TOPIC: STATISTICAL QUALITY CONTROL (SQC)

Q 1: A certain molded part is examined for conformity in diameter, and for the presence of excessive flashing. Equal-sized samples are taken throughout the eight-hour working day. For each sample, the mean and the range of the diameters are recorded, and a notion is made if any of the sampled parts has excessive flashing. The results for 14 samples are shown below.

Sample	Mean	Range	Sample	Mean	Range	Sample	Mean	Range
1	1.975	0.2	6	2	0.3	11	2.05	0.1
2	2.025	0.3	7	1.925	0.2	12	1.975	0.1
3	1.9	0.3	8	1.975	0.2	13	1.975	0.3
4	1.95	0.1	9	2	0.2	14	2.025	0.2
5	2.1	0.2	10	2.025	0.2			

- The sum of the mean and range of the 14 samples shown above are 27.9 and 2.9 respectively.
- The fifteen sample is: {2.00, 2.00, 1.90, 2.10, 2.40, 2.10, 1.95, 2.00}

Construct the appropriate process control chart(s) for the above process using all the information given. Explain if the process appears to be in control?

Solution:

- The appropriate process control charts are $\bar{x}$-chart and R-chart as they deal with the mean and the range of the samples.
- Note that, we have the information about 15 samples and hence we have to use all the 15 samples for calculating the control limits.
- Sample size = n = 8 (see sample 15 above)

The mean, $\bar{x}$ for the 15th sample is 2.0563 and the range R is 2.40-1.90 = 0.50.

$$\text{Therefore, } \bar{\bar{x}} = \frac{27.90 + 2.05625}{15} = 1.9971 \quad \text{and } \bar{R} = \frac{2.90 + 0.05}{15} = 0.2267$$

- Control Limits for the $\bar{x}$-chart can be computed using $UCL = \bar{\bar{x}} + A_2\bar{R}$ and $LCL = \bar{\bar{x}} - A_2\bar{R}$ as follows: (A_2 = 0.37 for n=8)

  Upper Control Limit, $UCL = \bar{\bar{x}} + A_2\bar{R} = 1.9971 + 0.37(0.2267) = 2.0810$

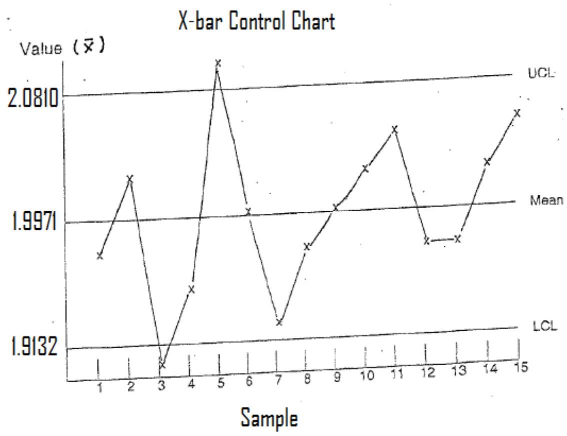
  Lower Control Limit, $LCL = \bar{\bar{x}} - A_2\bar{R} = 1.9971 - 0.37(0.2267) = 1.9132$

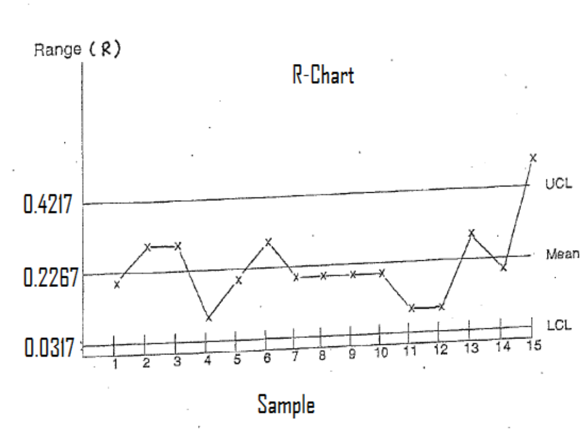
- Control Limits for R-charts can be computed using $UCL = D_4\bar{R}$, and $LCL = D_3\bar{R}$ as follows:

  Upper Control Limit, $UCL = D_4\bar{R} = 1.86(0.2267) = 0.4217$ (D_4 = 1.86 for n=8)

  Lower Control Limit, $LCL = D_3\bar{R} = 0.14(0.2267) = 0.0317$ (D_3 = 0.14 for n=8)

Both control charts are shown on the following page.





