MATH 1300-MIDTERM-2005

NAME and I.D.# Solutions, Version 2

Instructions– This exam consists of 6 multiple choice questions and 2 long answer questions. The multiple choice questions are worth 6 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages. **If you need additional scrap paper, it will be provided by the proctors.**

NO CALCULATORS. NO BOOKS. NO NOTES.

On the long answer questions, you must show your work.

Answers:

#1	#2	#3	#4
C	B	B	D

#5	#6
A	C

Multiple Choice Section-Question 1-
Consider the following function:

$$f(x) = \begin{cases} x+3 & \text{if } x < 1 \\ x^2+x+2 & \text{if } 1 \le x < 2 \\ x^2+3 & \text{if } 2 \le x \end{cases}$$

For what values of x is $f(x)$ continuous?
A) All real numbers. B) All real numbers except 1. C) All real numbers except 2.
D) All real numbers except 1 and 2. E) All real numbers except 0 and 1.

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x+3) = 4$
$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^2+x+2) = 4$
} continuous at $x=1$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x^2+x+2) = 8$
$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x^2+3) = 7$
} discontinuous at $x=2$

Question 2- Find the equation of the tangent line of the function $f(x) = x^3 - x + 7$ at $x = 2$.

A) $y = \frac{x}{3} + \frac{11}{3}$ B) $y = 11x - 9$ C) $y = 11x - 13$
D) $y = 2x + 2$ E) $y = 11x + 3$

$f'(x) = 3x^2 - 1$
$f'(2) = 11$
$f(2) = 13$

tangent:
$y = 11x + q$
$13 = 11 \cdot 2 + q$
$q = 13 - 22 = -9$

$y = 11x - 9$

Question 3-
Calculate:

$$\lim_{x\to -2} \frac{x^2 - x - 6}{x^2 + x - 2}$$

A) The limit does not exist. B) $\frac{5}{3}$ C) $\frac{-3}{7}$ D) $\frac{-1}{3}$ E) $\frac{1}{5}$

$x^2 - x - 6 = (x+2)(x-3)$

$x^2 + x - 2 = (x+2)(x-1)$

$$\lim_{x\to -2} \frac{(x+2)(x-3)}{(x+2)(x-1)} = \lim_{x\to -2} \frac{x-3}{x-1} = \frac{-2-3}{-2-1} = \frac{-5}{-3} = \boxed{\frac{5}{3}}$$

Question 4-Solve

$$3^x = 2^{x-2}$$

A) $x = \frac{2\ln(2)}{3}$ B) $x = \frac{2}{5}$ C) $x = \frac{\ln(2)}{\ln(3)}$ D) $x = \frac{2\ln(2)}{\ln(3)-\ln(2)}$ E) $x = \frac{\ln(3)-\ln(2)}{2\ln(2)}$

$\ln(3^x) = \ln(2^{x+2})$

$x\ln(3) = (x+2)\ln(2)$

$x(\ln(3) - \ln(2)) = 2\ln(2)$

$$\boxed{x = \frac{2\ln(2)}{\ln(3)-\ln(2)}}$$

Question 5

Let $f(x) = 2\sqrt{5-x^2}$. Find $f'(x)$ at $x = 1$.

A) -1 B) -2 C) -3 D) -4 E) -5

$$f(x) = 2(5-x^2)^{1/2}$$

$$f'(x) = 2\,\frac{1}{2}(5-x^2)^{-\frac{1}{2}} \cdot (-2x)$$

$$= \frac{-2x}{\sqrt{5-x^2}}$$

$$f'(1) = \frac{-2}{\sqrt{5-1}} = \boxed{-1}$$

Question 6 Let $g(x) = (x^2+3x+1)^7$. Find $g'(x)$ at $x = 0$.

A) 11 B) 19 C) 21 D) 34 E) 17

$$g'(x) = 7(x^2+3x+1)^6(2x+3)$$

$$g'(0) = 7 \cdot 1^6 \cdot 3 = 21$$

Long Answer Questions-Question 1 (12 points)

Suppose that a deposit of 4,000 dollars is made into a bank that gives 5% interest. Suppose that interest is compounded 5 times per year.

- (2 points) Write a formula for $A(t)$, the value of the investment, after t years.
- (4 points) How much will the investment be worth after 5 years?
- (6 points) How long will it take for the investment to double?

- $A(t) = 4000\left(1 + \frac{0.05}{5}\right)^{5t} = \boxed{4000\,(1.01)^{5t}}$

- $A(5) = 4000(1.01)^{25}$

- $A(t) = 8000$

  $4000(1.01)^{5t} = 8000$

  $5t \ln(1.01) = \ln(2)$

  $\boxed{t = \frac{\ln(2)}{5\ln(1.01)}}$

Question 2 (12 points)
Find $\frac{dy}{dx}$ at the point (1,2) for the following equation:

$$x^3y^2 + 4x^2 = y + 6$$

Implicit differentiation:

$$3x^2y^2 + x^3 2yy' + 8x = y'$$

$$y'(1 - 2x^3y) = 3x^2y^2 + 8x$$

$$y' = \frac{3x^2y^2 + 8x}{1 - 2x^3y}$$

In (1,2):

$$y' = \frac{3 \cdot 1^2 \cdot 2^2 + 8 \cdot 1}{1 - 2 \cdot 1^3 \cdot 2} = \frac{12 + 8}{1 - 4} = \boxed{-\frac{20}{3}}$$