Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

Mid-Term test for MAT2377, Spring-Summer 2017
Probability and Statistics for Engineers

Time : 80 minutes

Student Number :______________________________________

First Name : ____

Last Name : ____

- A
- This is a closed book exam. Formula sheets are attached to this exam.
- Only the calculators TI 30, TI 34, Casio fx-260 and Casio fx-300 are allowed.
- Write your detailed (as much as possible) responses in the space provided.
- At the end of exam you need to submit the complete exam booklet.
- If you see any error in this exam, please report it on your paper.
- There are 3 detailed answer questions (40 points) and 6 multiple choice questions (60 points).
  The exam will be marked on a total of 100 points.

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Cellular phones, unauthorized electronic devices or course notes
(unless an open-book exam) are not allowed during this exam.
Phones and devices must be turned off and put away in your bag.
Do not keep them in your possession, such as in your pockets.
If caught with such a device or document, the following may occur:
you will be asked to leave immediately the exam and academic fraud
allegations will be filed which may result in you obtaining a
0 (zero) for the exam.
By signing below, you acknowledge that you have ensured that you
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are complying with the above statement.

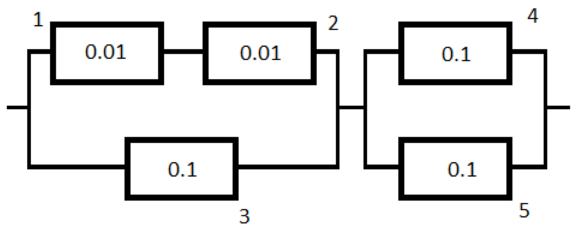
X___________________________________

Submit your answers for the multiple choice questions in the following table. Only this page will be marked for the mutiple choice questions (10 points for each correct answers and there is no penalty for wrong answers).

Question	Answer
1	
2	
3	
4	
5	
6	

Answer the following questions in details in the space provided

[12] 1. The following circuit operates only if there is a path of functional devices from left to right. For each device the probability it is **NOT** functional is indicated on the following diagram. Assume that devices fails or work independently. What is the probability that the circuit operates?



Solution. Let

$$A_i = \text{if the circuit } i \text{ operates}, i = 1, \ldots, 5.$$

We need to calculate

$$P\left(((A_1 \cap A_2) \cup A_3) \cap (A_4 \cup A_5)\right) = P((A_1 \cap A_2) \cup A_3) P(A_4 \cup A_5).$$

We have

$$P((A_1 \cap A_2) \cup A_3) = P(A_1 \cap A_2) + P(A_3) - P(A_1 \cap A_2 \cap A_3)$$
$$= 0.99^2 + 0.9 - 0.99^2(0.9) = 0.99801.$$

Similarly

$$P(A_4 \cup A_5) = P(A_4) + P(A_5) - P(A_4)P(A_5) = 0.9 + 0.9 - (0.9)(0.9) = 0.99.$$

Therefore

$$P(\text{circuit operates}) = (0.99801)(0.99) = 0.9880299.$$

[14] 2. The random variable X, the tensile strength of a certain cement (the measurment scale is $10000lb/in^2$) is being studied. Assume the p.d.f. for this random varaible is given by :

$$f(x) = \begin{cases} cx(1-x)^2, & \text{if } \ 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find c.

$$c\int_0^1 x(1-x)^2 dx = 1 = c\int_0^1 x(1+x^2-2x)dx = c(1/2+1/4-2/3) = c/12.$$

Therefore $c = 12$.

(ii) Compute

$$P(X > 0.5|X > 0.2).$$

$$P(X > 0.5|X > 0.2) = \frac{P(X > 0.5, X > 0.2)}{P(X > 0.2)} = \frac{P(X > 0.5)}{P(X > 0.2)}.$$

We have

$$P(X > a) = \int_a^1 f(x)dx = 1-12\int_0^a x(1-x)^2 dx = 1-12(a^2/2+a^4/4-2a^3/3).$$

Therefore

$$P(X > 0.5) = 1 - 12(1/8 + 1/64 - 1/12) = 0.3125$$

and

$$P(X > 0.2) = 1 - 12(0.2^2/2 + 0.2^4/4 - 2(0.2^3)/3) = 0.8192.$$

Therefore

$$P(X > 0.5|X > 0.2) = 0.3125/0.8192 \approx 0.3815.$$

Note from Calculus : When $k \neq -1$, we have

$$\int x^k dx = \frac{x^{k+1}}{k+1} + c.$$

[14] 3. For the events A, B and C let

$$P(A) = 0.3, P(B|A) = 0.75, P(B|A^{'}) = 0.2, P(C|A \cap B) = 0.2, P(C|A^{'} \cap B) = 0.15,$$

$$P(C|A \cap B^{'}) = 0.8, P(C|A^{'} \cap B^{'}) = 0.9.$$

Calculate $P(B^{'} \cap C)$.

