Assignment 4 - Retaining Walls

Question 1

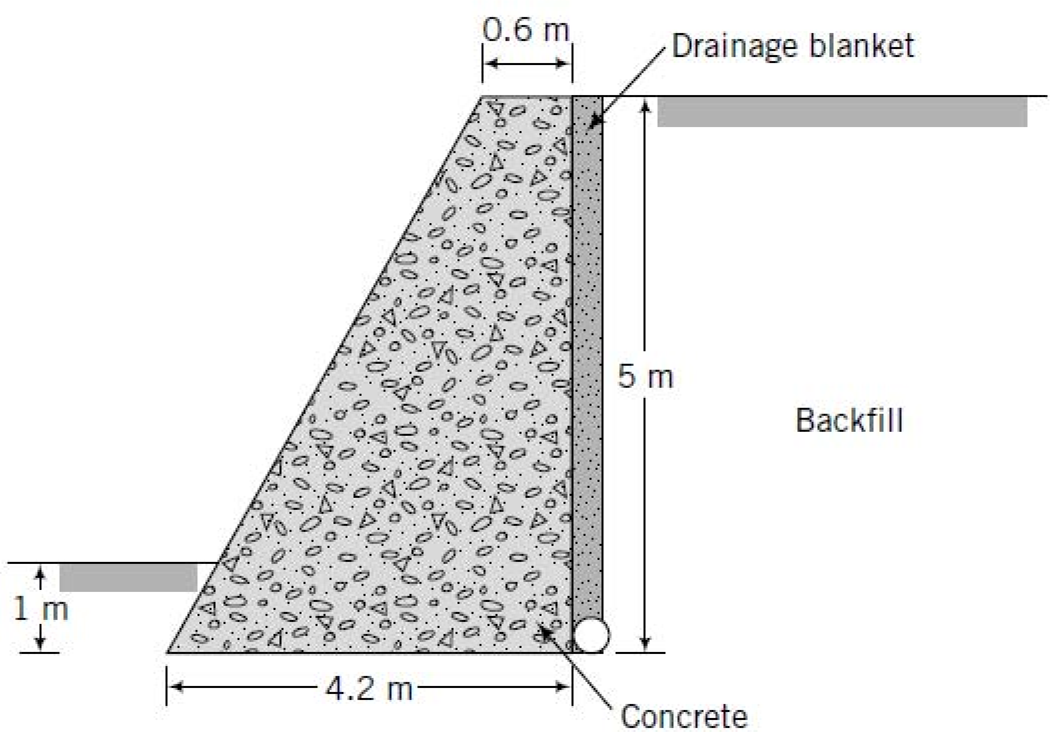
A gravity retaining wall, shown in the figure below, is required to retain 5 m of soil. The backfill is a coarse-grained soil with γ = 18 kN/m^3, γ_{sat} = 19 kN/m^3, ϕ' = 30° and c' = 0. The existing soil (below the base) has the following properties: γ = 19 kN/m^3, γ_{sat} = 20 kN/m^3, ϕ' = 36° and c' = 0. The wall is embedded 1 m into the existing soil and a drainage system is provided, as shown. The groundwater level is 4.5 m below the base of the wall. Determine the stability of the wall for the following conditions (assume k_1 = 2/3 for both base and wall friction):

a) Wall friction is zero.

b) Wall friction is $\delta' = 2/3\phi'$.

c) The drainage system becomes clogged during several days of a rainstorm and the groundwater rises to the surface. Neglect wall friction.

Neglect the passive resistance of the 1.0 m soil above the toe for all cases. The unit weight of concrete is $\gamma_{concr.}$ = 23.58 kN/m^3.



CVG 3106: Assignment # 4

Solution:

a) For zero wall friction, use Rankine's method

ϕ_1' (backfill) $= 30°$

$$K_a = \tan^2\left(45° - \frac{\phi_1'}{2}\right) = \tan^2\left(45° - \frac{30}{2}\right) = 0.33$$

$$P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2}(0.33)(18)(5.0)^2 = 75 \text{ kN/m}$$

P_a is horizontal as the backfill is horizontal and acts at $\frac{H}{3} = \frac{5.0}{3} = 1.67$ m from the base of wall.

