Faculté des sciences sociales | Faculty of Social Sciences

uOttawa

Département de science économique
Department of Economics

ECO2144 B Microeconomic Theory I
Midterm- Spring 2016
Paul Makdissi

Instructions: This midterm exam consists of three sections. No one is allowed to exit the exam room during the first two hours of the exam. A student wishing to use the washroom during the first hour of the exam has to hand back Section 1. Section 2 will be handed to her/him upon her/his return to the examination room. A student wishing to use the washroom during the second hour of the exam has to hand back Sections 1 and 2. Section 3 will be handed to her/him upon her/his return to the examination room. A student has the right to keep all sections as long as she/he does not go to the washrooms. A student also has the right to ask for all sections of the exam at any time. However, if the student receives Section 2 during the first hour, she/he has to wait until the second hour of the exam to use the washroom. As soon as a student asks for Section 3 she/he does not have the right to use the washroom anymore. No one is allowed to use the washroom during the last hour of the exam.

SECTION 1 IS ON THE REVERSE SIDE

Section 1

Problem 1: Market equilibrium (20 marks)
Moschini and Meike (1992) have estimated the following market demand for processed pork in Canada: $Q^d = 171 - 20p + 20p_b + 3p_c + 2Y$, where p represents the price of processed pork, p_b, the price of beef, p_c, the price of chicken and Y, income of consumers in thousands of dollars. The supply of processed pork is given by $Q^s = 178 + 40p - 60p_h$, where p_h is the price of hogs. Assume that $p_b = 4$, $p_c = 3.\overline{3}$, $Y = 12.5$ and $p_h = 1.50$.

a) Find the equilibrium price and quantity for processed pork. (4 marks)

b) What is the value of the price elasticity of demand at this equilibrium? (4 marks)

c) What is the value of the cross-price elasticity of processed pork with respect to the price of beef at this equilibrium? (4 marks)

d) What is the value of income elasticity of demand at this equilibrium? (4 marks)

e) What is the value of the price elasticity of supply at this equilibrium? (4 marks)

Problem 2: Market equilibrium and taxation (12 marks)
Assume that the daily market demand for marijuana in Ottawa in 2020 is $Q^d = 20000p^{-0.5}$ (where p is the price per gram) and that the inverse market supply curve of marijuana in the city is $p = 1.5p_w$, where p_w is the wholesale price of a gram of marijuana.

a) What is the price elasticity of demand? (4 marks)

b) Assume that the Ottawa retail market for marijuana is competitive. Calculate the equilibrium price and quantity of cigarettes as a function of wholesale price. Let Q^* represent the equilibrium quantity. Find $\frac{dQ^*}{dp_w}$. (4 marks)

c) Assume that $p_w = 5$ and assume that the mayor of Ottawa imposes a tax $\tau = 2$ per gram of marijuana. How does the introduction of this tax affect the equilibrium retail price and quantity of marijuana? (4 marks)

END OF THE FIRST SECTION OF THE MIDTERM.

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Problem 3: Indifference curves (20 marks)

a) Explain what is an indifference curve. (4 marks)

b) Totally differentiate the mathematical expression of an indifference curve and use the result to find an expression for the marginal rate of substitution of q_1 and q_2. (4 marks)

c) Explain why indifference curves have a negative slope. (4 marks)

d) Explain why indifference curves cannot cross. (4 marks)

e) Explain why indifference curves are not thick. (4 marks)

Problem 4: Consumer's individual demands (16 marks)
Assume that a consumer's preferences are represented by the following utility function $U(q_1, q_2) = q_1^{\beta} q_2^{1-\beta}$. Let p_1 represent the price of good 1, p_2, the price of good 2 and Y, consumer's income.

a) Find the individual demands of good 1 and 2. (Display all the details of the constrained optimization problem). (12 marks)

b) What is the economic interpretation of β? (4 marks)

END OF THE SECOND SECTION OF THE MIDTERM.

