MATH 151 _ Midterm1 _ 2014 Spring

1: $\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \lim_{x\to 2^+} \frac{x-2}{x-2} = 1$

$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \lim_{x\to 2^-} \frac{2-x}{x-2} = -1 \neq 1$.

Then $\lim_{x\to 2} \frac{|x-2|}{x-2}$ does not exist.

2: $\lim_{x\to -2} \left[ \frac{1}{x+2} + \frac{4}{x^2-4} \right]$

$= \lim_{x\to -2} \frac{x-2+4}{x^2-4}$

$= \lim_{x\to -2} \frac{1}{x-2}$

$= \frac{1}{-4}$.

Answer: d

3:

a): $4x^3-4x+1 = \frac{dy}{dx}$

b): $\frac{dy}{dx} = -\sin(\sqrt{x^2+1})\,(\sqrt{x^2+1})'$

$= -\sin(\sqrt{x^2+1})\, \frac{2x}{2\sqrt{x^2+1}}$

$= \frac{-x\sin(\sqrt{x^2+1})}{\sqrt{x^2+1}}$

c): $2x+y+xy'+2yy'=0$

Thus $(2y+x)y' = -(2x+y)$

$y' = \frac{-(2x+y)}{2y+x}$.

4: $\lim_{x\to 2^-} f(x) = c^2 2^2 = c^2-4$.

$\lim_{x\to 2^+} f(x) = 2(2+c) = 4+2c$

By $c^2-4 = 2c+4$, we get $c^2-2c-8=0$

$(c-4)(c+2)=0$

Thus $c=4$ and $c=-2$.

The Answer is e).

5): a) $\forall \varepsilon>0$, $\exists \delta>0$, such that for each $x\in [a-\delta, a+\delta]$,

$|f(x)-L| < \varepsilon$.

b): If $||2x+1|-5| < \varepsilon$.

Then $|x-2| < \frac{\varepsilon}{2}$. Thus $\delta = \frac{\varepsilon}{2} = \frac{1}{6}$.

The Answer is d).

6: Answer d

$\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\frac{1}{\cos^2 x}}{1} = \frac{1}{\cos^2 0} = 1$.

7: Answer: d

By $3x^2+3y^2y'=0$ and $y'=-1$, we get

$x^2-y^2=0$.

Then by $x^3+y^3=1$, we get

$x=y=\frac{1}{\sqrt[3]{2}}$.

8: The top of the ladder is

$y=\sqrt{9-x^2}$.

Then $\frac{dy}{dt} = \frac{-2x}{2\sqrt{9-x^2}} \cdot \frac{dx}{dt}$

$= \frac{-x}{\sqrt{9-x^2}} \cdot \frac{2}{3}$.

For $x=2$, we get $\frac{dy}{dt} = \frac{-2}{\sqrt{5}} \cdot \frac{2}{3} = \frac{-4}{3\sqrt{5}}$ m/sec.