Université d'Ottawa
Faculté de génie
École de science informatique
et de génie électrique

uOttawa
L'Université canadienne
Canada's university

University of Ottawa
Faculty of Engineering
School of Electrical Engineering
and Computer Science

ELG 3126x **RANDOM SIGNALS AND SYSTEMS, Instructor:** J. Lostracco, SITE 0110B, [jlostrac@uottawa.ca](mailto:jlostrac@uottawa.ca)

**Assignment 4: pdf, cdf, Normal, Exponential** Due Fri 10 Jun 2016 –upload photos or scans to BBoard

Q1. Let Y = 5X + 3.

(a) Find the mean and variance of Y in terms of the mean and variance of X.
(b) Evaluate the mean and variance of Y if X is an arbitrary Gaussian random variable.
(c) Evaluate the mean and variance of Y if X= d cos ($2\pi$U) where U is a uniform random variable in the unit interval.

Q2. Let X be a Gaussian random variable with m=4 and $\sigma^2$=25.

(a) Find : P[X > 4], P[X >7], P[6.72 < X < 10.16], P[2 < X < 7], P[6 < X < 8]
(b) P[X < a] = .8869 , Find a.
(c) P[ X > b] = .11131, find b.
(d) P[13 < X <c ] = 0.0123 , find c.

Q3.

Two chips are being considered for use in a certain system. The lifetime of chip 1 is modeled by a Gaussian random variable with mean 20,000 hours and standard deviation 6000 hours. (The probability of negative lifetime is negligible.) The lifetime of chip 2 is also a Gaussian random variable but with mean 22,000 hours and standard deviation 2000 hours. Which chip is preferred if the target lifetime of the system is;

(i) 20,000 hours? (ii) 24,000 hours?

Q4.

The exam grades in a certain class have a Gaussian pdf with mean m and standard deviation $\sigma$.

- Find the constants a and b so that the random variable Y = aX + b has a Gaussian pdf with mean 3m and standard deviation $2\sigma$.

Q5.

The lifetime X of a light bulb is a random variable with P[X > t] = 2/(2+t) for t >0. Suppose three new light bulbs are installed at time t = 0. At time t = 1 all three light bulbs are still working.

- Find the probability that at least one light bulb is still working at time t = 8.

Q6.

A sample of 25 bulbs is taken from a very large lot of light bulbs and tested until they fail to determine the burn life of each bulb. The average value is found to be 1500 hours. The sample standard deviation is found to be 200 hours.

- Assuming that the bulb lifetime is well approximated by a Gaussian distribution, find the 95% confidence interval for the average life of bulbs in the lot.

Q7.

In passive sonar, after processing the sound received, the receiver produces an output which is described by a random variable R that is always a zero mean Gaussian random variable. When there is only background noise, R has variance 4. When a target is present to produce noise, the variance of R is greater by the noise variance $(\sigma_T)^2$. The sonar system must use R to decide between

- H0 : there is no target present;
- H1 : a target is present.

The operation of the sonar system is such that a target is said to be present when $|R| > 2$, and is otherwise said to be absent.

(a) What is the probability of a false alarm (the probability a target is said to be present when there is none)?

(b) What is the probability of a target being detected as a function of the target noise variance $(\sigma_T)^2$. ?

Q8.

A random variable X has density $f_X(x) = ax^2$ on the interval [0, b].

- Find the density of $Y = X^3$.

Q9.

Is there a value of c that makes the following function a density function?
(Either find c, or explain why there isn't such a c).

$f(x) = c(x^2 - 3x + 2)$ if $0 \le x \le 3$ ; and $f(x)=0$ otherwise.

Q10.

The lifetime of a car's brakes is exponentially distributed with $\lambda = 1/30$ (units are thousands of km). After 22 thousand km of operation, your car's brakes are still working. What is the probability that the brakes will fail between 35 thousand and 45 thousand km of operation? (Hint: lifetime is exponentially distributed)

**BONUS**

Q11.

A random variable X has density function

$f(x) = (9-x^2)/18$ if $0 \le x \le 3$,

and $f(x) = 0$ otherwise.

a) Calculate E(X) and
b) Var(X).