From the above two charts, we see that the process is OUT of control. Samples 3 and 5 for the $\bar{x}$ chart and sample 15 for R-chart are out of control for the control limits. The causes must be investigated.

Q 2: The following table lists the number of defective 40-watt light bulbs found in samples of 100 light bulbs selected over 25 days from a manufacturing process.

Day	Defective	Day	Defective	Day	Defective	Day	Defective	Day	Defective
1	3	6	4	11	2	16	2	21	2
2	2	7	4	12	4	17	3	22	2
3	5	8	5	13	4	18	1	23	3
4	8	9	6	14	4	19	4	24	5
5	3	10	1	15	0	20	0	25	3

(a) Construct an appropriate chart to monitor the process and plot the data using the 3-sigma limits. Comment on the process.
(b) How large should the number of defectives be in a sample selected from the manufacturing process before the process is assumed to be out of control?

Solution:

(a) Number of samples = 25 and sample size n = 100

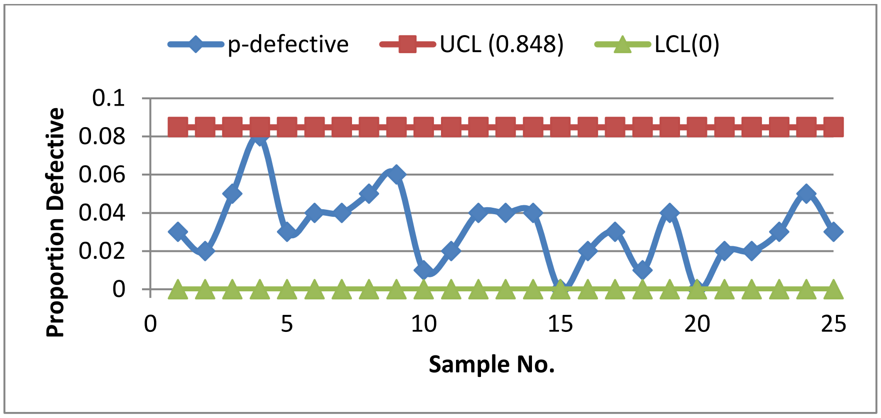
Proportion defective in the first sample = 3/100 = 0.03
Proportion defective in the second sample = 2/100 = 0.02
… …
Proportion defective in the 25th sample = 3/100 = 0.03

- Average proportion defective for the group of 13 samples = $\bar{p} = \dfrac{80}{25*100} = .032$
- The sample standard deviation can be computed as $\sigma_p = \sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\dfrac{0.032(1-0.032)}{100}} = 0.0176$
- The control limits can be computed as follows:

$$UCL = \bar{p} + z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.032 + 3\sqrt{\frac{0.032(1-0.032)}{100}} = 0.0848$$

$$LCL = \bar{p} - z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.032 - 3\sqrt{\frac{\overline{0.032}(1-\overline{0.032})}{100}} = 0\ (cannot\ be - ve)$$

An UCL of 0.0848 indicates that the number of defectives is 8.48 in a sample size of 100 bulbs. Therefore, the process seems to be in control although sample 8 plots close to the UCL and may need further investigation.



(b) For the process to be out of control, the number of defective should be 9 or more in a sample size of 100.

