ITI1100

Addition of Chapter 1 Notes: Complements

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One’s Complement Representation

- **The one’s complement of a binary number involves inverting all bits.**
  - **1’s comp of 00110010 is 11001101**
  - **1’s comp of 10101011 is 01010100**
- **For an n bit number N the 1’s complement is $(2^n-1) - N$.**
- **Called diminished radix complement by Mano since 1’s complement for base (radix 2).**
- **To find negative of 1’s complement number take the 1’s complement.**

$0\underline{0001100}_2 = 12_{10}$

$1\underline{1110011}_2 = -12_{10}$

Two’s Complement Representation

- The two’s complement of a binary number involves **inverting all bits and adding 1.**
  - 2’s comp of 00110011 is **11001101**
  - 2’s comp of 10101010 is **01010110**
- For an n bit number N the 2’s complement is $(2^n-1) – N + 1$.
- Called radix complement by Mano since 2’s complement for base (radix 2).
- To find negative of 2’s complement number take the 2’s complement.

$0\underline{0001100}_2 = 12_{10}$

Sign bit

Magnitude

$1\underline{1110100}_2 = -12_{10}$

Sign bit

Magnitude

Two’s Complement Shortcuts

- Algorithm 1 **– Simply complement each bit and then add 1 to the result.**
  - **Finding the 2’s complement of $(01100101)_2$ and of its 2’s complement…**

N	**= 01100101**	**[N] =**	**10011011**
	10011010		**01100100**
+	**1**	**+**	**1**
	---------------		**---------------**
	10011011		**01100101**

- Algorithm 2 **– Starting with the least significant bit, copy all of the bits up to and including the first 1 bit and then complementing the remaining bits.**
  - **N = 0 1 1 0 0 1 0 1**

    [N] = 1 0 0 1 1 0 1 1

Complements of Numbers

- **The complement of the complement = the original number.**

$$[\boldsymbol{N}] = \boldsymbol{r^n} - \boldsymbol{N}$$

$$[\boldsymbol{r^n} - \boldsymbol{N}] = \boldsymbol{r^n} - (\boldsymbol{r^n} - \boldsymbol{N}) = \boldsymbol{N}$$

- **For a number N with a radix point, remove the point temporarily to form the r's or (r-1)'s complement. The radix point is restored to the complemented number in the same position.**

Subtraction of Unsigned Numbers with r’s Complements

The subtraction of two n-digit unsigned numbers M-N in base r can be done as follows:

$$\begin{aligned} \boldsymbol{M + r's\ complement\ of}\ \mathbf{N} \\ = \mathbf{M} + (\boldsymbol{r^n} - \boldsymbol{N}) \\ = \mathbf{M} - \mathbf{N} + \boldsymbol{r^n} \end{aligned}$$

1. When M>=N, sum produces an end carry “1” . Discard the end carry. The left is a positive number, M-N.
2. When M<N, the end carry is “0” (or no end carry). The result is a negative number. Take the r’s complement of the sum and place a negative sign in front.

Subtraction of Unsigned Numbers with (r-1)'s Complements

The subtraction of two n-digit unsigned number M-N in base r can be done by means of the (r-1)'s complement:

$$\boldsymbol{M + (r-1)'s\ complement\ of}\ \mathbf{N}$$
$$= \mathbf{M} + (\boldsymbol{r}^{\boldsymbol{n}} - \mathbf{1}) - \boldsymbol{N}$$
$$= (\mathbf{M} - \mathbf{N} - \mathbf{1}) + \boldsymbol{r}^{\boldsymbol{n}}$$

1. When M>=N, add the end carry "1" to low order bit. The result is a positive number.
2. When M<N, the end carry is "0" (or no end carry). The result is a negative number. Take the (r-1)'s complement of the sum and place a negative sign in front.

How To Represent Signed Numbers

- **Plus and minus sign used for decimal numbers: 25 (or +25), -16, etc.**
- **For computers, desirable to represent everything as *bits.***
- **Three types of signed binary number representations: signed magnitude, signed 1's complement, signed 2's complement.**
- **In each case: left-most bit indicates sign: positive (0) or negative (1).**

Consider ***signed magnitude***:

$0\underline{0000011}_2 = 3_{10}$

$1\underline{0000011}_2 = -3_{10}$

Signed-Complements

- Computer arithmetic uses the signed-complements to representation the binary number.
  - Signed-complement representation of “+ve”:

    = sign bit “0”+ “ve”
  - Signed-complement representation of “-ve”:

    = the 1’s or 2’s complement of +ve (including the sign bit);

0 0 1 1 (ve)

0 0 0 1 1 (+ve)

1 1 1 0 1 (−ve)

- Signed-2’s complement representation is commonly used in computer arithmetic (only 1 representation for 0).
- When X is in signed complement form:
  - 1’s or 2’s complement of X (including the sign bit) = -X

$0\underline{0000011}_2 = 3_{10}$

Sign bit

$1\underline{1111101}_2 = -3_{10}$

Sign bit

Signed-2’s Complement Addition and Subtraction

- Addition of two signed binary numbers represented in signed-2’s complement form:
  - Is an addition of the two binary numbers (including sign bits). A carry out of the sign-bit is discarded.
  - The sum is in signed- 2’s complement form. The sign bit decides if the result is positive or negative.

