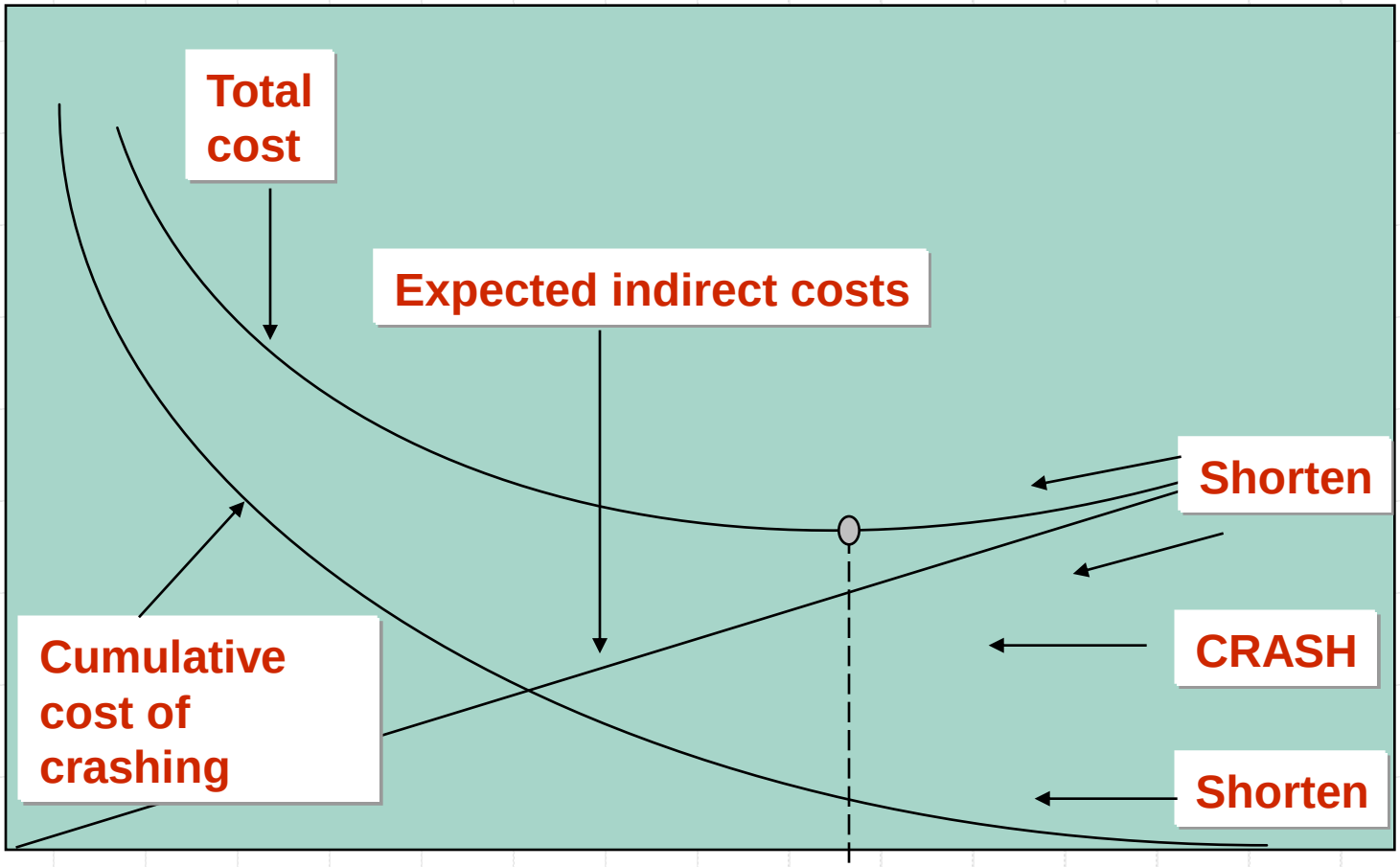
COMM 225: Production/Operations Management

Fall 2013

Project Crashing

Time-Cost Trade-Offs: Crashing

- Project Crashing is reducing the total project time (i.e., duration) while minimizing the cost to do so



Project Crashing

- Crashing
  - reducing project time by expending additional resources
- Crash time
  - the reduced duration for performing an activity (achieved through the use of additional resources)
- Crash cost
  - cost of reducing activity time
- Goal
  - reduce project duration at minimum cost (until a certain stopping criterion is met)

Crashing

- General procedure:
  - Obtain estimates of **regular and crash durations** and **direct and indirect costs per period** for each activity
  - Determine the lengths of all paths
  - Determine which are the critical paths(s)
  - Crash critical activities one period at a time, starting from the cheapest, as long as crashing cost per period does not exceed the benefits (or as long as a certain stopping criterion is met)
  - Recalculate the critical path after every crashing step
  - If there is more than one critical path, crash both simultaneously by either crashing the cheapest activity on each path or by crashing the cheapest common activity, depending on whichever option is cheaper.

