Final Examination
Tuesday, April 22, 2014 - 9:00 am

ENGR233 – Applied Advanced Calculus
Time: 3 Hours

Course Coordinator
A. Sebak

Instructors:
H. Akbari, C. David,
N. Rossokhata, A. Shnirelman

Instructions:

- ✓ Books, Notes and Calculators are NOT allowed.
- ✓ Submit all sheets and papers at the end of the exam.
- ✓ Credit will NOT be given for correct answers unless the full details of the calculation are shown.

Problem 1 (10 Marks) - Find an equation of the plane containing (1, 5, -1) that is perpendicular to the line of intersection of the two planes: $-x + y - 5z = 4$ and $2x - y - 2z = 0$.

Problem 2 (10 Marks) - A thin wire has the shape of the curve C traced by

$$\vec{r}(t) = \cos(\pi t)\,\hat{\mathbf{i}} + t\,\hat{\mathbf{j}} + (\sin(\pi t) + 5)\,\hat{\mathbf{k}} \quad \text{on the interval } 0 \le t \le 1.$$

(a) Find the length of the curved wire. **(b)** Find the mass $\int_C \rho\, ds$ of the wire if its mass density $\rho(x,y,z) = xz$.

Problem 3 (10 Marks) - Use Green's theorem to evaluate the line integral $\int_C (3x^2 - 4xy)\,dx + (y^3 - x^2)\,dy$ where C is the rectangle with vertices $(2,1)$, $(3,1)$, $(3,6)$, and $(2,6)$ traversed in a **clockwise direction**.

Problem 4 (10 Marks) - Using spherical coordinates, find the volume of the solid inside $z = (x^2 + y^2)^{1/2}$ and bounded by $z^2 + x^2 + y^2 = 1$, and $z^2 + x^2 + y^2 = 4$.

Problem 5 (10 Marks) - Evaluate the surface integral $\iint_S y^3 z\, dS$

where S is the portion of the surface $x = 4 + z^2$ bounded by $z = 0$, $z = 1$, $y = 0$ and $y = 2$.

Problem 6 (10 Marks) - Use the divergence theorem to compute the outward flux $\oiint_S (\vec{F} \bullet \hat{n})\,dS$ of the vector field $\vec{F} = 3x^2\,\hat{\mathbf{i}} + (2x + y)\,\hat{\mathbf{j}} + (x^2 - z^2)\,\hat{\mathbf{k}}$ through the finite cylinder $x^2 + y^2 = 4$, and $-1 \le z \le 3$.

Problem 7 (10 Marks) - Use Stokes' Theorem to evaluate $\oint_C \vec{F} \bullet d\vec{r}$ where

$\vec{F} = (4z + x)\,\hat{\mathbf{i}} - 2xy\,\hat{\mathbf{j}} + (x^2 - y)\,\hat{\mathbf{k}}$ and C is a triangle in the first octant defined by the plane of $x + 3y + z = 3$ and C is oriented **counter clockwise** direction when viewed from above.

Problem 8 (10 Marks) - Find the center of mass of the half cylinder whose shape is described by $x^2 + y^2 \le 4$, $x \ge 0$, and $-1 \le z \le 1$ and whose density is $\rho(x,y,z) = x^2$. **Hint**: Use and apply symmetry.

Problem 9 (10 Marks) - Find the function $g(x,z)$ so that the vector

$\vec{F} = (8x\cos(y)\sin^2(z) + 2ze^{2xz})\,\hat{\mathbf{i}} - 4x^2\sin(y)\sin^2(z)\,\hat{\mathbf{j}} + (g(x,z)\cos(y) + 2xe^{2xz})\,\hat{\mathbf{k}}$ is a conservative field.

Problem 10 (10 Marks) - Use the change of variable technique to evaluate the integral

$\iint_R (x^2 + y^2)(x^2 - y^2)^5\, dxdy$ where R is the region bounded by the graphs of x = 0, x = 1, y =0, y =1 by means of the change of variables $u = 2xy$ and $v = x^2 - y^2$.
