

Question (1)

The range of signed integers N expressed in 2's complement representation that can be stored in a 10-bit register is:

- (a) $-1024 \leq N \leq +1023$ (b) $-1023 \leq N \leq +1024$
(c) $-512 \leq N \leq +511$ (d) $-511 \leq N \leq +512$
(e) None of the above

SOLUTION

1-bit for the Sign and 9 bits represents the number, so
 $-2^9 \leq N \leq 2^9 - 1$

(c) $-512 \leq N \leq +511$

Question (2)

Identify the decimal number which is represented next in floating point with the IEEE 754 standard:

11000010100010101100000000000000

- (a) $(- 133.375)_{10}$ (b) $(- 69.375)_{10}$
(c) $(- 138.750)_{10}$ (d) $(- 34.6875)_{10}$
(e) $(- 8.671875)_{10}$

SOLUTION

1 10000101 000101011000000000000000
S E M

Question (2) (Cont.)

SOLUTION

<u>1</u>	<u>10000101</u>	<u>000101011000000000000000</u>
S	E	M

✓ Find "real" exponent, n

$$\begin{aligned}n &= E - 127 \\ &= 10000101_2 - 127 = 133 - 127 = 6\end{aligned}$$

✓ Put S, M, and n together to form binary result
(Don't forget the implied "1." on the left of the mantissa.)

$$-1.000101011_2 \times 2^6 = (-1000101.011)_2$$

-69

$2^{-2} = 0.25$
 $2^{-3} = 0.125$
0.375

(b) $(-69.375)_{10}$

Question (3)

Give the best binary approximation of $A = (26.6)_{10}$ and $B = - (23.4)_{10}$ employing signed **2's-complement representation** with 2 bits for the fractional part.

SOLUTION

$$A = (26.6)_{10} \approx +(11010.10)_2 = \underline{(011010.10)_2} = (26.5)_{10}$$

$$B = - (23.4)_{10} = -(010111.10)_2 = -b \text{ with } b = +23.5 \text{ (closest to 23.4)}$$

Let's assume that we don't know how to find the **2's-complement** of a fractional **negative** number, so we'll consider

- $b = (010111.10)_2 = (010111.10)_2 \times 2^2/2^2 = (01011110)_2/2^2$

- First we'll find the **2's-complement** of $(01011110)_2 = (10100010)_2$

- then we'll divide it by 2^2 to scale back, such that

$$\Rightarrow \text{2's-complement of } (010111.10)_2 = (10100010)_2/2^2 = \underline{(101000.10)_2}$$

Question (4)

Two 6-bit register contain two numbers $X = 01010$ and $Y = 10101$ (expressed in 2's complement representation). Calculate the sum ($S = X+Y$) and the difference ($D=X-Y$) of these numbers using additions and 2's complementation only. Since both S and D have to be represented with 6 bits, indicate if overflow occurs, and explain how a circuit can detect these situations. Convert in decimal and write each intermediate and final result, to check the correctness of your assertions.

SOLUTION

Since both X AND Y ARE ALREADY IN 2'S COMPLEMENT REPRESENTATION, their value is

$$X = (01010)_2 = +10_{10} \text{ and}$$

$$Y = (10101)_2 = - \text{2's complement of } (0101)_2 = - (1011)_2 = -11_{10}$$

$$\Rightarrow -11_{10} = -(+11)_{10} \text{ represented with 6 bits is: } -(001011)_2 = \text{2's complement}(001011)_2 = (110101)_2$$

$$\text{As } -Y \text{ is needed too: } -Y = \text{2's complement of } (110101)_2 = (001011)_2 = (\text{just for verification}) = +11_{10}$$

$S = X+Y$	2's complement representation	Base 10 representation	$D=X-Y$	2's complement representation	Base 10 representation																								
$X +$	CY: <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	0	0	0	0	0	0	+10	$X +$	CY: <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td></tr></table>	0	0	1	0	1	0	+10												
0	0	0	0	0	0																								
0	0	1	0	1	0																								
Y	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td><td>0</td><td>1</td></tr></table>	0	0	1	0	1	0	1	1	0	1	0	1	-11	$-Y$	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td></tr><tr><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td></tr></table>	0	0	1	0	1	0	0	0	1	0	1	1	+11
0	0	1	0	1	0																								
1	1	0	1	0	1																								
0	0	1	0	1	0																								
0	0	1	0	1	1																								
S	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr></table>	1	1	1	1	1	1	-1	D	<table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td><td>1</td></tr></table>	0	1	0	1	0	1	$+21$												
1	1	1	1	1	1																								
0	1	0	1	0	1																								