Determine the resisting moment:

$\gamma_{concr.} = 23.58$ kN/m³

Section #	Area (m²)	Weight (kN/m)	Moment arm vs C	Moment (kN.m/m)
1	8.00	70.74	3.9	275.89
2	9.00	212.22	2.4	509.33

$\Sigma V = 282.96$ kN/m $\qquad \Sigma M_R = 785.22$ kN.m/m

The overturning moment is:

$$M_o = P_a\left(\frac{H}{3}\right) = 75\left(\frac{5.0}{3}\right) = 125 \text{ kN.m/m}$$

$$FS_{(overturning)} = \frac{\Sigma M_R}{M_o} = \frac{785.22}{125.0} = 6.28 > 2.0, \text{ OK}$$

The passive resistance of the soil above toe is ignored!

$$\delta'_{base} = k_1 \phi_2' = \frac{2}{3} \times 36 = 24°$$

$$FS_{(sliding)} = \frac{(\Sigma V)\tan\delta'_{base} + \cancel{B k_2 c_2'}^{\,0} + \cancel{P_P}^{\,0}}{P_a}$$

$$FS\,(\text{sliding}) = \frac{282.96 \tan 24^\circ}{75} = 1.68 > 1.5 \text{ OK}$$

$$M_{net} = \Sigma M_R - M_o = 785.22 - 125.0 = 660.22 \text{ kN.m/m}$$

$$\bar{x} = \frac{M_{net}}{\Sigma V} = \frac{660.22}{282.96} = 2.33 \text{ m}$$

$$e = \left|\frac{B}{2} - \bar{x}\right| = \left|\frac{4.2}{2} - 2.33\right| = 0.23 \text{ m} < \frac{B}{6} = \frac{4.2}{6} = 0.70 \text{ m}$$

The maximum stress occurs at the heel rather than at the toe:

$$q_{max} = q_{heel} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right) = \frac{282.96}{4.2}\left(1 + \frac{6 \times 0.23}{4.2}\right) = 89.51 \text{ kN/m}^2$$

$$q_{min} = q_{toe} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right) = \frac{282.96}{4.2}\left(1 - \frac{6 \times 0.23}{4.2}\right) = 45.24 \text{ kN/m}^2$$

$$q_u = \cancel{c_2' N_c F_{cd} F_{ci}}^{\,0} + q N_q F_{qd} F_{qi} + \tfrac{1}{2}\gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i}$$

$$q = \gamma_2 D = 19 \times 1.0 = 19 \text{ kN/m}^2$$

GWT @ 4.5 m > B = 4.2 m → GW has <u>no effect</u> on BC!

$$B' = B - 2e = 4.2 - 2(0.23) = \underline{3.74 \text{ m}}$$

for $\phi_2' = 36^\circ \rightarrow N_q = 37.75 \qquad N_\gamma = 56.31$

$$\frac{D_f}{B} = \frac{1.0}{4.2} = 0.24 < 1 \text{ and } \phi_2' = 36^\circ > 0$$

$$F_{qd} = 1 + 2\tan\phi_2'(1 - \sin\phi_2')^2\left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1 + 2\tan 36^\circ(1 - \sin 36^\circ)^2\left(\frac{1.0}{4.2}\right) = 1.06$$

$$F_{\gamma d} = 1.0$$

$$\psi^\circ = \tan^{-1}\left(\frac{P_a}{\Sigma V}\right) = \tan^{-1}\left(\frac{75}{282.96}\right) = 14.8^\circ$$

$$F_{qi} = \left(1 - \frac{\psi^\circ}{90^\circ}\right)^2 = \left(1 - \frac{14.8}{90}\right)^2 = 0.70$$