- Subtraction of two signed numbers in signed-2’s complement form:

  A - B = A + (-B)

  = A + 2’s complement of B(including the sign bit)

  = A+ $[B]_2$

Signed-2’s Complement Addition

- **Using 2’s complement numbers, adding numbers is easy.**
- **For example, suppose we wish to add +$(1100)_2$ and +$(0001)_2$.**
- **Let’s compute $(12)_{10}$ + $(1)_{10}$.**
  - **$(12)_{10}$ = +$(1100)_2$ = 01100_2 in 2’s comp.**
  - **$(1)_{10}$ = +$(0001)_2$ = 00001_2 in 2’s comp.**

Step 1: Add binary numbers
(*including the sign bit*)
Step 2: Ignore carry bit

```
         0 1 1 0 0
Add  +   0 0 0 0 1
     --------------
Final  0 0 1 1 0 1
Result
```

Ignore

Signed-2’s Complement Subtraction

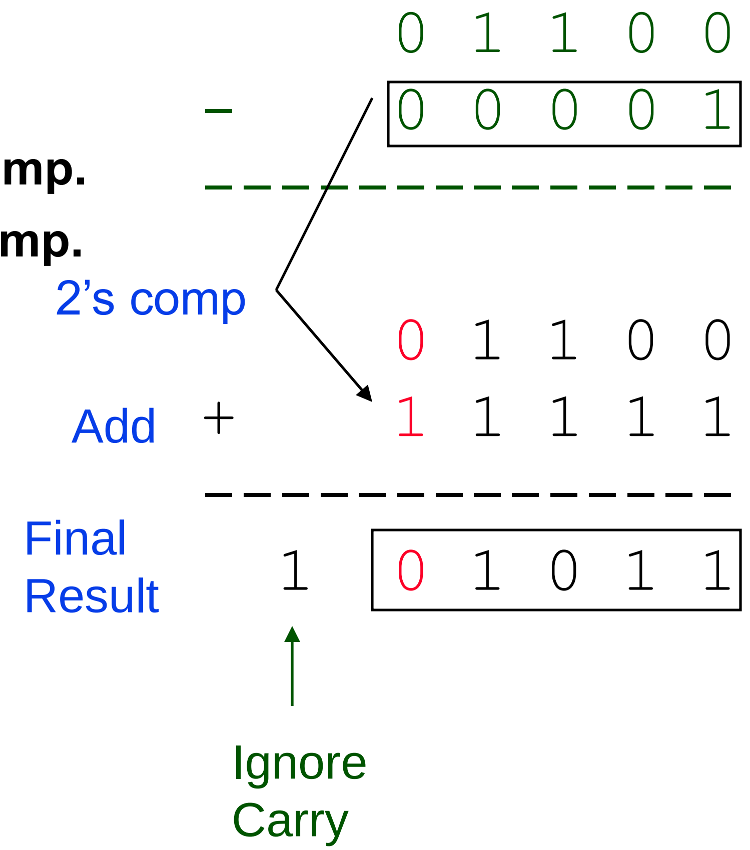
- Using 2’s complement numbers, follow steps for subtraction
- For example, suppose we wish to subtract $+(0001)_2$ from $+(1100)_2$.
- Let’s compute $(12)_{10}$ - $(1)_{10}$.
  - $(12)_{10} = +(1100)_2 = 01100_2$ in 2’s comp.
  - $(-1)_{10} = -(0001)_2 = 11111_2$ in 2’s comp.

Step 1: Take 2’s complement of 2nd operand

Step 2: Add binary numbers

(including the sign bit)

Step 3: Ignore carry bit



$$\begin{array}{r} 0\ 1\ 1\ 0\ 0 \\ -\quad \boxed{0\ 0\ 0\ 0\ 1} \\ \hline \end{array}$$

2’s comp

$$\begin{array}{r} 0\ 1\ 1\ 0\ 0 \\ +\quad 1\ 1\ 1\ 1\ 1 \\ \hline 1\ \ \boxed{0\ 1\ 0\ 1\ 1} \end{array}$$

Add

Final Result

Ignore Carry

Signed-2's Complement Subtraction: Example #2

- **Let's compute $(13)_{10} - (5)_{10}$.**
  - $(13)_{10} = +(1101)_2 = (01101)_2$
  - $(-5)_{10} = -(0101)_2 = (11011)_2$
- **Adding these two 5-bit codes…**

```
                    0 1 1 0 1
carry          +    1 1 0 1 1
                --------------
                 1  0 1 0 0 0
```

- **Discarding the carry bit, the sign bit is seen to be zero, indicating a correct result. Indeed,**

$$(01000)_2 = +(1000)_2 = +(8)_{10}.$$

Signed-2’s Complement Subtraction: Example #3

- **Let’s compute $(5)_{10} – (12)_{10}$.**
  - **$(-12)_{10}$ = $-(1100)_2$ = $(10100)_2$**
  - **$(5)_{10}$ = $+(0101)_2$ = $(00101)_2$**
- **Adding these two 5-bit codes…**

```
          0  0  1  0  1
+         1  0  1  0  0
-----------------------
          1  1  0  0  1
```

- **Here, there is no carry bit and the sign bit is 1. This indicates a negative result, which is what we expect. $(11001)_2 = -(7)_{10}$.**

Signed-1’s Complement Addition and Subtraction

- Addition of two signed binary numbers represented in signed-1’s complement form:
  - Is an addition of the two binary numbers (including sign bits). A carry out of the sign-bit is added to the low order bit.
  - The sum is in signed- 2’s complement form. The sign bit decides if the result is positive or negative.