No Overflow

Overflow Detection Expressions

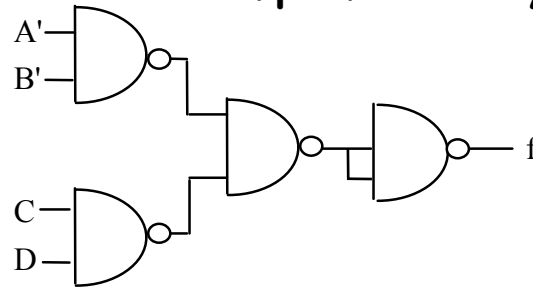
$$\text{OFL} = \begin{cases} \text{if } (\text{Sgn}X = \text{Sgn}Y) \neq \text{Sgn}C, \text{ i.e.,} \\ \text{or} \\ \text{if either of the last two carry bits is 1} \end{cases}$$

$$(\text{Sgn}X \cdot \text{Sgn}Y \cdot \overline{\text{Sign}C} + \overline{\text{Sgn}X} \cdot \overline{\text{Sgn}Y} \cdot \text{Sgn}C)$$

$$(C_7 \cdot \overline{C_6} + \overline{C_7} \cdot C_6) = C_7 \oplus C_6$$

Question (5)

Which of the logic functions is implemented by the following circuit?



SOLUTION

$$F = \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} = \overline{A \cdot B} \cdot \overline{C \cdot D} = \overline{(A+B) \cdot (C+D)} = A \cdot \overline{C} + A \cdot \overline{D} + B \cdot \overline{C} + B \cdot \overline{D}$$

$$\begin{aligned} & (8,9,12,13) + (8,10,12,14) + \\ & + (4,5,12,13) + (4,6,12,14) = \\ & = \text{Sm}(4,5,6,8,9,10,11,12,13,14) \end{aligned}$$

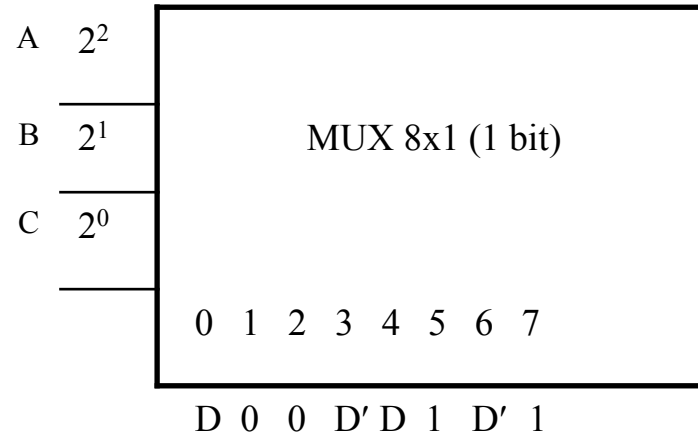
$$\begin{aligned} (e) f(A,B,C,D) &= \\ & \text{Sm}(4,5,6,8,9,10,12,13,14) + X(7,11) \end{aligned}$$

		<u>CD</u>			
		00	01	<u>11</u>	10
A	<u>AB</u>				
	00	0	1	3	2
	01	4	5	7	6
	11	12	13	15	14
10	8	9	11	10	
		<u>D</u>			

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Question (6)

Which of the following logic functions is implemented by the given circuit?



