NAME: Instructor's Solutions STUDENT ID# : ____________

LAKEHEAD UNIVERSITY

FACULTY OF ENGINEERING

Midterm Exam -2016

Engineering	3021	AA	Summer Transition Program
SUBJECT	COURSE NO.	SECTION	

Engineering Analysis A — COURSE TITLE

Drs. M.N. Uddin and H. Naser — INSTRUCTOR

Friday, July 15, 2016	9:00 a.m. - 11 a.m.	2 hours	AT1003/UC2011
EXAM DATE	EXAM TIME	DURATION	ROOM

- **NO CALCULATOR OR ELECTRONIC GADGETS ALLOWED**
- **SHOW YOUR STEPS – NO STEPS NO MARKS**
- **CLOSED BOOK EXAM**

This exam **MAY NOT** be removed from the room

STUDENTS PLEASE NOTE

YOU MUST count the number of pages in this examination question paper **BEFORE** beginning to write, and report any discrepancy immediately to a proctor. **MARKS** are indicated in the right hand margin.

This is page 1 of 11.

This section is for faculty use only

Q	1	2	3	4	5	6	7	8	9	10	Total
M											

1. Solve the inequality $x^2 > 2x + 8$ in terms of intervals and illustrate the solution set on the real number line. 10

4

$x^2 - 2x - 8 > 0$

$x^2 - 4x + 2x - 8 > 0$

$(x-4)(x+2) > 0$

If $(x-4)(x+2) = 0$ then $x = -2, 4$

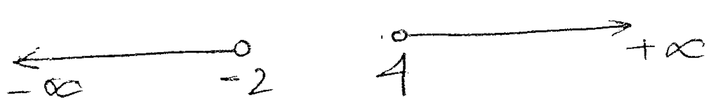
4

	$(x+2)$	$(x-4)$	$(x+2)(x-4)$	
$x < -2$	$(-)$	$(-)$	$(+)$	✓
$-2 < x < 4$	$(+)$	$(-)$	$(-)$	
$x > 4$	$(+)$	$(+)$	$(+)$	✓

2

$\therefore$ $x < -2$ and $x > 4$

$\therefore$ $x = (-\infty, -2) \cup (4, \infty)$



2. Find the functions, $f \circ g$, $g \circ g$, f/g, and their domains. 10

$$f(x) = 2x^2 - 1, \quad g(x) = \sqrt{4-x}$$

2 $\quad f\circ g\,(x) = f(g(x)) = 2g^2(x) - 1 = 2(4-x) - 1 = 7 - 2x$

2 $\quad D_{f\circ g} = \{x \mid x \text{ in the domain of } g \text{ and } g(x) \text{ in the domain of } f\}$

$= \{x \mid 4 - x \geq 0\} = \{x \mid x \leq 4\} = (-\infty, 4].$

2 $\quad g\circ g\,(x) = g(g(x)) = \sqrt{4 - \sqrt{4-x}}$

2 $\quad D_{g\circ g} = \{x \mid x \text{ in the domain of } g \text{ and } g(x) \text{ in the domain of } g\}$

$= \{x \mid 4 - x \geq 0 \text{ and } 4 - \sqrt{4-x} \geq 0\}$

$= \{x \mid x \leq 4 \text{ and } \sqrt{4-x} \leq 4\} = \{x \mid x \leq 4 \text{ and } 4 - x \leq 16\}$

$= \{x \mid x \leq 4 \text{ and } x \geq -12\} = [-12, 4].$

1 $\quad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2 - 1}{\sqrt{4-x}}$

1 $\quad D_{f/g} = \{x \mid x \text{ in domain } f \text{ and } g\text{, and } g(x) \neq 0\}$

$= \{x \mid 4 - x \geq 0 \text{ and } \sqrt{4-x} \neq 0\}$

$= \{x \mid x < 4\} = (-\infty, 4)$

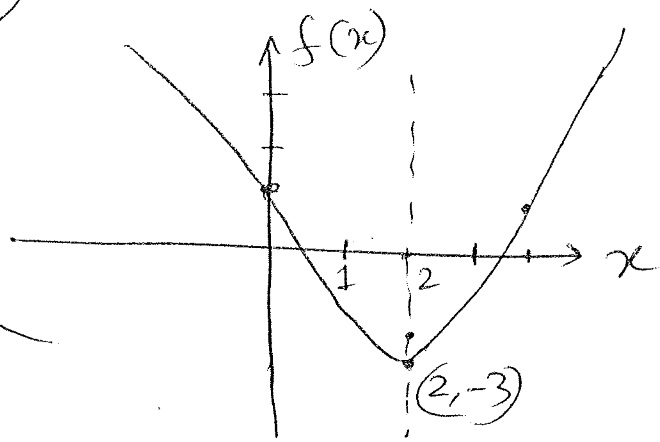
3. Sketch the graph of the following function and determine whether it is one-to-one. If not, what should be the condition to make it a one-to-one function. Consider it as a one-to-one function, find a formula for the inverse of the function. 10