- Subtraction of two signed numbers in signed-1’s complement form:

  A - B = A + (-B)

  = A + 1’s complement of B(including the sign bit)

  = A+ $[B]_1$

Signed-1's Complement Addition

- Using 1's complement numbers, adding numbers is easy.
- For example, suppose we wish to add $+(1100)_2$ and $+(0001)_2$.
- Let's compute $(12)_{10} + (1)_{10}$.
  - $(12)_{10} = +(1100)_2 = 01100_2$ in 1's comp.
  - $(1)_{10} = +(0001)_2 = 00001_2$ in 1's comp.

$$\begin{array}{llcccccc}
 & & & 0 & 1 & 1 & 0 & 0 \\
\text{Add} & + & & 0 & 0 & 0 & 0 & 1 \\
\hline
 & & 0 & 0 & 1 & 1 & 0 & 1 \\
\text{Add carry} & & \multicolumn{5}{l}{\llcorner\!\longrightarrow} & 0 \\
\hline
\text{Final Result} & & & 0 & 1 & 1 & 0 & 1
\end{array}$$

Step 1: Add binary numbers (*including the sign bit*)

Step 2: Add carry to low-order bit

Signed-1's Complement Subtraction

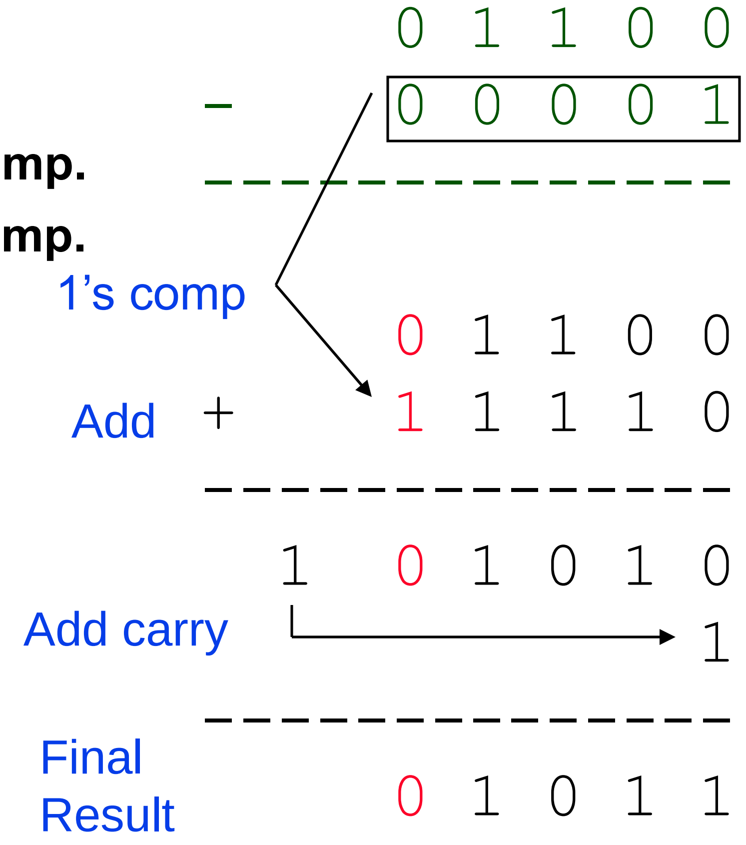
- **Using 1's complement numbers, subtracting numbers is also easy.**
- **For example, suppose we wish to subtract $+(0001)_2$ from $+(1100)_2$.**
- **Let's compute $(12)_{10}$ - $(1)_{10}$.**
  - **$(12)_{10}$ = $+(1100)_2$ = 01100_2 in 1's comp.**
  - **$(-1)_{10}$ = $-(0001)_2$ = 11110_2 in 1's comp.**

Step 1: Take 1's complement of 2nd operand

Step 2: Add binary numbers

(including the sign bit)

Step 3: Add carry to low order bit



$$\begin{array}{lrccccc}
 & & 0 & 1 & 1 & 0 & 0 \\
- & & \boxed{0} & \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} \\
\hline
\text{1's comp} & & & & & & \\
 & & 0 & 1 & 1 & 0 & 0 \\
\text{Add} \quad + & & 1 & 1 & 1 & 1 & 0 \\
\hline
 & 1 & 0 & 1 & 0 & 1 & 0 \\
\text{Add carry} & & & & & & 1 \\
\hline
\text{Final Result} & & 0 & 1 & 0 & 1 & 1
\end{array}$$