UNIVERSITY OF WINDSOR
DEPARTMENT OF MATHEMATICS AND STATISTICS

03-62-139/140 Differential Calculus
Midterm Exam 1
Saturday, October 22, 2016, 9:00-10:20 AM

Last name (PRINT): SOLUTION

First name: _____

Student No.: _____

Section No.: _____

Instructions :

- This test has 9 problems and a total of 7 pages, including this cover page. You have 80 minutes.
- Read carefully and answer ALL questions. Show ALL your work to receive FULL credit.
- You must give **exact** answers (and not decimal approximations).
- Answer each question in the space provided. If you need more space you can continue on the backside of the page. No paper, other than this exam package, is allowed.
- Only non programmable and non graphic calculators are permitted.
- No cell phones or any other electronic devices are allowed. Headphones may not be worn.

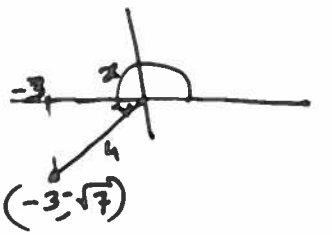
(5) 1. If $\cos x = -\frac{3}{4}$ and $\cot y = \frac{4}{3}$, where x and y lie between π and 2π, evaluate $\sin(x+y)$

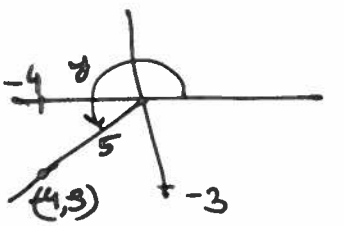
$\sin(x+y)$

$= \sin x \cos y + \cos x \sin y$

$= -\frac{\sqrt{7}}{4}\cdot\frac{-4}{5} + \frac{-3}{4}\cdot\frac{-3}{5}$

$= \frac{+4\sqrt{7}+9}{20}$





(5) 2. Find all values of x in the interval $[0, \pi]$ that satisfy the inequality

$$2\sin^2 x - 1 < 0$$

$2\sin^2 x - 1 = 0 \Rightarrow \sin x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

Interval	Test point	$2\sin^2 x - 1 < 0$
$(0, \frac{\pi}{4})$	$\pi/6$	T
$(\frac{\pi}{4}, \frac{3\pi}{4})$	$\pi/2$	F
$(\frac{3\pi}{4}, \pi)$	$\frac{5\pi}{6}$	T

Test point	$2\sin^2 x - 1 < 0$
0	T
$\pi/4$	F
$3\pi/4$	F
π	T

$x \in [0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \pi]$

(6) 3. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation

$$7\cos^2 x + 5\sin^2 x = 11\cos x$$

$7\cos^2 x + 5(1-\cos^2 x) = 11\cos x$

$2\cos^2 x - 11\cos x + 5 = 0$

$(2\cos x - 1)(\cos x - 5) = 0$

$\cos x = \frac{1}{2}$ or $\cos x = 5$

But $\cos x \neq 5$.

If $\cos x = \frac{1}{2}$ then

$x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$= \frac{\pi}{3}, \frac{5\pi}{3}$

(3) 4. Prove the identity: $\dfrac{\cos 2x}{\sin x} = \csc x - 2\sin x$

$L.H.S. = \dfrac{\cos 2x}{\sin x}$

$= \dfrac{1 - 2\sin^2 x}{\sin x}$

$= \dfrac{1}{\sin x} - \dfrac{2\sin^2 x}{\sin x}$

$= \csc x - 2\sin x$

$= R.H.S.$

(6) 5. Find a formula for the inverse of the given function. Also find the domain and range of f and f^{-1}.

$$f(x) = \frac{-4x+3}{8x+5}$$

Let $y = \frac{-4x+3}{8x+5}$

$8xy + 5y = -4x + 3$

$8xy + 4x = 3 - 5y$

$(8y+4)x = 3 - 5y$

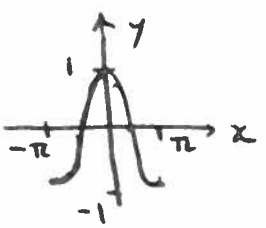
$x = \frac{3-5y}{8y+4}$

$x \leftrightarrow y$: $y = \frac{3-5x}{8x+4}$

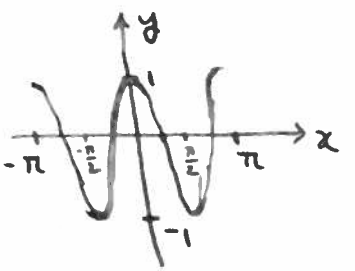
$$f^{-1}(x) = \frac{3-5x}{8x+4}$$

	Domain	Range
f	$\mathbb{R} \setminus \{-\frac{5}{8}\}$	$\mathbb{R} \setminus \{-\frac{1}{2}\}$
f^{-1}	$\mathbb{R} \setminus \{-\frac{1}{2}\}$	$\mathbb{R} \setminus \{-\frac{5}{8}\}$

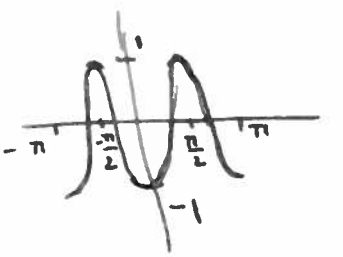
(4) 6. Graph the function $y = 4 - \cos 2x$, where $x \in [-\pi, \pi]$ not by plotting points, but by starting with the graph of one of the standard functions and applying the appropriate transformations.