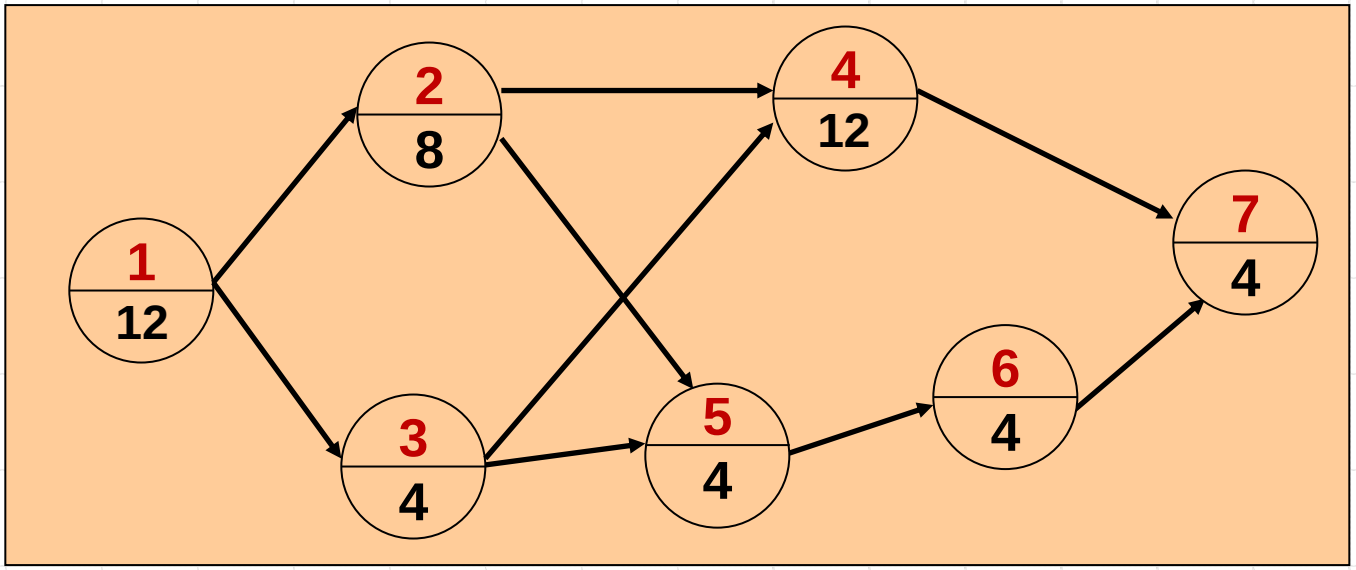
Crashing cost per unit time

- In some instances, the normal (regular) cost and the crash cost for each activity may be given.
- It is important to note that, when deciding on which critical activity to crash, we pick the activity with the least expensive crash cost per unit time, or the marginal cost.
- The marginal cost, is calculated as follows:

$$\text{Marginal Cost} = \frac{\Delta Cost}{\Delta Time} = \frac{(\text{Crash Cost} - \text{Normal Cost})}{(\text{Normal Time} - \text{Crash Time})}$$

Example 1 – Building a House

Consider the following project network involving 7 activities for constructing a house.