$$F_{\gamma i} = \left(1 - \frac{\psi^\circ}{\phi_2'}\right)^2 = \left(1 - \frac{14.8}{36^\circ}\right)^2 = 0.35$$

$$q_u = (19)(37.75)(1.06)(0.70) + \tfrac{1}{2}(19)(3.74)(56.31)(1.0)(0.35) = 1232.44 \text{ kN/m}^2$$

$$FS_{(BC)} = \frac{q_u}{q_{max}} = \frac{1232.44}{89.57} = 13.8 > 3 \quad , \underline{OK}$$

b) For wall friction $\delta'_{wall} = 2/3\,\phi_1'$, use Coulomb's method:

For $\alpha = 0^\circ$ (horizontal backfill), $\beta = 90^\circ$ (vertical back face) and $\phi_1' = 30^\circ$ from Table on p. 12 (Lecture notes)
$K_a = 0.2973 \approx \underline{0.30}$

$$P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2}(0.30)(18)(5.0)^2 = 67.5 \text{ kN/m}$$

P_a is inclined at an angle $\delta'_{wall} = \frac{2}{3}\phi_1'$ to the horizontal!

$$P_h = P_a \cos\left(\tfrac{2}{3}\phi_1'\right) = 67.5 \cos\left(\tfrac{2}{3} \times 30\right) = 63.43 \text{ kN/m}$$

$$P_v = P_a \sin\left(\tfrac{2}{3}\phi_1'\right) = 67.5 \sin\left(\tfrac{2}{3} \times 30\right) = 23.09 \text{ kN/m}$$

Determine the resisting moment:

Section #	Area (m^2)	Weight (kN/m)	Moment arm vs C	Moment (kN.m/m)
1	3.00	70.74	3.9	275.89
2	9.00	212.22	2.4	509.33
		P_v = 23.09	4.2	96.98
		ΣV = 306.05 kN/m		ΣM_R = 882.20 kN.m/m

The overturning moment is:

$$M_o = P_h\left(\frac{H}{3}\right) = 63.43\left(\frac{5}{3}\right) = 105.72 \text{ kN.m/m}$$

$$FS(\text{overturning}) = \frac{\Sigma M_R}{M_o} = \frac{882.20}{105.72} = 8.34 > 2.0, \text{ OK}$$

$$\delta'_{base} = k_1 \phi_2' = \frac{2}{3} \times 36 = 24°$$

$$FS(\text{sliding}) = \frac{\Sigma V \tan \delta'_{base}}{P_h} = \frac{306.05 \tan 24°}{63.43} = 2.14 > 1.5, \text{OK}$$

$$M_{net} = \Sigma M_R - M_o = 882.20 - 105.72 = 776.48 \text{ kN.m/m}$$

$$\bar{x} = \frac{M_{net}}{\Sigma V} = \frac{776.48}{306.05} = 2.54 \text{ m}$$

$$e = \left|\frac{B}{2} - \bar{x}\right| = \left|\frac{4.2}{2} - 2.54\right| = 0.44 \text{ m} < \frac{B}{6} = 0.70 \text{ m}$$

The maximum stress is at the heel!

$$q_{max} = q_{heel} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right) = \frac{306.05}{4.2}\left(1 + \frac{6 \times 0.44}{4.2}\right) = 118.67 \text{ kN/m}^2$$

$$q_{min} = q_{toe} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right) = \frac{306.05}{4.2}\left(1 - \frac{6 \times 0.44}{4.2}\right) = 27.07 \text{ kN/m}^2$$

$$q_u = q N_q F_{qd} F_{qi} + \frac{1}{2} \gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i}$$

$q = 19 \text{ kN/m}^2$ (see a)

