Advanced Functions MHF4U-C
Practice Test

Time: 2 hours

Total Marks: 100

Instructions

- The Practice Test uses the same format, type of questions, marking scheme, and length as the Final Test.
- To get the most out of this Practice Test, allot yourself two uninterrupted hours, and don't use notes or books. You may use a scientific calculator.
- Write your answers in the space provided.
- The test has four (4) parts. An approximate time is given for each part. Look over the test before you begin, and leave some time to review your work at the end.

Part	Category	Marks	Time (min)
Preview			5
A	Logs and Exponents	25	30
B	Polynomials and Rational Functions	24	25
C	Trigonometry	25	25
D	Properties of Functions	26	30
Review			5
Total		**100**	**120**

Sept. 2016

Formula Sheet

Compound-angle formulas:

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

Double-angle formulas:

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos(2x) = 1 - 2\sin^2 x$$

$$\cos(2x) = 2\cos^2 x - 1$$

Pythagorean identity:

$$\cos^2 x + \sin^2 x = 1$$

Trig identities:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

Part A: Logs and Exponents (25 marks) *(approximate time: 30 minutes)*

1. Simplify each of the following:

a) $\dfrac{\left(-45x^2y^3\right)^0\left(8x^3y^2z^5\right)^5}{\left(2x^5y^3z^2\right)^3}$ **(2 marks)**

b) $x^{a-3b}x^{2a+4b}$ **(2 marks)**

2. Solve for x in each of the following:

a) $\log_x\left(\dfrac{1}{64}\right) = 3$ **(1 mark)**

b) $\log_7(3x + 4) = 2$ **(2 marks)**

3. State the equation of the inverse of $y = 3^{x+5}$. **(2 marks)**

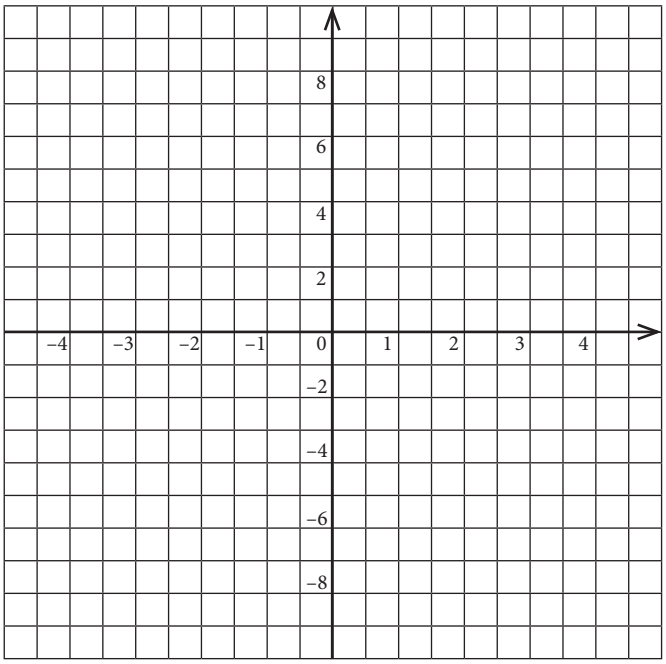
4. Given the function $y = 3(2^{4x-8}) - 7$

a) State the significance of 3. **(1 mark)**

b) State the significance of $4x - 8$. **(1 mark)**

c) State the significance of -7. **(1 mark)**

d) The parent function's y-intercept (0, 1) is transformed to (2, –4). Sketch the function $y = 3(2^{4x-8}) - 7$ and its parent function including any relevant asymptotes. **(3 marks)**



5. Express the relationship $\sqrt[3]{125} = 5$ (without using fractions) in the following forms:

a) exponential **(1 mark)**

b) log notation **(1 mark)**

6. Your friend has trouble solving the equation $2\log_6 x = \log_6 72 - \log_6 2$. Solve the equation for your friend and explain every step in detail as you proceed. **(2 marks)**

7. Four litres of an acid has pH 3. How many litres of pure water must be added to the 4 L to create a solution of pH 5? **(3 marks)**