$$f(x) = x^2 - 4x + 1$$

4

$f(x) = x^2 - 4x + 4 - 3$

$= (x-2)^2 - 3$



1 It is not a one-to-one function since the horizontal line test fails for the graph.

1 It will be a one-to-one function if $x \geq 2$

4

$y = (x-2)^2 - 3$

$\Rightarrow (x-2)^2 = y+3$

$\Rightarrow x-2 = \sqrt{y+3}$

$\Rightarrow x = 2\sqrt{y+3}$

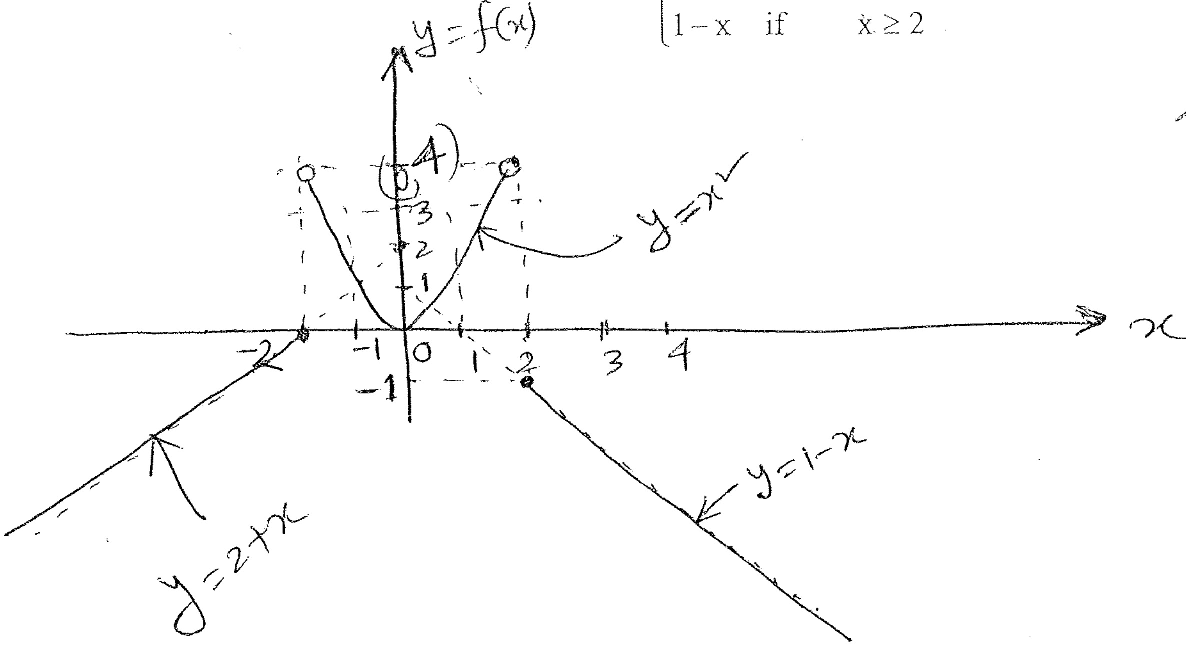
Interchanging x and y,

$y = 2\sqrt{x+3} = f^{-1}(x)$

4. Find $\lim_{x\to -2^-} f(x)$, $\lim_{x\to -2^+} f(x)$, $\lim_{x\to -2} f(x)$, $f(-2)$, $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$, and $f(2)$ and hence determine at which points the function is continous. If not continutous, is it it continuous from left, right or neither side? Explain why by using the definition of continuty.

10

$$f(x) = \begin{cases} 2+x & \text{if} \quad x \le -2 \\ x^2 & \text{if} \quad -2 < x < 2 \\ 1-x & \text{if} \quad x \ge 2 \end{cases}$$



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3

$\lim_{x\to -2^-} f(x) = 0$, $\lim_{x\to -2^+} f(x) = 4$, $\lim_{x\to -2} f(x)$ does not exis since, $\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$

$f(-2) = 0$, $\lim_{x\to 2^-} f(x) = 4$, $\lim_{x\to 2^+} f(x) = -1$

$f(2) = -1$

1

The function is continuous everywhere except at $x = -2, 2$.

2

But $f(x)$ is continuous from left at $x = -2$ since $\lim_{x\to -2^-} f(x) = f(-2)$

$f(x)$ is continuous from right at $x = +2$ since $\lim_{x\to 2^+} f(x) = f(2)$

5. Find the limit if it exists. If the limit does not exist explain why.

10

$$\lim_{x \to 1} \frac{\sqrt{8+x^2}-3}{x-1}$$

$x=1$, $\frac{0}{0}$, undefined form.

$$= \lim_{x\to 1} \frac{(\sqrt{8+x^2}-3)(\sqrt{8+x^2}+3)}{(x-1)(\sqrt{8+x^2}+3)}$$

$$= \lim_{x\to 1} \frac{8+x^2-9}{(x-1)(\sqrt{8+x^2}+3)}$$

$$= \lim_{x\to 1} \frac{x^2-1}{(x-1)(\sqrt{8+x^2}+3)}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(\sqrt{8+x^2}+3)}$$

$$= \lim_{x\to 1} \frac{x+1}{\sqrt{8+x^2}+3}$$

$$= \frac{1+1}{\sqrt{8+1}+3} = \frac{2}{3+3} = \frac{2}{6} = \frac{1}{3}$$

6. Find the following limits for the given function $f(x) = \dfrac{x^2 - 2x}{x^2 - 4x + 4}$. Does the function have any horizontal or vertical asymptotes? 10

$$\lim_{x \to \infty} f(x), \quad \lim_{x \to 2^-} f(x), \quad \lim_{x \to 2^+} f(x)$$

