}}}{s(s+1)}\right\} = -2U\left(t - \frac{1}{2}\right)\left(1 - e^{-(t-\frac{1}{2})}\right)$$

$$y(t) = (1 - e^{-t}) - 2U\left(t - \frac{1}{2}\right)\left(1 - e^{-(t-\frac{1}{2})}\right) \quad [3]$$