ELG3106 Electromagnetic Engineering

Assignment # 1

Text, Problem 1.14
Text, Problem 1.21, (a), (c), (e)
Text, Problem 1.26
Text, Problem 1.27, (a), (c), (e)
Text, Problem 6.26
Text, Problem 6.29
Text, Problem 7.2
Text, Problem 7.8

Problem 1.14 A certain electromagnetic wave traveling in seawater was observed to have an amplitude of 98.02 (V/m) at a depth of 10 m, and an amplitude of 81.87 (V/m) at a depth of 100 m. What is the attenuation constant of seawater?

Solution: The amplitude has the form $Ae^{\alpha z}$. At $z = 10$ m,

$$Ae^{-10\alpha} = 98.02$$

and at $z = 100$ m,

$$Ae^{-100\alpha} = 81.87$$

The ratio gives

$$\frac{e^{-10\alpha}}{e^{-100\alpha}} = \frac{98.02}{81.87} = 1.20$$

or

$$e^{-10\alpha} = 1.2e^{-100\alpha}.$$

Taking the natural log of both sides gives

$$\begin{aligned} \ln(e^{-10\alpha}) &= \ln(1.2e^{-100\alpha}), \\ -10\alpha &= \ln(1.2) - 100\alpha, \\ 90\alpha &= \ln(1.2) = 0.18. \end{aligned}$$

Hence,

$$\alpha = \frac{0.18}{90} = 2 \times 10^{-3} \quad \text{(Np/m)}.$$

Problem 1.21 Complex numbers z_1 and z_2 are given by

$$z_1 = 5\angle{-60^\circ}$$
$$z_2 = 4\angle{45^\circ}.$$

(a) Determine the product $z_1 z_2$ in polar form.
(b) Determine the product $z_1 z_2^*$ in polar form.
(c) Determine the ratio z_1/z_2 in polar form.
(d) Determine the ratio z_1^*/z_2^* in polar form.
(e) Determine $\sqrt{z_1}$ in polar form.

Solution:

(a) $z_1 z_2 = 5e^{-j60^\circ} \times 4e^{j45^\circ} = 20e^{-j15^\circ}$.

(b) $z_1 z_2^* = 5e^{-j60^\circ} \times 4e^{-j45^\circ} = 20e^{-j105^\circ}$.

(c) $\dfrac{z_1}{z_2} = \dfrac{5e^{-j60^\circ}}{4e^{j45^\circ}} = 1.25e^{-j105^\circ}$.

(d) $\dfrac{z_1^*}{z_2^*} = \left(\dfrac{z_1}{z_2}\right)^* = 1.25e^{j105^\circ}$.

(e) $\sqrt{z_1} = \sqrt{5e^{-j60^\circ}} = \pm\sqrt{5}\,e^{-j30^\circ}$.

Problem 1.26 Find the phasors of the following time functions:

(a) $\upsilon(t) = 9\cos(\omega t - \pi/3)$ (V)

(b) $\upsilon(t) = 12\sin(\omega t + \pi/4)$ (V)

(c) $i(x,t) = 5e^{-3x}\sin(\omega t + \pi/6)$ (A)

(d) $i(t) = -2\cos(\omega t + 3\pi/4)$ (A)

(e) $i(t) = 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6)$ (A)

Solution:

(a) $\widetilde{V} = 9e^{-j\pi/3}$ V.

(b) $v(t) = 12\sin(\omega t + \pi/4) = 12\cos(\pi/2 - (\omega t + \pi/4)) = 12\cos(\omega t - \pi/4)$ V, $\widetilde{V} = 12e^{-j\pi/4}$ V.