8. How much more intense is an earthquake of magnitude 6.3 than one of magnitude 3.9? **(3 marks)**

Part B: Polynomials and Rational Functions (24 marks) *(approximate time: 25 minutes)*

9. Given $f(x) = -2(3x - 4)^2(2x + 5)^3(5x - 1)$ determine

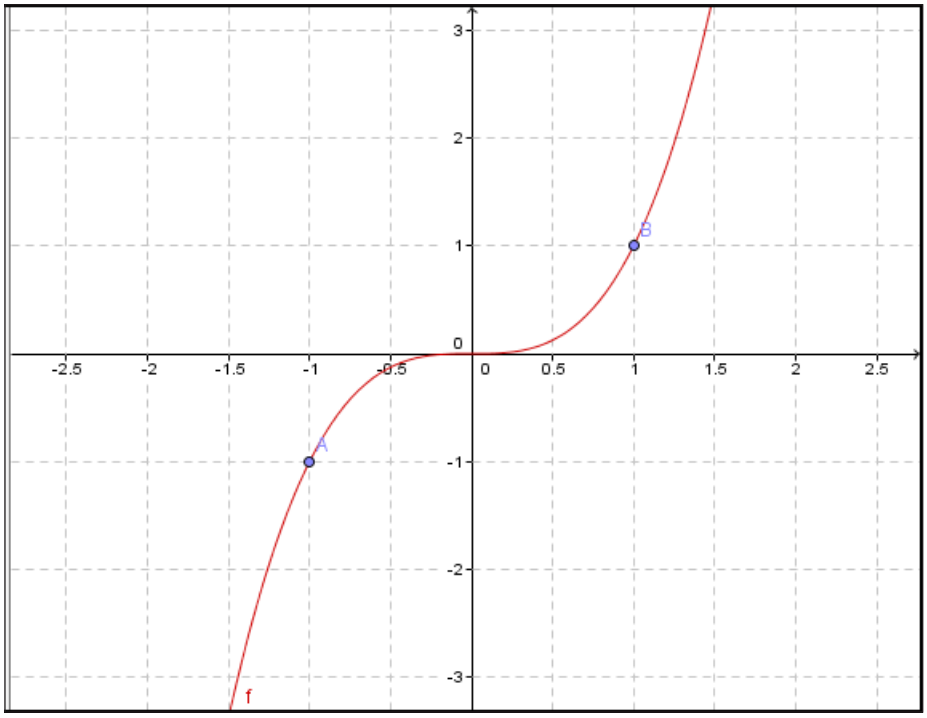
a) the degree of the polynomial. **(1 mark)**

b) the dominant term. **(1 mark)**

c) the y-intercept. **(1 mark)**

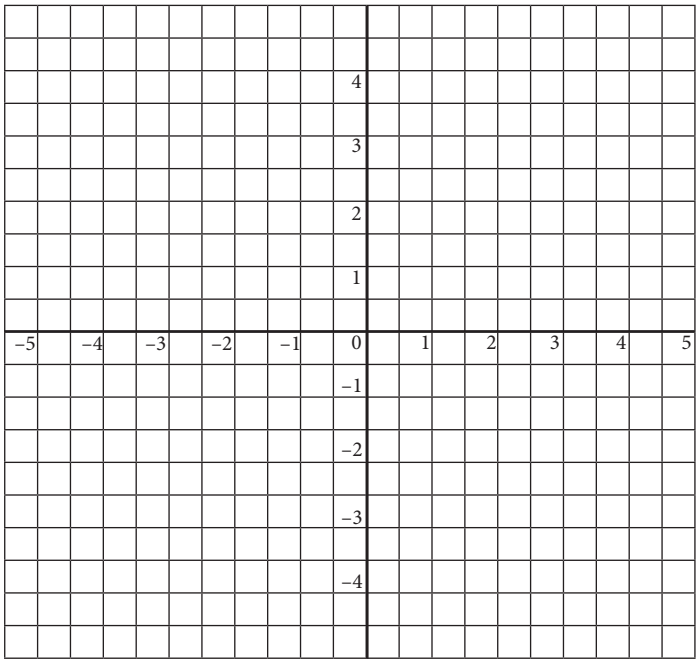
10. Determine the value of k if $x + 1$ is a factor of $f(x) = x^3 - kx^2 + kx - 7$. **(2 marks)**