Example 1 – Building a House

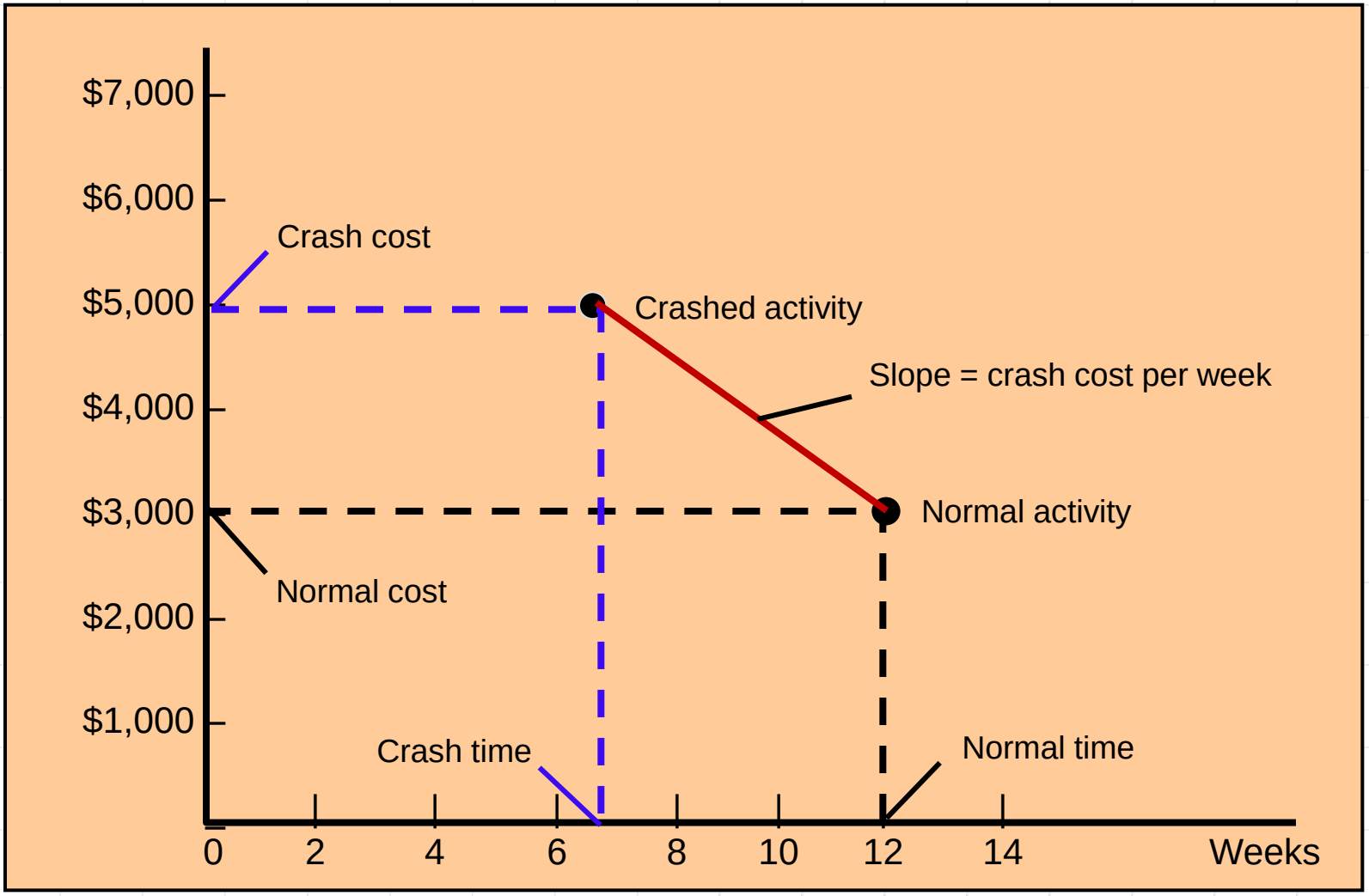
Relevant information on the normal and crash time and cost are given in the table.

ACTIVITY	NORMAL TIME (WEEKS)	CRASH TIME (WEEKS)	NORMAL COST	CRASH COST
1	12	7	$3,000	$5,000
2	8	5	2,000	3,500
3	4	3	4,000	7,000
4	12	9	50,000	71,000
5	4	1	500	1,100
6	4	1	500	1,100
7	4	3	15,000	22,000
			$75,000	$110,700

IMPORTANT:

We need to calculate crash cost per week since it is not directly given in the table

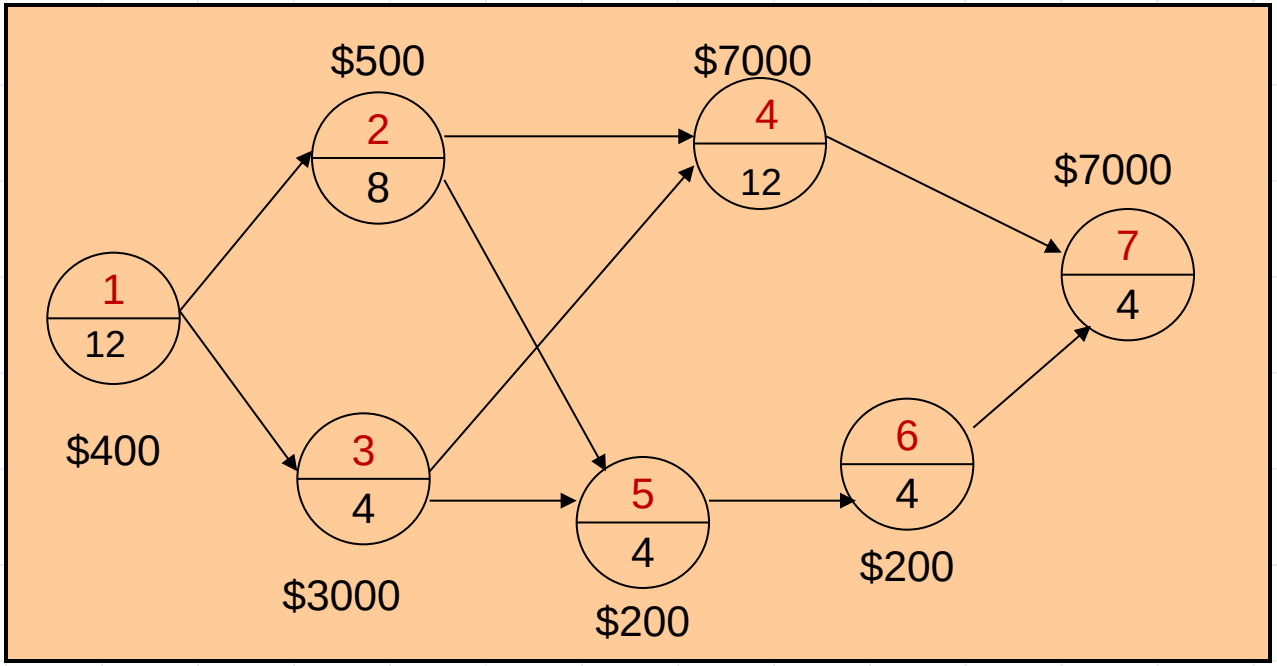
Example 1 – Calculating crash cost per week for activity 1