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Problem 5: CES Utility (16 marks)
Fatima's utility function is $U(q_1, q_2) = (q_1^\rho + q_2^\rho)^{\frac{1}{\rho}}$. Let p_1 represent the price of good 1, p_2, the price of good 2 and Y, Fatima's income.

a) Show that there is a positive monotonic transformation such that there is an equivalent utility function $U(q_1, q_2) = q_1^\rho + q_2^\rho$. (4 marks)

b) Find the individual demands of good 1 and 2. (Display all the details of the constrained optimization problem). (12 marks)

Problem 6: Quasi-linear utility (16 marks)
Satochi's utility function is $U(q_1, q_2) = 4q_1^{0.5} + q_2$. Let p_1 represent the price of good 1, p_2, the price of good 2 and Y, Satochi's income.

a) Find the individual demands of good 1 and 2. You should identify under which condition there is a corner solution where Satochi consumes only good 1. (Display all the details of the constrained optimization problem). (12 marks)

b) Assume that originally the prices are $p_1 = p_2 = 1$. However, the price of good 1 increases to $p_1^* = 2$. Discuss the substitution, income and total effect on the demand for good 1. (4 marks)

END OF THE MIDTERM.

MIDTERM solutions

#1

a) $Q^d = 171 - 20p + (20)(4) + (3)(3\tfrac{1}{3}) + 25$
$Q^d = 286 - 20p$ (1 mark)

$Q^s = 178 + 40p - (60)(1.50)$
$Q^s = 88 + 40p$ (1 mark)

Equilibrium: $286 - 20p = 88 + 40p$
$60p = 198$
$p^* = 3.30$ (1 mark)
$Q^* = 286 - (20)(3.30)$
$Q^* = 220$ (1 mark)

b) $\varepsilon = \dfrac{\partial Q^d}{\partial p} \cdot \dfrac{P^*}{Q^*}$ (2 marks)

$\varepsilon = (-20)\left(\dfrac{3.30}{220}\right)$ (1 mark)

$\varepsilon = -0.3$ (1 mark)

c) $\varepsilon_b = \dfrac{\partial Q^d}{\partial p_b} \cdot \dfrac{P_b}{Q^*}$ (2 marks)

$\varepsilon_b = (20)\left(\dfrac{4}{220}\right)$ (1 mark)

$\varepsilon_b = 0.364$ (1 mark)

d) $\xi = \frac{\partial Q^d}{\partial Y} \cdot \frac{Y}{Q^*}$ (2 marks)

$\xi = (2) \cdot \left(\frac{12.5}{220}\right)$ (1 mark)

$\xi = 0.114$ (1 mark)

e) $\eta = \frac{\partial Q^s}{\partial P} \cdot \frac{P^*}{Q^*}$ (2 marks)

$\eta = (40)\left(\frac{3.30}{220}\right)$ (1 mark)

$\eta = 0.6$ (1 mark)

#2

a) $\varepsilon = -0.5$ (constant) (4 marks)

b) Supply is an horizontal curve at $p = 1.5 p_w$

$\Rightarrow p^* = 1.5\, p_w$ (1 mark)

$Q^* = 20{,}000\,(1.5\, p_w)^{-0.5}$

$Q^* = 16{,}329.93\, p_w^{-0.5}$ (1 mark)

$\frac{dQ^*}{dp_w} = -8{,}164.97\, p_w^{-1.5}$ (2 marks)

c) $Q^* = 20{,}000\ (1.5 \times 5)^{-0.5}$
$Q^* = 7{,}302.97$

If $\tau = 2 \Rightarrow P_\tau = 1.5\, P_W + 2$
$P_\tau = 9.5$ (2 marks)

$Q_\tau = 20{,}000\ (9.5)^{-0.5}$
$Q_\tau = 6{,}488.86$ (2 marks)