(c)

$$\begin{aligned} i(t) &= 5e^{-3x}\sin(\omega t + \pi/6) \text{ A} = 5e^{-3x}\cos[\pi/2 - (\omega t + \pi/6)] \text{ A} \\ &= 5e^{-3x}\cos(\omega t - \pi/3) \text{ A}, \\ \widetilde{I} &= 5e^{-3x}e^{-j\pi/3} \text{ A}. \end{aligned}$$

(d)

$$\begin{aligned} i(t) &= -2\cos(\omega t + 3\pi/4), \\ \widetilde{I} &= -2e^{j3\pi/4} = 2e^{-j\pi}e^{j3\pi/4} = 2e^{-j\pi/4} \text{ A}. \end{aligned}$$

(e)

$$\begin{aligned} i(t) &= 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6) \\ &= 4\cos[\pi/2 - (\omega t + \pi/3)] + 3\cos(\omega t - \pi/6) \\ &= 4\cos(-\omega t + \pi/6) + 3\cos(\omega t - \pi/6) \\ &= 4\cos(\omega t - \pi/6) + 3\cos(\omega t - \pi/6) = 7\cos(\omega t - \pi/6), \\ \widetilde{I} &= 7e^{-j\pi/6} \text{ A}. \end{aligned}$$

Problem 1.27 Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) $\widetilde{V} = -5e^{j\pi/3}$ (V)
(b) $\widetilde{V} = j6e^{-j\pi/4}$ (V)
(c) $\widetilde{I} = (6 + j8)$ (A)
(d) $\widetilde{I} = -3 + j2$ (A)
(e) $\widetilde{I} = j$ (A)
(f) $\widetilde{I} = 2e^{j\pi/6}$ (A)

Solution:

(a)

$$\widetilde{V} = -5e^{j\pi/3} \text{ V} = 5e^{j(\pi/3-\pi)} \text{ V} = 5e^{-j2\pi/3} \text{ V},$$
$$v(t) = 5\cos(\omega t - 2\pi/3) \text{ V}.$$

(b)

$$\widetilde{V} = j6e^{-j\pi/4} \text{ V} = 6e^{j(-\pi/4+\pi/2)} \text{ V} = 6e^{j\pi/4} \text{ V},$$
$$v(t) = 6\cos(\omega t + \pi/4) \text{ V}.$$

(c)

$$\widetilde{I} = (6 + j8) \text{ A} = 10e^{j53.1^\circ} \text{ A},$$
$$i(t) = 10\cos(\omega t + 53.1^\circ) \text{ A}.$$

(d)

$$\widetilde{I} = -3 + j2 = 3.61\, e^{j146.31^\circ},$$
$$i(t) = \mathfrak{Re}\{3.61\, e^{j146.31^\circ} e^{j\omega t}\} = 3.61\, \cos(\omega t + 146.31^\circ) \text{ A}.$$

(e)

$$\widetilde{I} = j = e^{j\pi/2},$$
$$i(t) = \mathfrak{Re}\{e^{j\pi/2} e^{j\omega t}\} = \cos(\omega t + \pi/2) = -\sin\omega t \text{ A}.$$

(f)

$$\widetilde{I} = 2e^{j\pi/6},$$
$$i(t) = \mathfrak{Re}\{2e^{j\pi/6} e^{j\omega t}\} = 2\cos(\omega t + \pi/6) \text{ A}.$$

Problem 6.26 The electric field radiated by a short dipole antenna is given in spherical coordinates by

$$\mathbf{E}(R,\theta;t) = \hat{\boldsymbol{\theta}}\,\frac{2\times 10^{-2}}{R}\sin\theta\,\cos(6\pi\times 10^{8}t - 2\pi R) \quad \text{(V/m)}.$$

Find $\mathbf{H}(R,\theta;t)$.