Solution. We have

$$P(B^{'} \cap C) = P(B^{'} \cap C \cap A) + P(B^{'} \cap C \cap A^{'}) = P(C|A \cap B^{'})P(A \cap B^{'}) + P(C|A^{'} \cap B^{'})P(A^{'} \cap B^{'})$$

$$= 0.8P(B^{'} \cap A) + 0.9P(A^{'} \cap B^{'}) = 0.8P(B^{'}|A)P(A) + 0.9P(B^{'}|A^{'})P(A^{'})$$

$$= 0.8(1 - 0.75)(0.3) + 0.9(1 - 0.2)(1 - 0.3) = 0.06 + 0.504 = 0.564.$$

[10] 1. Let A and B be two independent events such that $P(A) = 0.6$ and $P(B) = 0.2$. Determine $P(A' \cup B)$.

(a) 0.92
(b) 0.52
(c) 0.68
(d) 0.72
(e) 0.62

Solution.

$$P(A' \cup B) = P(A') + P(B) - P(A')P(B) = (1-0.6) + 0.2 - (1-0.6)(0.2) = 0.52.$$

[10] 2. Consider the random variables X and Y with joint probability mass function :

$$f(x, y) = \begin{cases} c(x+y), & x \in \{1, 2, 3\},\ y \in \{1, 2\} \\ 0, & \text{otherwise.} \end{cases}$$

What is the value of c and are X and Y independent or not ?

(a) $c = 1/9$, independent
(b) $c = 1/21$, independent
(c) $c = 1/9$, dependent
(d) $c = 1/25$, dependent
(e) $c = 1/21$, dependent.

Solution. We have

$$1 = \sum_x \sum_y f(x, y) = c((1+1)+(1+2)+(2+1)+(2+2)+(3+1)+(3+2)) = 21c.$$

Therefore $c = 1/21$. Notice that

$$f_1(1) = P(X = 1) = f(1, 1) + f(1, 2) = (1+1)/21 + (1+2)/21 = 5/21$$

and similarly

$$P(Y = 1) = f_2(1) = f(1, 2) + f(2, 2) + f(3, 2) = (3+4+5)/21 = 12/21.$$

On the other hand

$$P(X = 1, Y = 1) = f(1, 1) = 2/21 \neq (5/21)(12/21).$$

Therefore X and Y are not independent. Answer is (e).

[10] 3. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $p = 0.7$ be the probability that the student knows the answer and $q = 0.3$ the probability that the student guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{5} = 0.2$. What is the conditional probability that a student knew the answer to a question, given that he or she answered it correctly ?

(a) 1/6
(b) 50/86
(c) 70/76
(d) 20/30
(e) 1/2.

Solution. Let E_1 be the even that student knows the answer and E_2 be the event that the student does not know the answer. Also let A dento the even that student answers question correctly. We have

$$P(E_1) = 0.7, P(E_2) = 0.3, P(A|E_1) = 1, P(A|E_2) = 0.2.$$

Therefore

$$P(E_1|A) = \frac{P(A|E_1)P(E_1)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2)} = \frac{1(0.7)}{1(0.7) + 0.2(0.3)} = \frac{0.7}{0.76} = \frac{70}{76}.$$

Answer is (c).

[10] 4. A company rents 35% of the cars for its employees from agency I and 65% from agency II. If 8% of the cars of agency I and 5% of the cars of agency II break down during the rental periods, what is the probability that a car rented by this company breaks down ?

(a) 0.1123
(b) 0.0015
(c) 0.7450

(d) 0.0605
(e) 0.0105

Let E_1 and E_2 denote renting a car from agencies I and II, respectively. Denote B the even that the rented car breaks down. We have

$$P(B) = P(B|E_1)P(E_1)+P(B|E_2)P(E_2) = (0.08)(0.35)+(0.05)(0.65) = 0.0605.$$

Answer is d.

[10] 5. Let the discrete random variable X have the p.m.f.

$$f(x) = \begin{cases} kx^2, & \text{if} \quad x = 1, 2, 3 \\ 0, & \text{otherwise.} \end{cases}$$

where k is a constant. Find $E(X)$.

(a) 2
(b) 13/7
(c) 19/7
(d) 3
(e) 18/7

Solution.

$$1 = f(1) + f(2) + f(3) = k + 4k + 9k$$

gives $k = 1/14$. Therefore

$$E(X) = \frac{1}{14}\sum_{x=1}^{3} x^3 = \frac{1+8+27}{14} = \frac{18}{7}.$$

Answer is (e).

[10] 6. Let the random variable X has a cumulative distribution function as follows.

$$F(x) = \begin{cases} 0, & \text{if} \quad x < 0 \\ 0.5, & \text{if} \quad 0 \leq x < 2 \\ 0.7, & \text{if} \quad 2 \leq x < 3 \\ 1 & \text{if} \quad x \geq 3. \end{cases}$$

Compute $P(X = 2|X \geq 2)$.

(a) 0.2
(b) 0.3
(c) 0.4
(d) 0.5
(e) 0.6

Solution.

$$P(X = 2|X \geq 2) = \frac{P(X = 2, X \geq 2)}{P(X \geq 2)} = \frac{P(X = 2)}{P(X \geq 2)} = \frac{0.2}{0.2 + 0.3} = 0.4.$$

Answer is (c).