$B' = B - 2e = 4.2 - 2(0.44) = 3.32$ m

$$\left.\begin{array}{ll} N_q = 37.75 & N_\gamma = 56.31 \\ F_{qd} = 1.06 & F_{\gamma d} = 1.0 \end{array}\right\} \text{ From a)}$$

$$\psi° = \tan^{-1}\left(\frac{P_h}{\Sigma V}\right) = \tan^{-1}\left(\frac{63.43}{306.05}\right) = 11.7°$$

$$F_{qi} = \left(1 - \frac{11.7°}{90°}\right)^2 = 0.76$$

$$F_{\gamma i} = \left(1 - \frac{11.7^\circ}{36^\circ}\right)^2 = 0.46$$

$$q_u = (19)(37.75)(1.06)(0.76) + \tfrac{1}{2}(19)(3.32)(56.31)(1.0)(0.46) = 1394.78\ kN/m^2$$

$$FS_{(BC)} = \frac{q_u}{q_{max}} = \frac{1394.78}{118.67} = 11.75 > 3, \quad OK$$

c) GWT @ surface and no wall friction - use Rankine's method!

$$\gamma_1' = \gamma_{sat,1} - \gamma_w = 19 - 9.81 = 9.19\ kN/m^3$$

$K_a = 0.33$ - From a)

$$P_a = \frac{1}{2} K_a \gamma' H^2 + \frac{1}{2}\gamma_w H^2 = \frac{1}{2}(0.33)(9.19)(5.0)^2 + \frac{1}{2}(9.81)(5.0)^2 = 160.53\ kN/m'$$

Pa is horizontal and acts at $\frac{H}{3} = 1.67\,m$ from the base.

$$\left.\begin{array}{l}\Sigma V = 282.96\ kN/m \\ \Sigma M_R = 785.22\ kN.m/m\end{array}\right\} \text{From a)}$$

The overturning moment is:

$$M_o = P_a\left(\frac{H}{3}\right) = 160.53\left(\frac{5.0}{3}\right) = 267.55\ kN.m/m$$

$$FS(\text{overturning}) = \frac{\Sigma M_R}{M_o} = \frac{785.22}{267.55} = 2.93 > 2.0, \ \underline{OK}$$

$$\delta'_{base} = 24^\circ$$

$$FS(\text{sliding}) = \frac{\Sigma V \tan \delta'_{base}}{P_a} = \frac{282.96 \tan 24^\circ}{160.53} = 0.78 < 1.5$$

<u>NOT OK</u>

$$M_{net} = \Sigma M_R - M_o = 785.22 - 267.55 = 517.67\ kN.m/m$$

$$\bar{x} = \frac{M_{net}}{\Sigma V} = \frac{517.67}{282.96} = 1.83\ m$$

$$e = \left|\frac{B}{2} - \bar{x}\right| = \left|\frac{4.2}{2} - 1.83\right| = 0.27\,m < \frac{B}{6} = 0.70\,m$$

The maximum stress occurs at the toe !

$$q_{max} = q_{toe} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right) = \frac{282.96}{4.2}\left(1 + \frac{6 \times 0.27}{4.2}\right) = 93.36\ kN/m^2$$

$$q_{min} = q_{heel} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right) = \frac{282.96}{4.2}\left(1 - \frac{6 \times 0.27}{4.2}\right) = 41.39\ kN/m^2$$

$$q_u = q\,N_q\,F_{qd}\,F_{qi} + \tfrac{1}{2}\gamma_2' B' N_\gamma F_{qd} F_{\gamma i}$$

$$q = \gamma_2' D = 10.19 \times 1.0 = 10.19\ kN/m^2$$

$$\gamma_2' = \gamma_{sat,2} - \gamma_w = 20 - 9.81 = 10.19\ kN/m^3$$

$$B' = B - 2e = 4.2 - 2(0.27) = 3.66\ m$$

$$\left.\begin{array}{ll} N_q = 37.75 & N_\gamma = 56.31 \\ F_{qd} = 1.06 & F_{\gamma d} = 1.0 \end{array}\right\} \text{ From a)}$$