Solution: Converting to phasor form, the electric field is given by

$$\widetilde{\mathbf{E}}(R,\theta) = \hat{\boldsymbol{\theta}}E_\theta = \hat{\boldsymbol{\theta}}\frac{2\times 10^{-2}}{R}\sin\theta e^{-j2\pi R} \quad \text{(V/m)},$$

which can be used with Eq. (6.87) to find the magnetic field:

$$\begin{aligned}
\widetilde{\mathbf{H}}(R,\theta) &= \frac{1}{-j\omega\mu}\nabla\times\widetilde{\mathbf{E}} = \frac{1}{-j\omega\mu}\left[\hat{\mathbf{R}}\frac{1}{R\sin\theta}\frac{\partial E_\theta}{\partial\phi} + \hat{\boldsymbol{\phi}}\frac{1}{R}\frac{\partial}{\partial R}(RE_\theta)\right] \\
&= \frac{1}{-j\omega\mu}\hat{\boldsymbol{\phi}}\frac{2\times 10^{-2}}{R}\sin\theta\,\frac{\partial}{\partial R}(e^{-j2\pi R}) \\
&= \hat{\boldsymbol{\phi}}\frac{2\pi}{6\pi\times 10^{8}\times 4\pi\times 10^{-7}}\frac{2\times 10^{-2}}{R}\sin\theta e^{-j2\pi R} \\
&= \hat{\boldsymbol{\phi}}\frac{53}{R}\sin\theta\, e^{-j2\pi R} \quad (\mu\text{A/m}).
\end{aligned}$$

Converting back to instantaneous value, this is

$$\mathbf{H}(R,\theta;t) = \hat{\boldsymbol{\phi}}\frac{53}{R}\sin\theta\cos(6\pi\times 10^{8}t - 2\pi R) \quad (\mu\text{A/m}).$$

Problem 6.29 The magnetic field in a given dielectric medium is given by

$$\mathbf{H} = \hat{\mathbf{y}}\, 6\cos 2z \sin(2\times 10^7 t - 0.1x) \quad \text{(A/m)},$$

where x and z are in meters. Determine:

(a) $\mathbf{E}$,

(b) the displacement current density $\mathbf{J}_\text{d}$, and

(c) the charge density ρ_v.

Solution:

(a)

$$\begin{aligned}
\mathbf{H} &= \hat{\mathbf{y}}\, 6\cos 2z \sin(2\times 10^7 t - 0.1x) = \hat{\mathbf{y}}\, 6\cos 2z \cos(2\times 10^7 t - 0.1x - \pi/2),\\
\widetilde{\mathbf{H}} &= \hat{\mathbf{y}}\, 6\cos 2z\, e^{-j0.1x} e^{-j\pi/2} = -\hat{\mathbf{y}}\, j6\cos 2z\, e^{-j0.1x},\\
\widetilde{\mathbf{E}} &= \frac{1}{j\omega\varepsilon} \nabla \times \widetilde{\mathbf{H}}\\
&= \frac{1}{j\omega\varepsilon} \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & -j6\cos 2z\, e^{-j0.1x} & 0 \end{vmatrix}\\
&= \frac{1}{j\omega\varepsilon} \left\{ \hat{\mathbf{x}} \left[-\frac{\partial}{\partial z} (-j6\cos 2z\, e^{-j0.1x}) \right] + \hat{\mathbf{z}} \left[\frac{\partial}{\partial x} (-j6\cos 2z\, e^{-j0.1x}) \right] \right\}\\
&= \hat{\mathbf{x}} \left(-\frac{12}{\omega\varepsilon} \sin 2z\, e^{-j0.1x} \right) + \hat{\mathbf{z}} \left(\frac{j0.6}{\omega\varepsilon} \cos 2z\, e^{-j0.1x} \right).
\end{aligned}$$

From the given expression for $\mathbf{H}$,

$$\omega = 2\times 10^7 \quad \text{(rad/s)},$$
$$\beta = 0.1 \quad \text{(rad/m)}.$$

Hence,

$$u_\text{p} = \frac{\omega}{\beta} = 2\times 10^8 \quad \text{(m/s)},$$

and

$$\varepsilon_\text{r} = \left(\frac{c}{u_\text{p}}\right)^2 = \left(\frac{3\times 10^8}{2\times 10^8}\right)^2 = 2.25.$$