#3

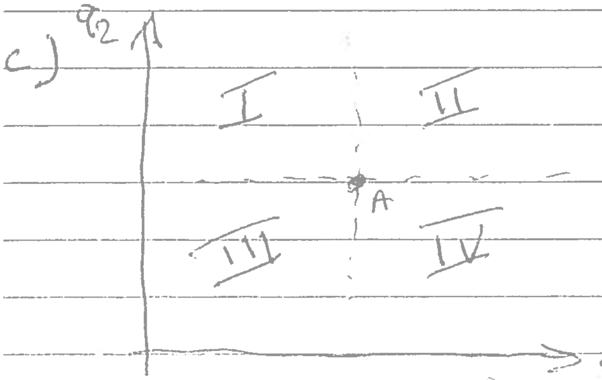
a) It is a curve representing all baskets (q_1, q_2) yielding a same level of utility (4 marks)

b) $u(q_1, q_2) = \bar{u}$ (1 mark)

$u_1\, dq_1 + u_2\, dq_2 = 0$ (1 mark)

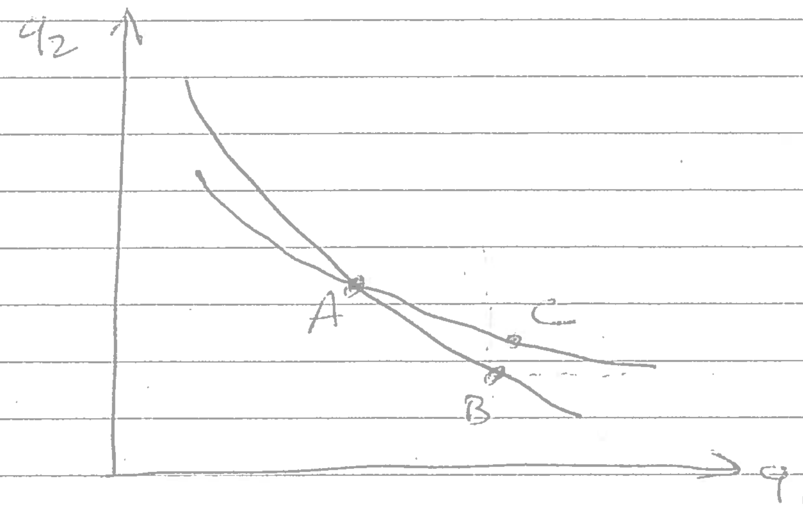
$MRS = \left.\frac{dq_2}{dq_1}\right|_{u(\cdot)=\bar{u}} = -\frac{u_1}{u_2}$ (2 marks)

c)



- Monotonicity implies that all baskets in II are preferred to A (1 mark)
- It also implies that A is preferred to all baskets in III (1 mark)
- The indifference curve has to go through A from I and IV implying a negative slope (2 marks)

d) Assume that they were



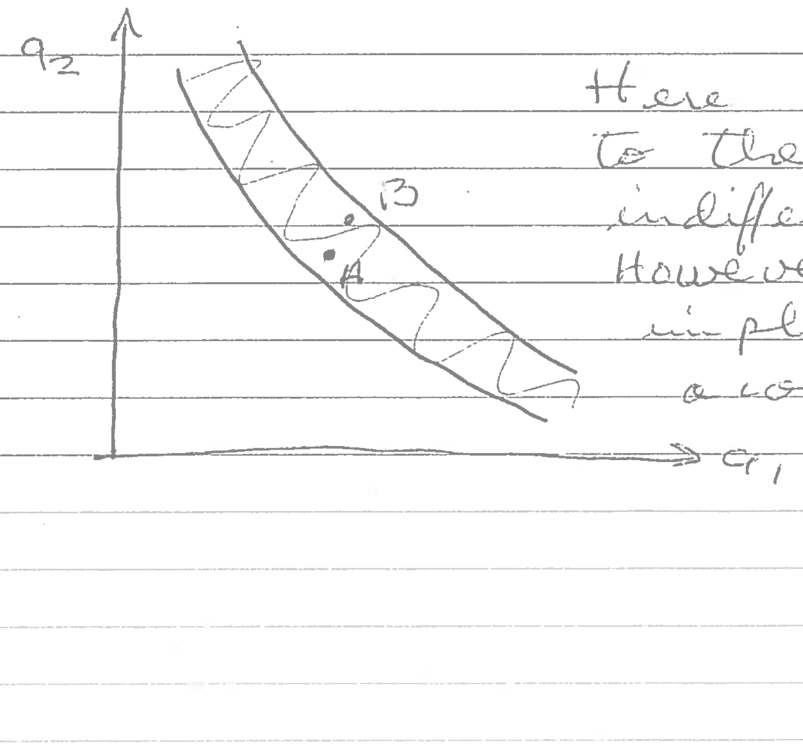
Here $A \sim C$

$C \succ B$ because of monotonicity

$\Rightarrow A \sim B$ violates transitivity because $A \sim C \succ B$

(4 marks)

e)



Here A and B belong to the same thick indifference curve. However, monotonicity implies $B \succ A$ ... a contradiction.