4

$$\lim_{x \to \infty} \frac{x^2 - 2x}{x^2 - 4x + 4} \; ; \; x = \infty, \; \frac{\infty}{\infty} \text{ form}, \text{ undefined}$$

$$= \lim_{x \to \infty} \frac{\frac{x^2 - 2x}{x^2}}{\frac{x^2 - 4x + 4}{x^2}} = \lim_{x \to \infty} \frac{1 - \frac{2}{x}}{1 - \frac{4}{x} + \frac{4}{x^2}} = \frac{1}{1} = 1$$

2

$$\lim_{x \to 2^-} \frac{x(x-2)}{(x-2)^2} = \lim_{x \to 2^-} \frac{x}{x-2} = -\infty$$

2

$$\lim_{x \to 2^+} \frac{x}{x-2} = +\infty$$

2

$\therefore$ $y = 1$ is the horizontal asymptote and

$x = 2$ is the vertical asymptote

7. Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $y = 2\sqrt{x}$. Then find the equation of the tangent line to the graph of $y = 2\sqrt{x}$ at point P(1, 2).

10

6

$$f'(x) = \lim_{h \to 0} \frac{2\sqrt{x+h} - 2\sqrt{x}}{h}$$

$$= 2 \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= 2 \lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$\therefore \frac{df}{dx} = \frac{2}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$$

1

$$\left.\frac{df}{dx}\right|_{x=1} = \frac{1}{\sqrt{1}} = 1$$

3

$\therefore$ The equation of tangent line,

$$y - 2 = 1(x-1)$$

$$\Rightarrow y = x + 1$$

8. Differentiate the following function

10

$f(x) = \sin^3\left(\dfrac{x-1}{2x^3-4x}\right)$ (Use chain rule and $\dfrac{d(u/v)}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$)

8

$$f'(x) = 3\sin^2\left(\frac{x-1}{2x^3-4x}\right)\cos\left(\frac{x-1}{2x^3-4x}\right) \times \frac{(2x^3-4x)\times 1 - (x-1)(6x^2-4)}{(2x^3-4x)^2}$$

$$= 3\sin^2\left(\frac{x-1}{2x^3-4x}\right)\cos\left(\frac{x-1}{2x^3-4x}\right) \times \frac{2x^3 - \cancel{4x} - 6x^3 + \cancel{4x} + 6x^2 - 4}{4x^2(x^2-2)^2}$$

2

$$= 3\times\frac{-4x^3+6x^2-4}{4x^2(x^2-2)^2}\sin^2\left(\frac{x-1}{2x^3-4x}\right)\cos\left(\frac{x-1}{2x^3-4x}\right)$$

$$= \frac{3(-2x^3+3x^2-2)}{2x^2(x^2-2)^2}\sin^2\left(\frac{x-1}{2x^3-4x}\right)\cos\left(\frac{x-1}{2x^3-4x}\right)$$

9. Differentiate the following function $y^x = \ln(\sqrt{x}e^{x^2})$ 10

(Hint: *ln (ab) = ln(a) + ln(b)* and, *ln (a^n^) = n ln(a)*)

$$\ln y^x = \ln\left[\ln(\sqrt{x}e^{x^2})\right]$$

$$\Rightarrow x\ln y = \ln\left[\ln\sqrt{x} + \ln e^{x^2}\right] = \ln\left[\frac{1}{2}\ln x + x^2\right]$$

Differentiating both sides w.r.t. x:

$$\ln y + x\frac{y'}{y} = \frac{\frac{1}{2}\cdot\frac{1}{x} + 2x}{\frac{1}{2}\ln x + x^2} = \frac{\frac{1}{2x} + 2x}{\frac{1}{2}\ln x + x^2}$$

$$\frac{x}{y}\cdot y' = \frac{\frac{1}{2x} + 2x}{\frac{1}{2}\ln x + x^2} - \ln y$$

$$\Rightarrow y' = \frac{y}{x}\left[\frac{\frac{1}{2x} + 2x}{\frac{1}{2}\ln x + x^2} - \ln y\right]$$

10. Use implicit differentiation to find $\frac{dy}{dx}$ from the following equation. 10

$$y^3 - 4xy^2 + 2y - 10 = 0$$

Differentiating both sides w.r. to x,

$$3y^2 \frac{dy}{dx} - 4y^2 - 4x \times 2y \frac{dy}{dx} + 2\frac{dy}{dx} - 0 = 0$$

$$\Rightarrow \frac{dy}{dx}\left(3y^2 - 8xy + 2\right) = 4y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{4y^2}{3y^2 - 8xy + 2}$$