Using the values for ω and ε, we have

$$\begin{aligned}
\widetilde{\mathbf{E}} &= (-\hat{\mathbf{x}}\, 30\sin 2z + \hat{\mathbf{z}}\, j1.5\cos 2z) \times 10^3 e^{-j0.1x} \quad \text{(V/m)},\\
\mathbf{E} &= \left[-\hat{\mathbf{x}}\, 30\sin 2z \cos(2\times 10^7 t - 0.1x) - \hat{\mathbf{z}}\, 1.5\cos 2z \sin(2\times 10^7 t - 0.1x)\right] \quad \text{(kV/m)}.
\end{aligned}$$

(b)

$$\widetilde{\mathbf{D}} = \varepsilon\widetilde{\mathbf{E}} = \varepsilon_r\varepsilon_0\widetilde{\mathbf{E}} = (-\hat{\mathbf{x}}\,0.6\sin 2z + \hat{\mathbf{z}}\,j0.03\cos 2z) \times 10^{-6} e^{-j0.1x} \quad (\text{C/m}^2),$$

$$\mathbf{J}_\text{d} = \frac{\partial \mathbf{D}}{\partial t},$$

or

$$\widetilde{\mathbf{J}}_\text{d} = j\omega\widetilde{\mathbf{D}} = (-\hat{\mathbf{x}}\,j12\sin 2z - \hat{\mathbf{z}}\,0.6\cos 2z)e^{-j0.1x},$$

$$\mathbf{J}_\text{d} = \mathfrak{Re}[\widetilde{\mathbf{J}}_\text{d} e^{j\omega t}]$$

$$= \left[\hat{\mathbf{x}}\,12\sin 2z\sin(2\times 10^7 t - 0.1x) - \hat{\mathbf{z}}\,0.6\cos 2z\cos(2\times 10^7 t - 0.1x)\right] \quad (\text{A/m}^2).$$

(c) We can find ρ_v from

$$\nabla\cdot\mathbf{D} = \rho_\text{v}$$

or from

$$\nabla\cdot\mathbf{J} = -\frac{\partial \rho_\text{v}}{\partial t}.$$

Applying Maxwell's equation,

$$\rho_\text{v} = \nabla\cdot\mathbf{D} = \varepsilon\nabla\cdot\mathbf{E} = \varepsilon_r\varepsilon_0\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z}\right)$$

yields

$$\rho_\text{v} = \varepsilon_r\varepsilon_0\left\{\frac{\partial}{\partial x}\left[-30\sin 2z\cos(2\times 10^7 t - 0.1x)\right] + \frac{\partial}{\partial z}\left[-1.5\cos 2z\sin(2\times 10^7 t - 0.1x)\right]\right\}$$

$$= \varepsilon_r\varepsilon_0\left[-3\sin 2z\sin(2\times 10^7 t - 0.1x) + 3\sin 2z\sin(2\times 10^7 t - 0.1x)\right] = 0.$$

Problem 7.2 Write general expressions for the electric and magnetic fields of a 1-GHz sinusoidal plane wave traveling in the $+y$-direction in a lossless nonmagnetic medium with relative permittivity $\varepsilon_r = 9$. The electric field is polarized along the x-direction, its peak value is 6 V/m, and its intensity is 4 V/m at $t = 0$ and $y = 2$ cm.