$\Rightarrow$ An indifference curve cannot be thick.

(4 marks)

#4

a) $\max_{q_1, q_2} \; q_1^{\beta} q_2^{1-\beta}$

s.t. $p_1 q_1 + p_2 q_2 = Y$

(2 marks)

Use a "ln" transformation

$\Rightarrow \max_{q_1, q_2, \lambda} \mathcal{L} = \beta \ln q_1 + (1-\beta) \ln q_2 + \lambda [Y - p_1 q_1 - p_2 q_2]$

(2 marks)

F.O.C.: $\frac{\beta}{q_1} - \lambda p_1 = 0$ (1)

$\frac{1-\beta}{q_2} - \lambda p_2 = 0$ (2)

$Y - p_1 q_1 - p_2 q_2 = 0$ (3)

(2 marks)

$\Rightarrow$ From (1) and (2) $\frac{\beta}{p_1 q_1} = \frac{1-\beta}{p_2 q_2}$

$\Rightarrow p_2 q_2 = \frac{1-\beta}{\beta} p_1 q_1$ (4)

(4) $\rightarrow$ (3) $\Rightarrow Y - p_1 q_1 - \frac{1-\beta}{\beta} p_1 q_1 = 0$

$\left(\frac{\beta + 1 - \beta}{\beta}\right) p_1 q_1 = Y$

$\boxed{q_1(p_1, p_2, Y) = \frac{\beta Y}{p_1}}$ (5)

(3 marks)

(5) → (4) ⟹ $q_2 = \frac{1-\beta}{\beta} \frac{P_1}{P_2} \frac{\beta Y}{P_1}$

$$\boxed{q_2(P_1, P_2, Y) = \frac{(1-\beta)Y}{P_2}}$$ (3 marks)

b) $q_1(P_1, P_2, Y) = \frac{\beta Y}{P_1}$

⟹ $\beta = \frac{P_1 q_1(P_1, P_2, Y)}{Y}$

⟹ β is the share of total expenditures (or income) spent on good 1.

(4 marks)

#5

a) $\left[\left(q_1^{\rho} + q_2^{\rho}\right)^{\frac{1}{\rho}}\right]^{\rho} = q_1^{\rho} + q_2^{\rho}$ (4 marks)

b) $\max_{q_1, q_2} q_1^{\rho} + q_2^{\rho}$

s.t. $P_1 q_1 + P_2 q_2 = Y$ (2 marks)

$$\max_{q_1, q_2, \lambda} \mathcal{L} = q_1^{\rho} + q_2^{\rho} + \lambda\left[Y - p_1 q_1 - p_2 q_2\right]$$

(2 marks)

F.O.C.:

$$\rho q_1^{\rho-1} - \lambda p_1 = 0 \quad (1)$$
$$\rho q_2^{\rho-1} - \lambda p_2 = 0 \quad (2)$$
$$Y - p_1 q_1 - p_2 q_2 = 0 \quad (3)$$

(2 marks)

From (1) and (2) $\dfrac{q_1^{\rho-1}}{p_1} = \dfrac{q_2^{\rho-1}}{p_2}$

$$q_2^{\rho-1} = \frac{p_2\, q_1^{\rho-1}}{p_1}$$

$$q_2 = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\rho-1}} q_1 \quad (4)$$

