# University of Ottawa

## Department of Mathematics and Statistics

**MAT 1320F-Winter 2016-Feb 10th, 17:30-18:50-Midterm 1**

Professor: Xinhou Hua

Surname ____________________ First Name ____________________ Student # __________

• Time: 80 min. Total points: 20.

• Only the following calculators are allowed during Faculty of Science examinations: Texas Instruments TI-30 and TI-34, Casio FX-260 and Casio FX-300 (scientific and non-programmable calculators).

• Notes or books are not permitted.

• Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.

• Write *only* in non-erasable ink (ball-point or pen), not in pencil. Cross out, if necessary, but do not erase or overwrite.

• **Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.**

**By signing below, you acknowledge that you have ensured that you are complying with the above statement.**

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| Question | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Maximum | 2 | 1 | 2 | 7 | 4 | 4 | 20 |
| Grade | | | | | | | |

**Part I: Short Answer Questions, only answers are marked.**

**1.** [2 points] Let $f(x) = \ln(x-1)$, $g(x) = e^x$. Then

$f \circ g(x) =$ ________________ The domain of $f \circ g(x)$ : ________________

**Solution:** $f \circ g(x) = \ln(e^x - 1)$.

The domain is: $e^x - 1 > 0$, $x > 0$.

**2.** [1 point] Let $\sin x = 0.8$, where $\frac{\pi}{2} \leq x \leq \pi$. Then

$\cos x =$ ________________

**Solution:** Since $\frac{\pi}{2} \leq x \leq \pi$, $\cos x = -\sqrt{1-\sin^2 x} = -\sqrt{1-0.8^2} = -\sqrt{0.36} = -0.6$.

**3.** [2 points] Let $y = \frac{3x^3-1}{x(x^2+1)}$. Find the vertical and horizontal asymptotes.

Vertical Asymptote(s): ________________ Horizontal asymptote: ________________

**Solution:** (a) From $x(x^2+1) = 0$ we have $x = 0$. So VA: $x = 0$.

(b) Since $\lim\limits_{x\to\infty} \frac{3x^3-1}{x(x^2+1)} = 3$. So HA: $y = 3$.

**Part II: Long Answer Questions, you have to show your work clearly.**

**4.** [7 points] Calculate the following limits:

(i) [2 points] $\lim\limits_{x\to 1} \frac{x^2-3x+2}{x^2-1}$.

(ii) [3 points] $\lim\limits_{x\to 4} \frac{\sqrt{x}-2}{x-4}$.

(iii) [2 points] $\lim\limits_{x\to 2} \arctan\left(\frac{1}{(x-2)^2}\right)$.

**Solution:** (i) $\lim\limits_{x\to 1} \frac{x^2-3x+2}{x^2-1} = \lim\limits_{x\to 1} \frac{(x-2)(x-1)}{(x+1)(x-1)} = \lim\limits_{x\to 1} \frac{x-2}{x+1} = -\frac{1}{2}$.

(ii) $\lim\limits_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \lim\limits_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)} = \lim\limits_{x\to 4} \frac{x-4}{(x-4)(\sqrt{x}+2)} = \lim\limits_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{4}$.

(iii) $\lim\limits_{x\to 2} \arctan\left(\frac{1}{(x-2)^2}\right) = \arctan\infty = \frac{\pi}{2}$.

**5.** [4 points] Let $f(x) = \begin{cases} x^2+2cx, & \text{if } x<1 \\ x^3+6x, & \text{if } x>1 \\ 1-2b, & \text{if } x=1 \end{cases}$. Find the values of the constants $b$ and $c$ such that $f(x)$ is continuous at $x=1$.

**Solution:** Since

$$\lim_{x\to 1^-} f(x) = 1+2c, \quad \lim_{x\to 1^+} f(x) = 7.$$

When the limit $\lim\limits_{x\to 2} f(x)$ exists, $1+2c=7, \Rightarrow c=3$.

When $f(x)$ is continuous at $x=1$: $\lim\limits_{x\to 1} f(x) = 7 = f(1) = 1-2b, \Rightarrow b=-3$.

**6.** [4 points] Let $f(x) = \frac{3x}{x+2}$. Find $f'(x)$ by the **definition** of the derivative.

**Solution:**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{3(x+h)}{x+h+2} - \frac{3x}{x+2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(3x+3h)(x+2)-(3x)(x+h+2)}{(x+h+2)(x+2)}}{h} = \lim_{h\to 0} \frac{6h}{h(x+h+2)(x+2)}$$

$$= \lim_{h\to 0} \frac{6}{(x+h+2)(x+2)} = \frac{6}{(x+2)^2}.$$