Assignment # 2
Linear Programing Formulation and Sensitivity Analysis

ADM2302 students are reminded that submitted assignments must be neat, readable, and well-organized. Assignment marks will be adjusted for sloppiness, poor grammar and spelling, as well as for technical errors. This assignment can be done in a group of one, two, three or four students. Plagiarism on assignments will not be accepted, *each student must sign the statement of integrity.* Solutions to the Case Study are to be prepared using the **Report to Management format provided in the ADM2302 Course Outline**.

This assignment can NOT be hand written.

The assignment is to be submitted electronically as a single Word document file via Brightspace by Sunday June 11th prior to 23:59. Front page of the word document has to include title of the assignment, course code and section, student name and student number. Second page is *the statement of integrity that must be signed.*

E-mail questions related to the assignment should be sent to the Teaching Assistant.

Problem 1: (16 points)
A renowned Norwegian toymaker is preparing for the holiday season. It produces two kinds of toys: toy police cars and fire trucks. For the production, the company utilizes two raw material, wood and plastic and possesses 26000 kg and 36000 kg respectively of raw material for its monthly production. The quantities of raw material needed to build each toy are presented in the table below:

Raw material	Quantity required	
	Police car (kg)	Fire truck (kg)
Wood	2	4
Plastic	6	3

The fabrication time required to produce a fire truck is three times as long as what is required to produce a police car. The total production capacity (for both toys) is equivalent to producing 6000 fire trucks. For technical reasons, we must produce at least twice as many more police cars than fire trucks. The unit contribution for each toy is as follows: 20$ per police cars and 25$ per fire trucks.

1. Formulate algebraically the LP problem that provides the optimal solution to help the company prepare for the holiday season. (11 points)
2. Find the optimal schedule that maximizes profits using Solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). Include "managerial statements" that communicate the results of the analyses (i.e. describe verbally the results). (3 points)
3. The company has the opportunity to hire a contractor that would allow the total production capacity to increase by 10% for a cost of 5000$ per month. Should the company hire the contractor? Justify. (2 points)

Problem 2 (15 points)

Morton and Monson Inc. is a small manufacturer of parts for the aerospace industry. The production capacity for the next four months is given as follows:

	Production Capacity in Units	
Month	**Regular Production**	**Overtime Production**
January	3,000	500
February	2,000	400
March	3,000	600
April	3,500	800

The regular cost of production is $500 per unit and the cost of overtime production is $150 per unit in addition to the regular cost of production.

The company can utilize inventories to reduce fluctuations in production, but carrying one unit of inventory costs the company $40 per unit per month. Currently there are no units in inventory. However, the company wants to maintain a minimum safety stock of 100 units of inventory during the months of January, February, and March and wants to have 300 units in inventory at the end of April.

The estimated demand for the next four months is as follows:

Month	**January**	**February**	**March**	**April**
Demand	2,800	3,000	3,500	3,000

The production manager is in the process of preparing a four-month production schedule.

a. Formulate algebraically the LP problem that provides the optimal production schedule and minimizes the total cost. (10 points)

b. Find the optimal schedule that minimizes total cost using Solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). Include "managerial statements" that communicate the results of the analyses (i.e. describe verbally the results). (5 points)

Problem 3 (14 points)

A classic linear programming problem involves minimizing trim loss. Here is one version of the problem:

A mill cuts 20-foot pieces of wood into several different lengths: 8-foot, 10-foot and 12-foot. The mill has a certain amount of 20-foot stock on hand and orders for the various sizes. The objective is to fill the orders with as little waste as possible. For example, if two 8-foot lengths are cut from a 20-foot piece, there will be a loss of 4 feet, the leftover amount.

Currently, the mill has 350 20-foot pieces of wood on hand and the following orders, which must be filled from stock on hand:

Size in feet	**Number Ordered**
8	276
10	100
12	250

a. Formulate algebraically an LP model that will enable the mill operator to satisfy the orders with minimum trim loss. (*Hint:* List the different ways the 20-foot pieces could be cut into the desired sizes.) (8 points)

b. Using your notation from *a*, write an equation for
   i. The amount of waste that would result given a solution. (1 point)
   ii. The number of pieces of each size (8-foot, 10-foot, and 12-foot). (1 point)

c. Formulate this same linear programming problem on a spreadsheet and SOLVE using Excel solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). Include "managerial statements" that communicate the results of the analyses (i.e. describe verbally the results). (4 points)

CASE STUDY: Shevchenko Farms (40 points)

Report to Management Format (13 points)

The Shevchenko family operates a farm in Odesa, Ukraine, under the collective farming system. The family raises beef cattle to be distributed by the central government as its main activity. Previously, the Shevchenko family was told how many cows to raise each year by the central planning agency and was allocated the necessary animal feed to raise its cows. With the new market-driven economy, the Shevchenko family receives no instructions on how to operate its farm and must survive as best it can on its own by buying animal feed, raising cows, and selling them to any buyers they can find. Cows were fed a combination of corn and hay mixed to assure that minimum amounts of two primary nutrients were met: crude protein and calories. Corn and hay supplies have become less certain, and the Shevchenko family has contracted to buy silage from a nearby farm to supplement the previous cow diet of corn and hay. The Shevchenko family has turned to you, a group of Telfer graduates who recently started working as United Nations consultants, for help in managing their farm in the new Ukrainian market economy.