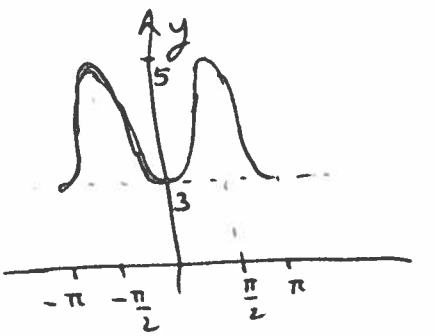
$y = \cos x$



$y = \cos 2x$



$y = -\cos 2x$



$y = 4 - \cos 2x$

(5) 7. Solve for x: $\log_3(x-2) + \log_3(x+6) = 2$

$\log_3 (x-2)(x+6) = 2$

$(x-2)(x+6) = 3^2$

$x^2 + 4x - 12 - 9 = 0$

$x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0$

$x = -7, 3$

But $x \neq -7$

$\boxed{x=3}$ is the only solution.

(13) 8. Evaluate the following limits, if they exist.

(a) $\lim_{x\to 6} \dfrac{x^2-5x-6}{x^2-2x-24}$

$= \lim_{x\to 6} \dfrac{(x-6)(x+1)}{(x-6)(x+4)}$

$= \lim_{x\to 6} \dfrac{x+1}{x+4} = \dfrac{6+1}{6+4} = \dfrac{7}{10}$

(b) $\lim_{x\to 2} \dfrac{x^2-2x-3}{\sqrt{x^2+12}+\cos(9\pi x)}$

$= \dfrac{2^2-2\cdot 2-3}{\sqrt{2^2+12}+\cos(9\pi\cdot 2)} = \dfrac{-3}{4+1} = -\dfrac{3}{5}$

(c) $\lim_{x\to 8} \frac{|x-8|}{x-8}$

$$\lim_{x\to 8^-} \frac{|x-8|}{x-8} = \lim_{x\to 8^-} \frac{-(x-8)}{x-8} = -1$$

$$\lim_{x\to 8^+} \frac{|x-8|}{x-8} = \lim_{x\to 8^+} \frac{x-8}{x-8} = 1$$

$$\lim_{x\to 8} \frac{|x-8|}{x-8} \quad \text{does not exist.}$$

(d) $\lim_{x\to 3} \frac{x^2-9}{\sqrt{3x+1}-\sqrt{2x+4}}$

$$= \lim_{x\to 3} \frac{x^2-9}{\sqrt{3x+1}-\sqrt{2x+4}} \cdot \frac{\sqrt{3x+1}+\sqrt{2x+4}}{\sqrt{3x+1}+\sqrt{2x+4}}$$

$$= \lim_{x\to 3} \frac{(x^2-9)(\sqrt{3x+1}+\sqrt{2x+4})}{3x+1-2x-4}$$

$$= \lim_{x\to 3} \frac{(x^2-9)(\sqrt{3x+1}+\sqrt{2x+4})(x+3)}{x-3}$$

$$= \lim_{x\to 3} (\sqrt{3x+1}+\sqrt{2x+4})(x+3)$$

$$= (\sqrt{3\cdot 9+1}+\sqrt{2\cdot 3+4})(6) = 12\sqrt{10}$$

(8) 9. For each of the following, circle either TRUE or FALSE.

(a) (T/F) The function $f(x) = x^3 + 5$ is an odd function.

(b) (T/F) The function $f(x) = x^2 - 4$ is a one to one function.

(c) (T/F) $f(x) = \left(\frac{1}{3}\right)^x$ is a decreasing function.

(d) (T/F) If $f(x) = \sqrt{2+x}$, and $g(x) = x^3$, then $(f \circ g)(x) = \sqrt{2+x^3}$.

(e) (T/F) The function $f(x) = \cos x$ is a periodic function on $\mathbb{R}$.

(f) (T/F) The domain of a function is always equal to the domain of its inverse.

(g) (T/F) The domain of the function $f(x) = \tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(h) (T/F) If both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then $\lim_{x \to a} \frac{f(x)}{g(x)}$ necessarily exists.

Question	Points	Score
1	5	
2	5	
3	6	
4	3	
5	6	
6	4	
7	5	
8	13	
9	8	
Total:	55	