Q 3: The accounts receivable department at Rick Wang Manufacturing has been having difficulty getting customers to pay the full amount of their bills. Many customers complain that the bills are not correct and do not reflect the materials that arrived to the receiving docks. The department has decided to implement SQC techniques to its billing process. To setup the appropriate control charts, 10 samples of 50 bills each were taken over a month's time and the items on the bills were checked against the bill of lading sent by the company's shipping department to determine the number of bills that were not correct. The results are as given in the table below. Determine the control limits for the appropriate chart using a 99.73% confidence level and construct the control charts. Is the process in control? If not, which sample(s) were out of control?

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of Incorrect Bills	6	5	11	4	0	5	3	4	7	2

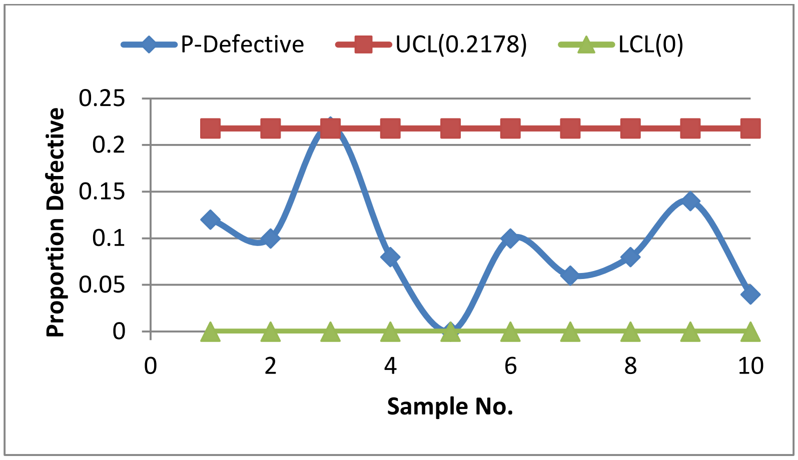
Solution:

- Average proportion defective = $\bar{p} = \frac{Total\ Number\ of\ Incorrect\ Bills}{Total\ Number\ of\ Records\ Examined} = \frac{47}{10*50} = 0.094$
- The control limits can be computed as follows:

$$UCL = \bar{p} + z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.094 + 3\sqrt{\frac{0.094(1-0.094)}{50}} = 0.21781$$

$$LCL = \bar{p} - z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.094 - 3\sqrt{\frac{0.094(1-0.094)}{50}} = 0\ (cannot\ be - ve)$$

- UCL corresponds to 10.89 incorrect bills per sample (i.e. 0.21781*50). Sample 3 is above the upper control limit. Hence, the process may not be in control and the cause must be investigated.



Q 4: An advertising agency tracks the complaints, by week received, about the billboards in its city:

Week	1	2	3	4	5	6
No. of Complaints	4	5	4	11	3	9

(a) What type of control chart would you use to monitor this process and why?
(b) What are the 3-sigma control limits for this process? Assume that the historical complaint rate is unknown. Is the process mean in control, according to the control limits?
(c) Assume that the historical complaint rate has been 4 calls a week. Is the process mean in control, according to the control limits?

Solution:

(a) The appropriate control chart here is c-chart. The goal here is to control the number of complaints per day. It is counting the number of occurrences per week.

(b) **If the process mean is unknown**, then using the information in the table, we get
Mean = $\bar{c}$ = 36/6 = 6 complaints per week.
Sample Standard deviation = $\sqrt{\bar{c}} = \sqrt{6} = 2.45$
The control limits can be computed as follows:

$$UCL \;= 6 + 3(2.45) = 13.35$$

$$LCL \;= 6 - 3(2.45) \Rightarrow 0$$

The process is in control.

(c) **If the process mean is known to be 4** complaints per week, then
Standard deviation = $\sqrt{\bar{c}} = \sqrt{4} = 2$
The control limits can be computed as follows:

$$UCL = 4 + 3(2.00) = 10.00$$

$$LCL \;= 4 - 3(2.00) \Rightarrow 0$$

The process is out of control (see sample 4).