(4) -> (3) $Y - p_1 q_1 - p_2 \left(\dfrac{p_2}{p_1}\right)^{\frac{1}{\rho-1}} q_1 = 0$

$$Y - p_1 \left(\frac{p_1}{p_1}\right)^{\frac{1}{\rho-1}} q_1 - p_2 \left(\frac{p_2}{p_1}\right)^{\frac{1}{\rho-1}} q_1 = 0$$

$$\left(\frac{p_1^{\frac{\rho}{\rho-1}} + p_2^{\frac{\rho}{\rho-1}}}{p_1^{\frac{1}{\rho-1}}}\right) q_1 = Y$$

$$\boxed{q_1(p_1, p_2, Y) = \frac{p_1^{\frac{1}{\rho-1}}\, Y}{p_1^{\frac{\rho}{\rho-1}} + p_2^{\frac{\rho}{\rho-1}}}} \quad (5)$$

(4 marks)

(5) → (4) $q_2 = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\rho-1}} \frac{P_1^{\frac{1}{\rho-1}} Y}{P_1^{\frac{\rho}{\rho-1}} + P_2^{\frac{\rho}{\rho-1}}}$

$$\boxed{q_2(P_1, P_2, Y) = \frac{P_2^{\frac{1}{\rho-1}} Y}{P_1^{\frac{\rho}{\rho-1}} + P_2^{\frac{\rho}{\rho-1}}}}$$

(4 marks)

NOTE: Some students may have substituted ρ using $\sigma = \frac{1}{\rho - 1}$

For these students the demands are

$$q_1(P_1, P_2, Y) = \frac{P_1^{\sigma} Y}{P_1^{\sigma+1} + P_2^{\sigma+1}}$$

$$q_2(P_1, P_2, Y) = \frac{P_2^{\sigma} Y}{P_1^{\sigma+1} + P_2^{\sigma+1}}$$

#6

a) $\max_{q_1, q_2} \; 4q_1^{0.5} + q_2$ (2 marks)

s.t. $p_1 q_1 + p_2 q_2 = Y$

$\Rightarrow$ Substitute $q_2 = \dfrac{Y - p_1 q_1}{p_2}$ (1) (2 marks)

$$\Rightarrow \max_{q_1} \; 4q_1^{0.5} + \frac{Y - p_1 q_1}{p_2}$$

F.O.C.: $2q_1^{-0.5} - \dfrac{p_1}{p_2} = 0$

$$q_1^{0.5} = \frac{2p_2}{p_1}$$

$$q_1 = \frac{4p_2^2}{p_1^2}$$

(2 marks)

This is valid only if we have an interior solution. An interior solution requires that $p_1 q_1 = \dfrac{4p_2^2}{p_1} \leq Y$

$$\Rightarrow q_1(p_1, p_2, Y) = \begin{cases} \dfrac{4p_2^2}{p_1^2} & \text{if } Y \geq \dfrac{4p_2^2}{p_1} \\[2ex] \dfrac{Y}{p_1} & \text{if } Y \leq \dfrac{4p_2^2}{p_1} \end{cases}$$

(2)

(4 marks)

) → (i)

$$q_2(P_1, P_2, Y) = \frac{Y - P_1 q_1(P_1, P_2, Y)}{P_2}$$

$$q_2(P_1, P_2, Y) = \begin{cases} \dfrac{Y - 4P_2^2/A}{P_2} & \text{if } Y \geq \dfrac{4P_2^2}{P_1} \\ 0 & \text{otherwise} \end{cases}$$

(4 marks)

) Since $\frac{4P_2^2}{P_1} = 4 < 10 = Y$

$$q_1 = \frac{4P_2^2}{P_1^2} \quad \text{when } P_1 = 1 \rightarrow q_1 = 4$$
$$P_1^* = 2 \rightarrow q_1^* = 1$$

Slutsky $\Rightarrow \dfrac{\partial q_1}{\partial P_1} = \dfrac{\partial q_1^H}{\partial p_1} - q_1 \dfrac{\partial q_1}{\partial y}$

$\dfrac{\partial q_1}{\partial y} = 0$ since y doesn't appear in $\dfrac{4P_2^2}{P_1^2}$

⇒ There is no income effect (2 marks)

$\Delta q_1 = -3$
substitution effect $= -3$
income effect $= 0$.
(2 marks)