Project Crashing

ACTIVITY	NORMAL TIME (WEEKS)	CRASH TIME (WEEKS)	NORMAL COST	CRASH COST	TOTAL ALLOWABLE CRASH TIME (WEEKS)	CRASH COST PER WEEK
1	12	7	$3,000	$5,000	5	$400
2	8	5	2,000	3,500	3	500
3	4	3	4,000	7,000	1	3,000
4	12	9	50,000	71,000	3	7,000
5	4	1	500	1,100	3	200
6	4	1	500	1,100	3	200
7	4	3	15,000	22,000	1	7,000
			$75,000	$110,700		

The home builder needs to have the house completed in 30 weeks. Which activities to crash and what is the associated extra cost?

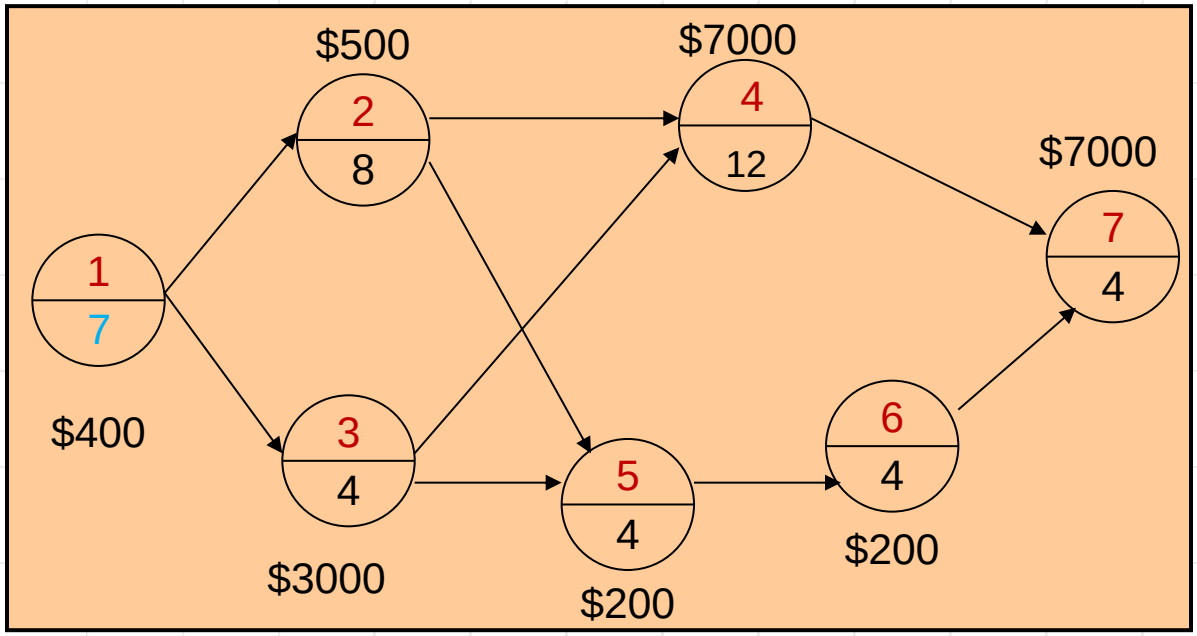


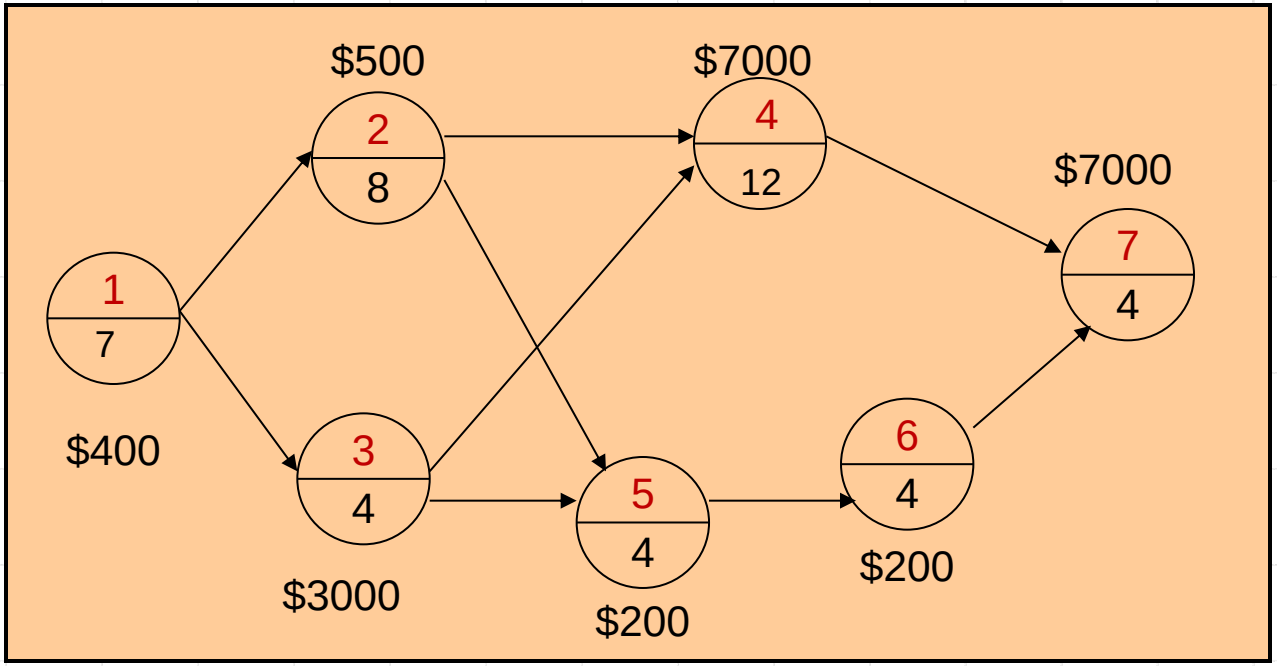
FROM …

Project Duration:
36 weeks

TO…

Project Duration:
31 weeks
Additional Cost:
$2000





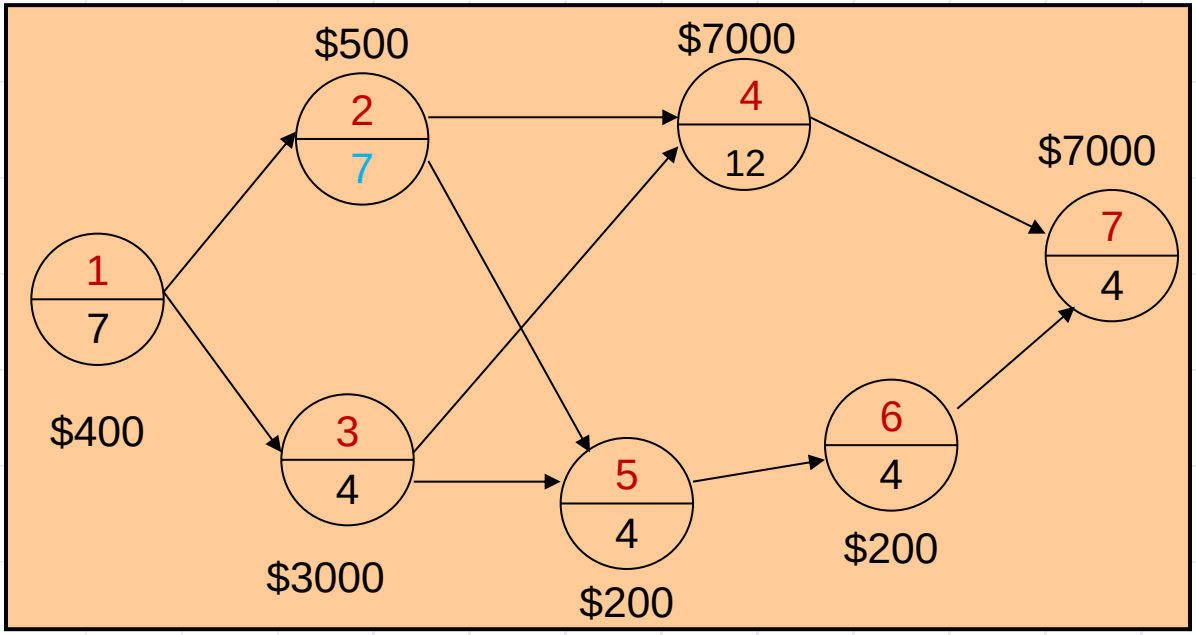
FROM …

Project Duration:
31 weeks

TO…

Project Duration:
30 weeks
Additional Cost:
$500

Total crashing cost:
$2500



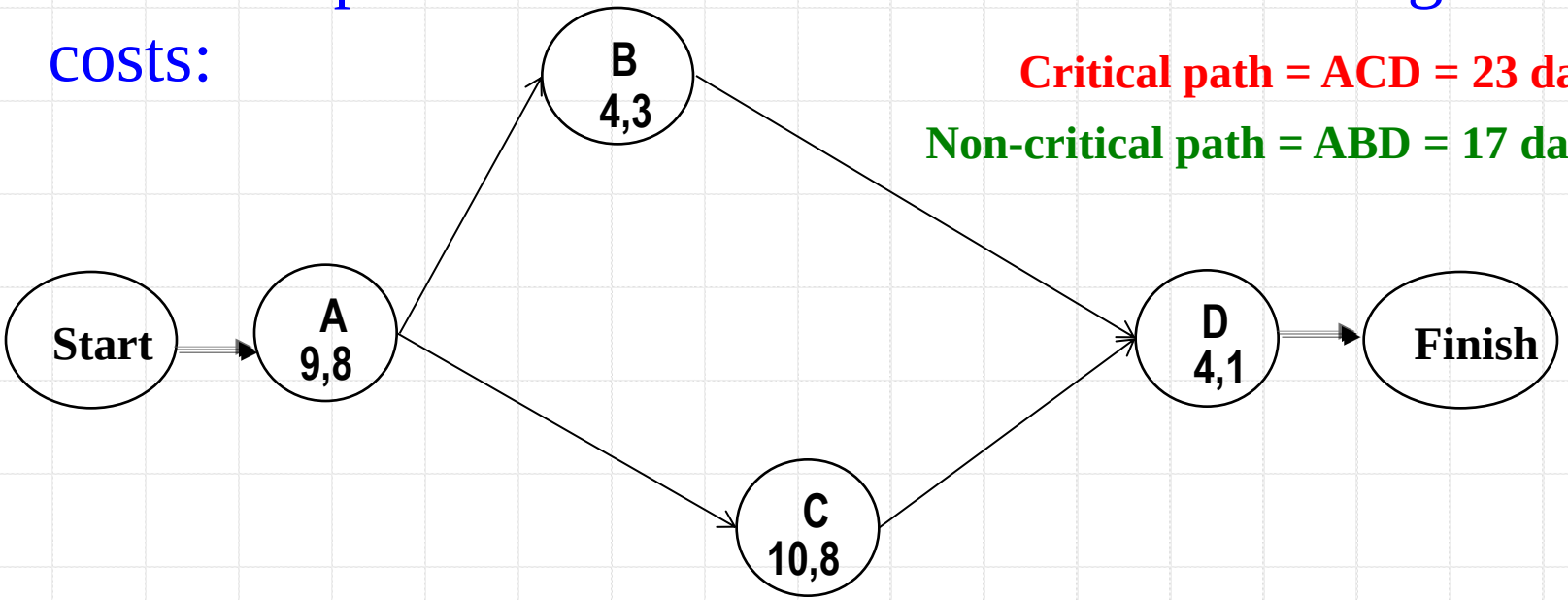
Crashing Example - 2

- Crash the following project:

		Time (days)		Cost (Dollars)	
Task	Predecessor	Normal	Crash	Normal	Crash
A	None	9	8	$ 9,000	$ 13,000
B	A	4	3	$ 4,000	$ 5,000
C	A	10	8	$ 4,000	$ 7,000
D	C,B	4	1	$ 4,000	$ 5,500

Crashing Example - 2

- Draw the precedence network and find the marginal costs:



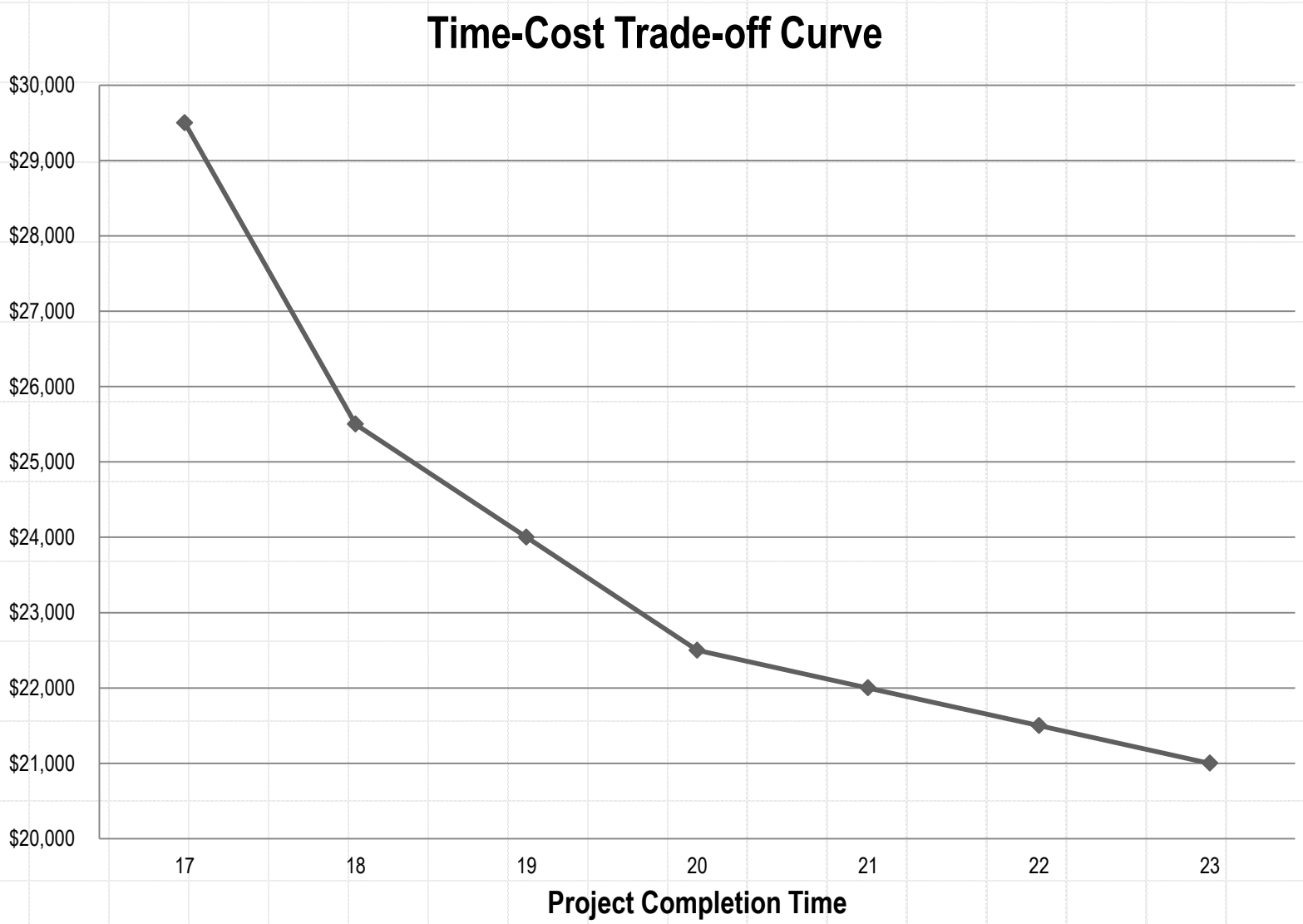
Critical path = ACD = 23 days

Non-critical path = ABD = 17 days

Task	Marginal Cost	# Days Task May be Shortened
A	$ 4,000	1
B	$ 1,000	1
C	$ 1,500	2
D	$ 500	3

Crashing Example - 2

Current Critical Path	Remaining # Days Task May be shortened	Cost per Day to Expedite Task	Least Cost task to Expedite	Total Cost of All tasks	Project Completion Time
A-C-D	All Activity times and costs are normal			$ 21,000	23
A-C-D	A-1, C-2, D-3	A-4000, C-1500, D-500	D	$ 21,500	22