SOLUTION

(e) $f(A,B,C,D) =$
 $S_m(1,6,9,10,11,12,14,15)$

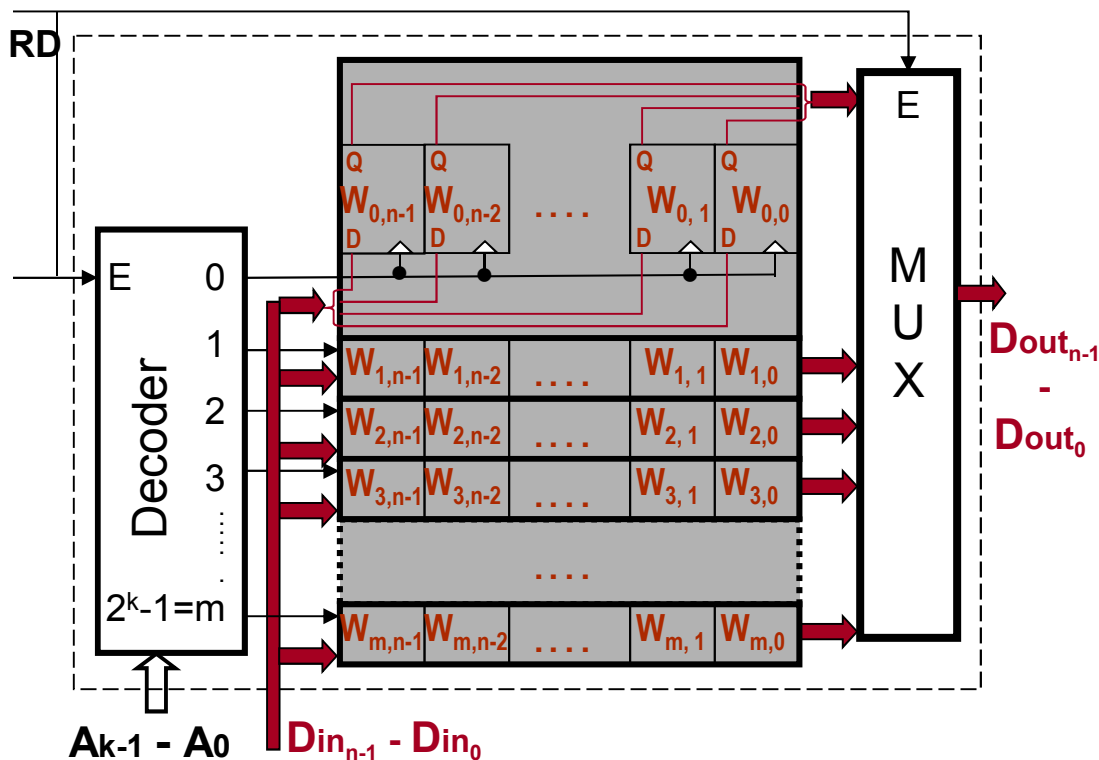
	CD	00	01	11	10
AB	00	0	1	0	0
01	00	0	0	0	1
11	00	1	0	1	1
10	00	0	1	1	1

Question (7)

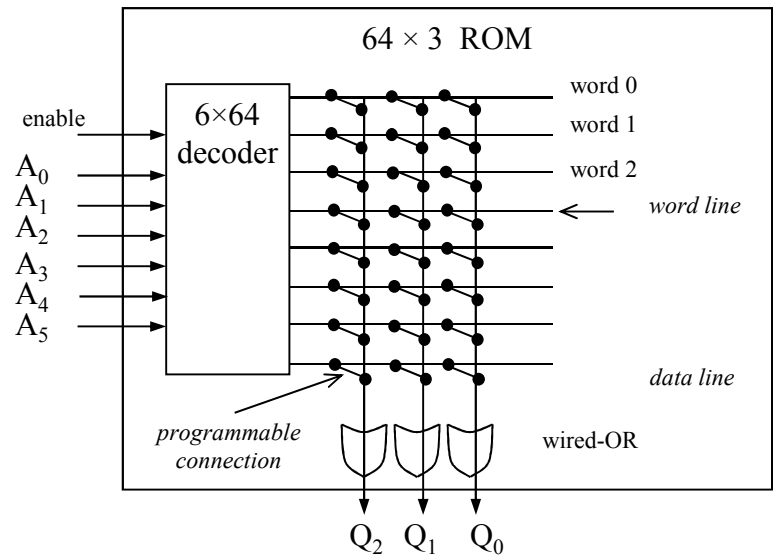
What is the capacity of a ROM, capable to implement three functions of six variables?

SOLUTION

Any combinational circuit of n functions of same k variables can be done with $2^k \times n$ ROM