Q 5: As a part of an insurance company's training program, participants learn how to conduct a fast but an effective analysis of client's insurability. The goal is to have participants achieve a time less than 45 minutes. There is no minimum time, but the quality of assessment should be acceptable. Test results for three participants were, Jerry, a mean of 37 minutes and a standard deviation of 2.5 minutes; Armand, a mean of 39 minutes and a standard deviation of 3 minutes; and Lau, a mean of 37.5 and standard deviation of 2.5 minutes. Which of the participants would you judge to be capable and why?

Solution:

Given: Upper Specification = 45 minutes, Lower Specification = 0.

We evaluate the process capability index of the three participants as follows:

- $Jerry\text{: } C_{pk} = min\left[\frac{\bar{\bar{x}}-Lower\ Spec.}{3\sigma}, \frac{Upper\ Spec.-\bar{\bar{x}}}{3\sigma}\right] = min\left[\frac{37-0}{3*2.5}, \frac{45-37}{3*2.5}\right] = 1.066$
- $Armand\text{: } C_{pk} = min\left[\frac{\bar{\bar{x}}-Lower\ Spec.}{3\sigma}, \frac{Upper\ Spec.-\bar{\bar{x}}}{3\sigma}\right] = min\left[\frac{39-0}{3*3}, \frac{45-39}{3*3}\right] = 0.066$
- $Lau\text{: } C_{pk} = min\left[\frac{\bar{\bar{x}}-Lower\ Spec.}{3\sigma}, \frac{Upper\ Spec.-\bar{\bar{x}}}{3\sigma}\right] = min\left[\frac{37.5-0}{3*2.5}, \frac{45-37.5}{3*2.5}\right] = 1$

Lau and Jerry have a process capability of 1 and 1.006, and hence they are capable of meeting design specifications. However, Armand is not capable.

Q 6: In a refinery, the octane rating of gasoline produced is measured by taking one observation from each batch. Twenty observations are shown below.

Observation Number	Octane rating	Observation number	Octane rating
1	89.2	11	85.4
2	86.5	12	91.6
3	88.4	13	87.7
4	91.8	14	85.0
5	90.3	15	91.5
6	87.5	16	90.3
7	92.6	17	85.6
8	87.0	18	90.9
9	89.8	19	82.1
10	92.2	20	85.8

(a) Construct appropriate control charts using three sigma control limits. Is the process in control?

(b) A new batch has octane rating of 94.0. Using the control charts in part (a), is the process still in control?

Solution:

(a) The appropriate control charts are the individual unit and the moving range charts.

The calculations are shown on the table in the following page.

For the individual unit or x chart:

$$UCL_x = \overline{X} + z\sigma = 88.56 + 3(2.91) = 97.29$$

$$LCL_x = \overline{X} - z\sigma = 88.56 - 3(2.91) = 79.83$$

For the moving range chart:

$$UCL_{MR} = D_4\overline{R} = 3.27(4.11) = 13.42$$
$$LCL_{MR} = D_3\overline{R} = 0(4.11) = 0$$

Yes, the process is in control because all individual observations and moving ranges fall in their respective control limits.

(b) The new moving range = |94.0 – 85.8| = 8.2.
Yes, the process is still in control because 79.83 ≤ 94 ≤ 97.29 and 0 ≤ 8.2 ≤ 13.42.

Obs. #	Octane Rating	Moving Range
1	89.20	
2	86.50	2.70
3	88.40	1.90
4	91.80	3.40
5	90.30	1.50
6	87.50	2.80
7	92.60	5.10
8	87.00	5.60
9	89.80	2.80
10	92.20	2.40
11	85.40	6.80
12	91.60	6.20
13	87.70	3.90
14	85.00	2.70
15	91.50	6.50
16	90.30	1.20
17	85.60	4.70
18	90.90	5.30
19	82.10	8.80
20	85.80	3.70
avg=	**88.56**	**4.11**
std dev=	**2.91**	