A-C-D	A-1, C-2, D-2	A-4000, C-1500, D-500	D	$ 22,000	21
A-C-D	A-1, C-2, D-1	A-4000, C-1500, D-500	D	$ 22,500	20
A-C-D	A-1, C-2	A-4000, C-1500	C	$ 24,000	19
A-C-D	A-1, C-1	A-4000, C-1500	C	$ 25,500	18
A-C-D	A-1	A-4000	A	$ 29,500	17
A-C-D, A-B-D	B-1			$ 29,500	17


Time-Cost Trade-off Curve
$30,000
$29,000
$28,000
$27,000
$26,000
$25,000
$24,000
$23,000
$22,000
$21,000
$20,000
17
18
19
20
21
22
23
Project Completion Time

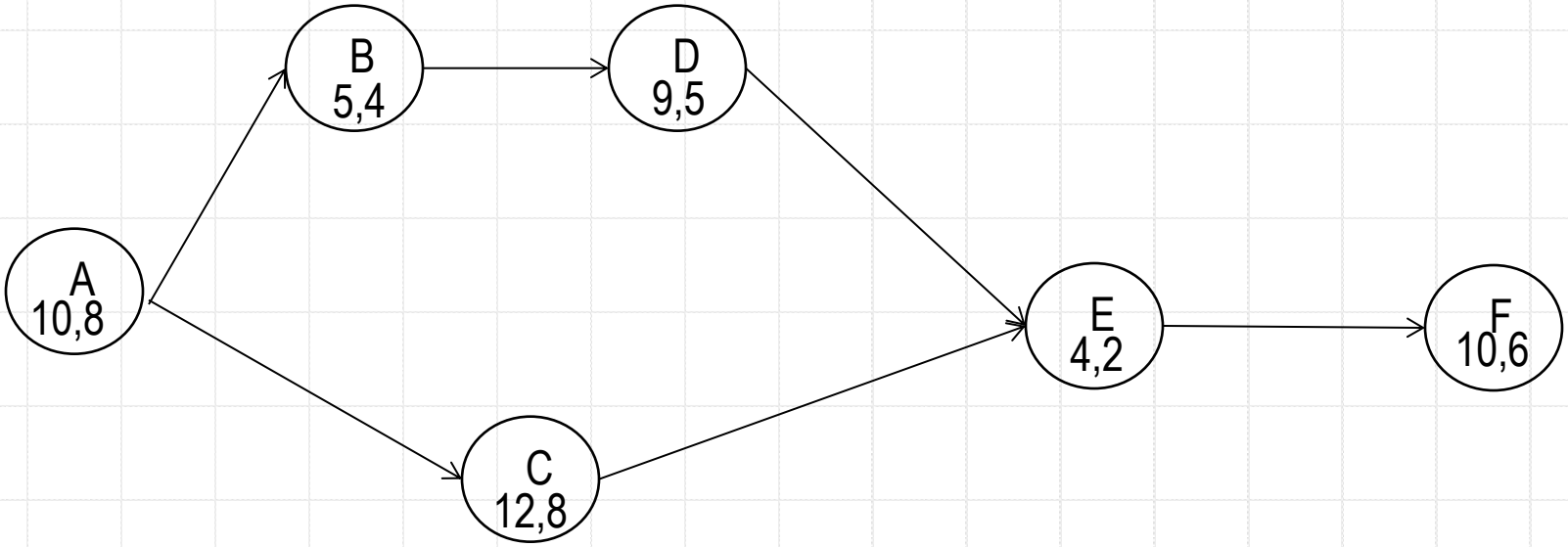
Example – 3 (Residential Home)