(d) 64 words of 3 bits
Internal view

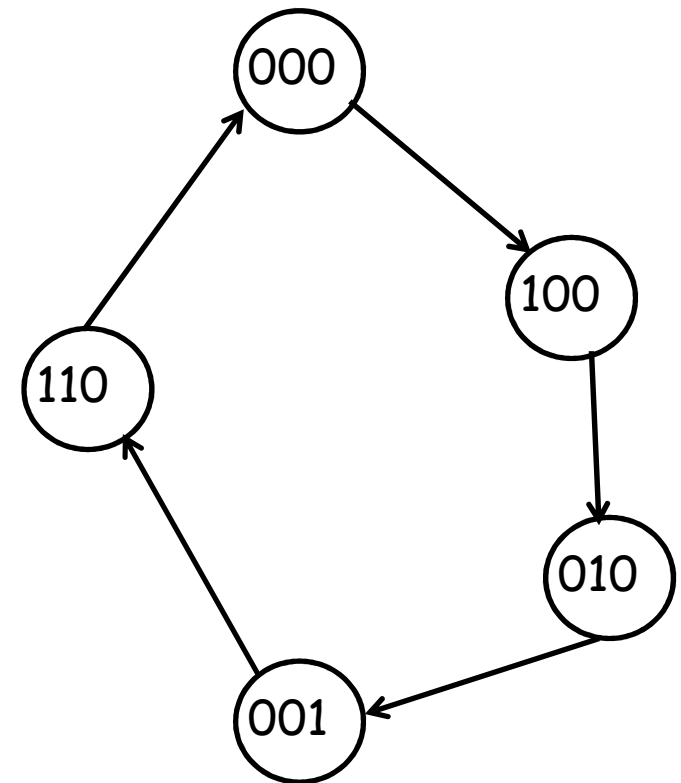


Question (8)

1) - Draw the state diagram of the sequential circuit whose state table is given below

SOLUTION

ABC	$A+B+C$	JA	KA	JB	KB	JC	KC
000	100	1	x	0	x	0	x
001	110	1	x	1	x	x	1
010	001	0	x	x	1	1	x
100	010	x	1	1	x	0	x
110	000	x	1	x	1	0	x



Question (8) (Cont.)

1) Indicate the correct set of minimized equations of the JK flip-flops inputs

SOLUTION

(c) $J_A = \overline{B}$, $K_A = 1$, $J_B = A+C$, $K_B = 1$, $J_C = \overline{A}.B$, $K_C = 1$.

A B C	$A^+B^+C^+$	J_A	K_A	J_B	K_B	J_C	K_C
0 0 0	1 0 0	1	x	0	x	0	x
0 0 1	1 1 0	1	x	1	x	x	1
0 1 0	0 0 1	0	x	x	1	1	x
0 1 1	xxx	x	x	x	x	x	x
1 0 0	0 1 0	x	1	1	x	0	x
1 0 1	xxx	x	x	x	x	x	x
1 1 0	0 0 0	x	1	x	1	0	x
1 1 1	xxx	x	x	x	x	x	x

Question (8) (Cont.)

2) If your circuit reaches by mistake any of the 3 states that are not used, determine their corresponding next state.

SOLUTION

To calculate the next states $A^+B^+C^+$ of the "don't care" states, use the excitation equations: $JA = B$, $KA = 1$, $JB = A+C$, $KB = 1$, $JC = \overline{A} \cdot B$, $KC = 1$

e.g., for $ABC = 011$ (the first row of the following table)

$JA=B'=1'=0$ (since $B = 1$, as given by the present state $ABC=011$) and $KA=1$

$\Rightarrow J=0$ & $K=1$ means A is RESET, i.e.,

$\Rightarrow A^+ = 0$ (where A^+ is A 's next state), \rightarrow the first cell of column A^+

$JB=A+C=0+1=1$ (because, from $ABC = 011 \Rightarrow A=0$ and $C=1$) and $KB=1$

$\Rightarrow J=1$ & $K=1$ means B 's next state is the complement of the present state

$\Rightarrow B^+ = B' = (1)' = 0$, \rightarrow the first cell of column B^+ of the same table

$JC=A'B=(0)' \cdot 1=1 \cdot 1=1$ and $KC=1$

$\Rightarrow J=1$ & $K=1$ means C 's next state is the complement of the present state

$\Rightarrow C^+ = C' = (1)' = 0$, \rightarrow the first cell of column C^+ of the same table

Copy results (i.e., the next state $A^+B^+C^+ = 000$) in the last column of the table:

$A B C$	JA	KA	A^+	JB	KB	B^+	JC	KC	C^+	$A^+B^+C^+$
011	0	1	0	1	1	0	1	1	0	000
101	1	1	0	1	1	1	0	1	0	010
111	0	1	0	1	1	0	0	1	0	000

2) Is your circuit auto-corrective? YES

Question (8) (Cont.)

3) Draw the time diagram and give the states of the flip-flops' outputs through the first 6 clock pulses, assuming that their initial state (at $t=0$) is 000 and they are triggered on the rising edge of the clock.

SOLUTION

