MAT 2377 (Spring-Summer 2017)

Solution to Assignment 5

Question 1. [5 points](a) We have

$$\bar{x} = 8.234, s_x = 0.02529822, s_x^2 = 0.00064.$$

[7.5 points] (b)

$$\bar{x} \pm t_{0.025}(14)s_x/\sqrt{n} = 8.234 \pm 2.145(0.02529822/\sqrt{15})$$

which gives $[8.21999, 8.24801]$.

[7.5 points] (c)

$$n = \frac{z_{\alpha/2}^2 \sigma^2}{E^2} = \frac{(1.96^2)(0.00064)}{0.01^2} = 24.58624 \approx 25.$$

Question 2. [7 points](a) We have

$$|Z_{observed}| = |\frac{\bar{x} - 155}{1.5/\sqrt{12}}| = |\frac{154.2 - 155}{1.5/\sqrt{12}}| = 1.847521.$$

We have

$$p - value = 2P(Z > 1.847521) = 0.06467 > 0.05$$

and we will not reject H_0.

[5 points] (b) The critical region is

$$|\frac{\bar{x} - 155}{1.5/\sqrt{12}}| > z_{\alpha/2} = 1.96 \Rightarrow \quad \text{Reject} \quad H_0.$$

This is equivalent to say We reject H_0 if

$$\bar{X} > 155.8487 \quad \text{or} \quad \bar{X} < 154.1513.$$

See above for the conclusion of the test.

[8 points] (c)

$$P\left(\text{Accepting} \quad H_0|H_1 \quad \text{is} \quad \text{true}\right) = P(154.1513 < \bar{X} < 155.8487|\mu 150).$$

Since $\bar{X} \sim N(150, 1.5^2/12)$ we get

$$P(154.1513 < \bar{X} < 155.8487|\mu 150, \sigma = 1.5) = P\left(\frac{154.1513 - 150}{1.5/\sqrt{12}} < Z < \frac{155.8487 - 150}{1.5/\sqrt{12}}\right)$$

$$= P(9.587 < Z < 13.50699) = 0.$$

Question 3 [15 points]. We have $n = 36$, $\mu = 302$, $\sigma^2 = 144$. Sample size is relatively large so we can apply central limit theorem, i.e.

$$P(\bar{X} < 300) \approx P\left(Z < \frac{300 - 302}{12/\sqrt{36}}\right) = 0.1586.$$

4.20. [4 points] (a)

$$n = 64, Var(\bar{X}) = \sigma/\sqrt{n} = 1.6/8 = 0.2.$$

This gives

$$P(\bar{X} < 2.7) = P(Z < -2.5) = 0.0062.$$

[3.5 points] (b)

$$P(\bar{X} > 3.5) = P(Z > 1.5) = 0.0668.$$

[3.5 points] (c)

$$P(3.2 < \bar{X} < 3.4) = P(0 < Z < 1) = 0.8413 - 0.5 = 0.3413.$$

5.40. [11 points] We have $n = 100, \hat{p} = 8/100$. Therefore $\hat{p} = 0.08, \hat{q} = 0.92$, and $z_0.01 = 2.33$. So,

$$0.08 \pm 2.33\sqrt{(0.08)(0.92)/100} = 0.08 \pm 0.063.$$

which yields $0.017 < p < 0.143$.

5.42. [11 points] We have $n = 500, \hat{p} = 485/500 = 0.97$. Therefore $\hat{p} = 0.97, \hat{q} = 0.03$, and $z_0.05 = 1.645$. So,

$$0.97 \pm 1.645\sqrt{(0.03)(0.97)/500} = 0.97 \pm 0.0133.$$

which yields $0.957 < p < 0.983$.

5.44[5.5 points] (a) We have $n = 100, \hat{p} = 0.24$. Therefore $\hat{p} = 0.24, \hat{q} = 0.76$, and $z_0.005 = 2.575$. So,

$$0.24 \pm 2.575\sqrt{(0.24)(0.76)/100} = 0.24 \pm 0.11.$$

which yields $0.13 < p < 0.35$.

[5.5 points] (b)

$$\text{Error} \leq 2.575\sqrt{(0.24)(0.76)/100} = 0.24 \pm 0.11.$$

5.46.

[11 points]

$$n = \frac{(2.557)^2(0.2278)(0.772)}{(0.05)^2} \approx 467$$

(Always round up).