Solution: For $f = 1$ GHz, $\mu_r = 1$, and $\varepsilon_r = 9$,

$$\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s},$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r} = \frac{2\pi f}{c}\sqrt{\varepsilon_r} = \frac{2\pi \times 10^9}{3 \times 10^8}\sqrt{9} = 20\pi \text{ rad/m},$$

$$\mathbf{E}(y,t) = \hat{\mathbf{x}} 6\cos(2\pi \times 10^9 t - 20\pi y + \phi_0) \quad \text{(V/m)}.$$

At $t = 0$ and $y = 2$ cm, $E = 4$ V/m:

$$4 = 6\cos(-20\pi \times 2 \times 10^{-2} + \phi_0) = 6\cos(-0.4\pi + \phi_0).$$

Hence,

$$\phi_0 - 0.4\pi = \cos^{-1}\left(\frac{4}{6}\right) = 0.84 \text{ rad},$$

which gives

$$\phi_0 = 2.1 \text{ rad} = 120.19°$$

and

$$\mathbf{E}(y,t) = \hat{\mathbf{x}}\, 6\cos(2\pi \times 10^9 t - 20\pi y + 120.19°) \quad \text{(V/m)}.$$

$$\tilde{H}(y) = \eta^{-1}\hat{k} \times \tilde{E}(y) \quad , \quad \hat{k} = \hat{y}$$

$$\eta = \frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{120\pi}{3} = 40\pi$$

$$\hat{k} \times \tilde{E} \Rightarrow \hat{y} \times \hat{x} = -\hat{z}$$

$$\vec{H}(y,t) = -\hat{z}\,\frac{6}{40\pi}\cos(2\pi \times 10^9 - 20\pi y + 120.19°) \quad \text{(A/m)}$$

Problem 7.8 A 60-MHz plane wave traveling in the $-x$-direction in dry soil with relative permittivity $\varepsilon_r = 4$ has an electric field polarized along the z-direction. Assuming dry soil to be approximately lossless, and given that the magnetic field has a peak value of 10 (mA/m) and that its value was measured to be 7 (mA/m) at $t = 0$ and $x = -0.75$ m, develop complete expressions for the wave's electric and magnetic fields.

Solution: For $f = 60$ MHz $= 6 \times 10^7$ Hz, $\varepsilon_r = 4$, $\mu_r = 1$,

$$k = \frac{\omega}{c}\sqrt{\varepsilon_r} = \frac{2\pi \times 6 \times 10^7}{3 \times 10^8}\sqrt{4} = 0.8\pi \quad \text{(rad/m)}.$$

Given that **E** points along $\hat{\mathbf{z}}$ and wave travel is along $-\hat{\mathbf{x}}$, we can write

$$\mathbf{E}(x,t) = \hat{\mathbf{z}} E_0 \cos(2\pi \times 60 \times 10^6 t + 0.8\pi x + \phi_0) \quad \text{(V/m)}$$

where E_0 and ϕ_0 are unknown constants at this time. The intrinsic impedance of the medium is

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{2} = 60\pi \quad (\Omega).$$

With **E** along $\hat{\mathbf{z}}$ and $\hat{\mathbf{k}}$ along $-\hat{\mathbf{x}}$, (7.39) gives

$$\mathbf{H} = \frac{1}{\eta}\,\hat{\mathbf{k}} \times \mathbf{E}$$

or

$$\mathbf{H}(x,t) = \hat{\mathbf{y}}\frac{E_0}{\eta}\cos(1.2\pi \times 10^8 t + 0.8\pi x + \phi_0) \quad \text{(A/m)}.$$

Hence,

$$\frac{E_0}{\eta} = 10 \quad \text{(mA/m)}$$
$$E_0 = 10 \times 60\pi \times 10^{-3} = 0.6\pi \quad \text{(V/m)}.$$

Also,

$$H(-0.75\text{ m}, 0) = 7 \times 10^{-3} = 10\cos(-0.8\pi \times 0.75 + \phi_0) \times 10^{-3}$$

which leads to $\phi_0 = 153.6°$.

Hence,

$$\mathbf{E}(x,t) = \hat{\mathbf{z}}\,0.6\pi\cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6°) \quad \text{(V/m)}.$$
$$\mathbf{H}(x,t) = \hat{\mathbf{y}}\,10\cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6°) \quad \text{(mA/m)}.$$