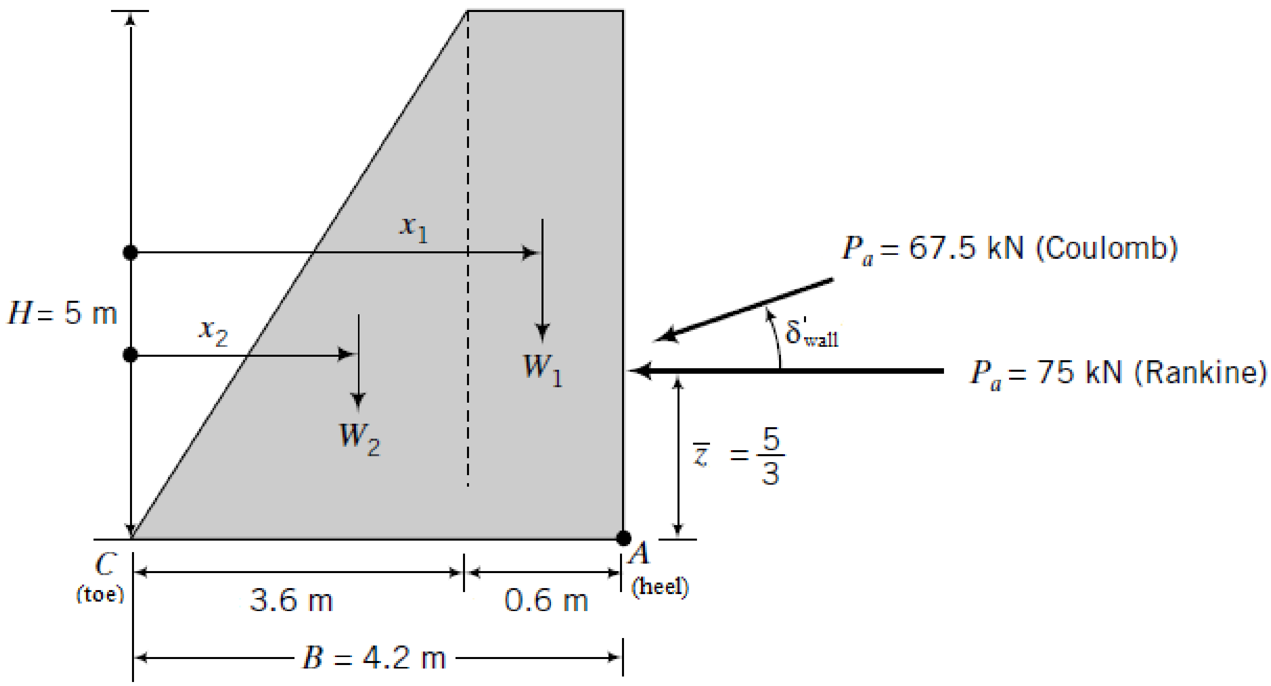
$$\psi^\circ = \tan^{-1}\left(\frac{P_a}{\Sigma V}\right) = \tan^{-1}\left(\frac{160.53}{282.96}\right) = 29.6^\circ$$

$$F_{qi} = \left(1 - \frac{\psi^\circ}{90^\circ}\right)^2 = \left(1 - \frac{29.6}{90}\right)^2 = 0.45$$

$$F_{\gamma i} = \left(1 - \frac{\psi^\circ}{\phi_2'}\right)^2 = \left(1 - \frac{29.6}{36}\right)^2 = 0.03$$

$$q_u = (10.19)(37.75)(1.06)(0.45) + \tfrac{1}{2}(10.19)(3.66)(56.31)(1.0)(0.03) = 214.99\ kN/m^2$$

$$FS_{(BC)} = \frac{q_u}{q_{max}} = \frac{214.99}{93.36} = 2.3 < 3.0$$, NOT OK


$H = 5$ m
x_1
x_2
W_1
W_2
$P_a = 67.5$ kN (Coulomb)
δ'_{wall}
$P_a = 75$ kN (Rankine)
$\bar{z} = \frac{5}{3}$
C
(toe)
A
(heel)
3.6 m
0.6 m
$B = 4.2$ m