- Crash the following project the maximum amount possible.

			Time (days)		Cost (Dollars)	
Task Description	**Task**	**Predecessor**	**Normal**	**Crash**	**Normal**	**Crash**
Framing	A	None	10	8	$ 10,000	$ 15,000
Windows and Doors	B	A	5	4	$ 5,000	$ 6,000
Roof	C	A	12	8	$ 5,000	$ 7,000
Trades	D	B	9	5	$ 9,000	$ 15,000
Insulation	E	D,C	4	2	$ 4,000	$ 6,000
Drywall	F	E	10	6	$ 7,000	$ 10,000

Example – 3 (Residential Home)

- The critical path is A-B-D-E-F (Duration = 38)
- The duration of the other path is 36 days



Paths	Remaining # Days Task May be shortened	Cost per Day to Expedite Task	Least Cost task to Expedite	Total Cost of All tasks	Project Completion Time
A-B-D-E-F (38)* A-C-E-F (36)	All Activity times and costs are normal			$ 40,000	38
A-B-D-E-F (38)* A-C-E-F (36)	A-2, B-1, D-4, E-2, F-4	A-2500, B-1000, D-1500, E-1000, F-750	F	$ 40,750	37
A-B-D-E-F (37)* A-C-E-F (35)	A-2, B-1, D-4, E-2, F-3	A-2500, B-1000, D-1500, E-1000, F-750	F	$ 41,500	36
A-B-D-E-F (36)* A-C-E-F (34)	A-2, B-1, D-4, E-2, F-2	A-2500, B-1000, D-1500, E-1000, F-750	F	$ 42,250	35
A-B-D-E-F (35)* A-C-E-F (33)	A-2, B-1, D-4, E-2, F-1	A-2500, B-1000, D-1500, E-1000, F-750	F	$ 43,000	34
A-B-D-E-F (34)* A-C-E-F (32)	A-2, B-1, D-4, E-2	A-2500, B-1000, D-1500, E-1000	E	$ 44,000	33
A-B-D-E-F (33)* A-C-E-F (31)	A-2, B-1, D-4, E-1	A-2500, B-1000, D-1500, E-1000	E	$ 45,000	32
A-B-D-E-F (32)* A-C-E-F (30)	A-2, B-1, D-4	A-2500, B-1000, D-1500	B	$ 46,000	31
A-B-D-E-F (31)* A-C-E-F (30)	A-2, D-4	A-2500, D-1500	D	$ 47,500	30
A-B-D-E-F (30)* **A-C-E-F (30)***					

Note: At this point, note that the duration of the other path (A-C-E-F) has decreased to 30. Hence there are two critical paths.

Let us continue…

Paths	Remaining # Days Task May be shortened	Cost per Day to Expedite Task	Least Cost task to Expedite	Total Cost of All tasks	Project Completion Time
A-B-D-E-F (30)* A-C-E-F (30)*	A-2, C-4, D-3	A-2500, C-500, D-1500	**C and D both	$ 49,500	29

** Note: At this point, we have two options: (1) To choose task A, and reduce the project completion time (using both the paths) to 29, and increase the total cost by $2500 or (2) To choose task C and task D, and reduce the project completion time to 29, and increase the total cost by $500+$1500 = $2000. We choose option (2) as it costs less.

A-B-D-E-F (29)* A-C-E-F (29)*	A-2, **C-3, D-2**	A-2500, C-500, D-1500	C,D	$ 51,500	28

We again choose to crash task C & task D as it costs less.

Current Critical Path	Remaining # Days Task May be shortened	Cost per Day to Expedite Task	Least Cost task to Expedite	Total Cost of All tasks	Project Completion Time
A-B-D-E-F (28)* A-C-E-F (28)*	A-2, **C-2, D-1**	A-2500, C-500, D-1500	C,D	$ 53,500	27
A-B-D-E-F (27)* A-C-E-F (27)*	A-2, C-1	A-2500, C-500	A	$ 56,000	26
A-B-D-E-F (26)* A-C-E-F (26)*	A-1, C-1	A-2500	A	$ 58,500	25
A-B-D-E-F (25)* A-C-E-F (25)*	C-1	None	None		