11. Given the following graph, state the regions where $y > 0$. **(2 marks)**



12. Determine the equation of the polynomial that crosses the x-axis at $x = -1$, bounces off the x-axis at $x = 3$, and has a y-intercept at $(0, -18)$. **(3 marks)**

13. Sketch the function $f(x) = (x - 1)^2(x - 3)$ and the function $g(x) = \frac{1}{f(x)}$. **(4 marks)**



14. Given the function $y = \sqrt{x}$, determine the equation of the curve after it has been vertically stretched by a factor of 2, reflected in the x-axis, horizontally compressed by a factor of 7, and translated left 3 units and down 7 units. **(4 marks)**

15. To maximize yearly revenue, a company wishes to determine the best selling price for their new line of shirts. They realize that if the price is too high or too low, they will not maximize profit. The equation $y = -10(x - 30)^2 + 6250$ models the scenario, where x represents the selling price per shirt and y represents the number of shirts sold.

a) Determine the selling price that maximizes the revenue. **(1 mark)**

b) Determine the maximum number of shirts sold. **(1 mark)**

c) What is the maximum revenue? **(1 mark)**

d) What are the x-intercepts for the function? **(3 marks)**

Part C: Trigonometry (25 marks) *(approximate time: 25 minutes)*

16. Convert $\frac{11\pi}{6}$ radians to degrees. **(1 mark)**

17. Convert 330° to radians (leave your answer in terms of π). **(1 mark)**

18. Determine the exact value of the following. In other words, your answer must not use decimals.

a) $\sin\left(\frac{5\pi}{4}\right)$ **(1 mark)**

b) $\cos\left(\frac{2\pi}{3}\right)$ **(1 mark)**

19. Determine the standard position angle formed by the line joining (0, 0) to the point (−4, −6). **(3 marks)**

20. Use the compound angle formula to determine $\cos\left(\frac{7\pi}{12}\right)$. **(3 marks)**

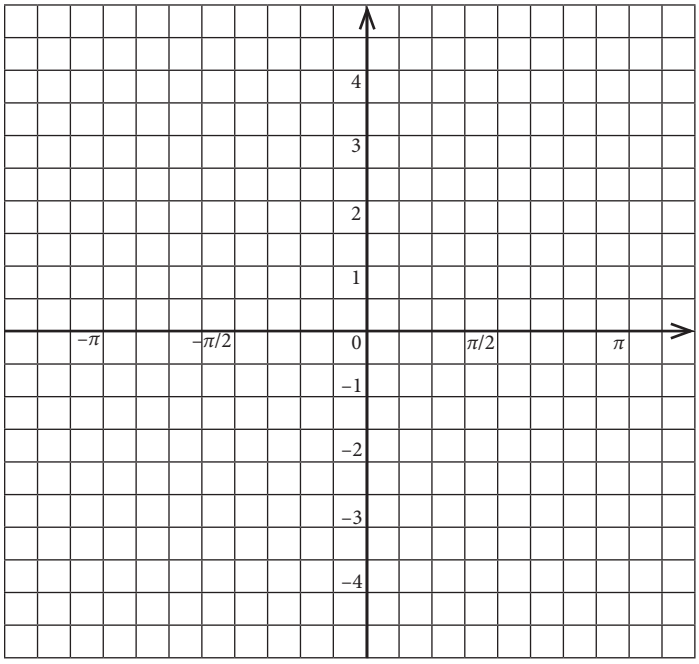
21. Determine the exact value of x. In other words, your answer must not use decimals.