On its own, the Shevchenko family contracted to sell up to 100 cows to an Odesa butcher for the next few months for a fixed price of 450 hryvnia per cow. In addition, the Shevchenko family contracted to buy up to 800 kilograms of silage from the nearby farm for a price of 10 hryvnia per kilogram. They believe they can buy any amount of corn for 19 hryvnia per kilogram and can purchase up to 600 kilograms of hay from another farm for 15 hryvnia per kilogram.

The table below presents the monthly requirements per cow in units of protein and thousands of calories (i.e. kilo-calories). It also presents, for the three sources of animal feed, the amount of crude protein and kilo-calories supplied per kilogram.

NUTRIENT	MINIMUM REQUIREMENT PER HOG PER MONTH	AMT. SUPPLIED PER KILOGRAM OF CORN	AMT. SUPPLIED PER KILOGRAM OF FOOD WASTE	AMT. SUPPLIED PER KILOGRAM OF POTATOES
Crude Protein	174	18	9	15
Kilo-calories	1400	30	120	80

To help the Shevchenko family understand how to run their farm, another United Nations consultant has built the following LP algebraic formulation and its corresponding Excel model:

H= number of Hogs Sold
C= Corn Purchased (in Kg)
F= Food scraped Purchased (in Kg)
P=Potatoes purchased (in Kg)

Objective function:
Maximize net revenue=450H-19C-10F-15P

Subject to:
H<=100
F<=800
P<=600
18C+9F+15P>= 174H (amount of crude protein)
30C+120F+80P>= 1400 H (amount of kilo-calories)
H,C,F,P>=0

	H	C	F	P			
	Hogs Sold	Corn purchased	Food scraped purchased	Potatoes purchased			
Decisions	100.000	95.238	742.857	600.000	26,761.905		
Price	$450.00	-$19.00	-$10.00	-$15.00			
Constraints:					**LHS**		**RHS**
Hogs supply limit	1				100.000	<=	100
Food scraped supply limit			1		742.857	<=	800
Potatoes supply limit				1	600.000	<=	600
Crude Protein Net Excess Nutrients	-174	18	9	15	0.000	>=	0
Kilo-Calories Net Excess Nutrients	-1400	30	120	80	0.000	>=	0

The Excel model above shown displays the optimum answer to maximize operating income. The corresponding solver Sensitivity Report is also presented:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B19	Decisions Hogs Sold	100	0	450	1E+30	261.048
C19	Decisions Corn purchased	95.238	0	-19	1.917	1
D19	Decisions Food scraped purchased	742.857	0	-10	0.5	24.694
E19	Decisions Potatoes purchased	600	0	-15	1E+30	1.095

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F22	Hogs supply limit LHS	100	261.048	100	5.405	21.739
F23	Food scraped supply limit LHS	742.857	0	800	1E+30	57.143
F24	Potatoes supply limit LHS	600	1.095	600	166.667	109.091
F25	Crude Protein Net Excess Nutrients LHS	0.000	-1.048	0	46800	1500
F26	Kilo-Calories Net Excess Nutrients LHS	0.000	-0.005	0	6000	78000

The family has many questions to ask about operating his farm, which you will expertly **answer by examining the Sensitivity Report and/or the Excel solver output provided above.**

Some of the Shevchenko family's questions cannot be answered from the Solver information given above. If so, answer as best you can from the information provided above, state why a more detailed answer is not available, and if appropriate, re-formulate the excel spreadsheet model and **re-run Solver** to produce the answers he seeks.

Questions:

1. If the Shevchenko family follows the recommended approach to managing the farm so as to maximize operating income next month, how much will they earn from selling the 100 hogs? How many hogs will they sell, and how much corn, food scraps, and potatoes should they purchase in that case? (3 points).

2. We are confused laments certain members of the family: "You are telling us to buy fractional kilograms of corn and food scraps for our hogs, but we have to make purchases in whole kilograms. What should we do?" (3 points).

3. One member of the family thinks he can persuade the butcher to buy another 5 hogs from the family farm at the same 450 hryvnia price. Would the family`s situation improve if they sold the butcher 105 hogs instead of 100? By how much would the operating income be improved, if at all? (3 points).

4. Another member of the family comments: "You have to understand that everything is very volatile right now. For example, it could easily happen that the cost of corn might suddenly change." In that case, the family would like to know by how much could the cost of corn increase before they would have to change their plans? (3 points).

5. "Business relationships are really uncertain in Ukraine just now. We are nervous that the farm manager selling potatoes will not honor his promise to deliver the 600 kilograms. If he delivers only 500 kilograms of potatoes, how much will we be hurt and what changes do we need to make in our decisions?" (5 points).

6. "Are there any alternate ways for us to achieve the same optimal operating income, involving, for example, different purchases of corn, food, or potatoes?" If so, what are they?" asks another member of the family. (3 points).

7. "An international salesman from the Ralston-Purina Company stopped by the farm and wants to sell us prepackaged dry hog feed imported from Poland. He wants to charge us 16 hryvnia per kilogram for the packaged hog food, which includes transportation from Poland. He says each kilogram of his product provides 16 units of crude protein and 150 kilocalories. Should we start importing hog feed from Poland? And how much should we buy, if any, and what would be the impact of this opportunity on our optimal operating income?" (7 points).