$\sin 2x - \cos x = 0$ **(4 marks)**

22. Given the function $y = 3\sin(2x + \pi) - 1$

a) Explain what each parameter does. **(2 marks)**

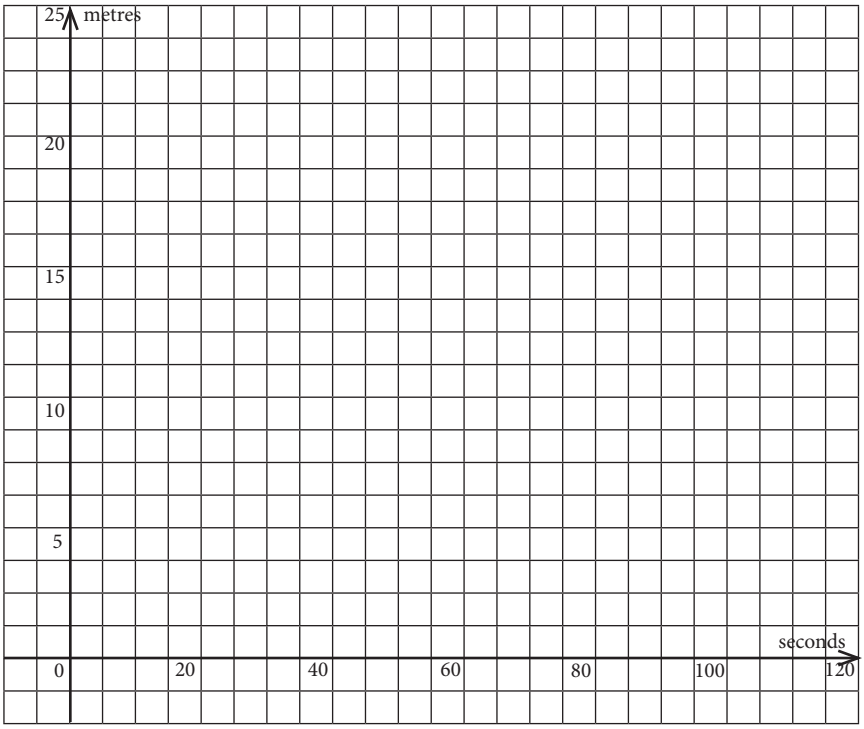
b) Sketch the curve. **(2 marks)**



23. Prove the identity $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$ **(3 marks)**

24. The chair on a Ferris wheel is 0.5 m from the ground at its lowest point and is 22.5 m from the ground at its highest point. It takes 60 seconds to make a full rotation.

a) Start the chair at its highest point and graph the path of the chair over a period of 120 seconds. **(2 marks)**



b) Determine the function that models the height of the chair. **(1 mark)**

c) Predict the height of the endpoint after 135 seconds. **(1 mark)**

Part D: Properties of Functions (26 marks) *(approximate time: 30 minutes)*

25. Determine $f(-1)$ if $f(x) = x^3 - 2x^2$ **(1 mark)**

26. Determine whether the function $g(x) = x^3 - 3$ is even, odd, or neither. **(2 marks)**

27. State the equation of the tangent to the curve $f(x) = \frac{3}{x^2} - 5$ at $x = 2$. **(5 marks)**

28. A ball is tossed in the air on the planet Mars. Its height is modelled by $h = -1.85(t - 4)^2 + 31$.

a) Determine the maximum height of the ball. **(2 marks)**

b) What velocity will the ball be travelling at when it hits the ground? **(5 marks)**

29. State whether each of the following is true or false. **(3 marks: 1 mark each)**

_____ **a)** The inverse of a linear function is also linear.

_____ **b)** A quadratic function is always an even function.

_____ **c)** The inverse of an odd function is not always a function.

30. List three factors that influence the domain of a combined function. **(3 marks)**

31. Determine the domain of the function $f(x) = \frac{1}{x^2 - 9}$ **(2 marks)**

32. (3 marks: 1 mark each)

Air pressure decreases as height increases. The following table shows the pressure at various heights.

Height (km)	Pressure (kPa)
0	100.0
4	70.0
8	49.0
12	34.3
16	24.0
20	16.8

a) Determine the function type.

b) Determine an equation that predicts the pressure at different heights.

c) Use your equation to predict the pressure in kPa after 10.5 km.