

MAT 1332, Winter 2017, Assignment 5

Due Wednesday March 22 in the math department dropboxes by 7:00pm.

Late assignments will not be accepted; nor will unstapled assignments.

Professors in the math department will not lend you a stapler; do not ask for one.

Please print double sided to save paper.

Instructor (circle one): Guy Beaulieu

Robert Smith?

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DGD (circle one): 1

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Name (Prime student) _____ Student Number _____

Student Name _____ Student Number _____

Student Name _____ Student Number _____

By signing below, we declare that this work is our own, that we have not copied from any other individual or other source and that all students contributed equally.

Signatures _____

QUESTION 1. Suppose A is an $n \times n$ matrix. Determine whether the following are TRUE or FALSE. If false, explain why not.

a) The characteristic equation is always an n th order polynomial. **TRUE.**

b) The characteristic equation can always be solved. **TRUE.**

c) 0 is never an eigenvalue. **FALSE. See Question 3.**

d) $\det A = \det A^T$. **TRUE.**

e) If A is invertible, so is A^T . **TRUE.**

f) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

FALSE. A might not be invertible.

QUESTION 2. Consider the system of equations

$$\begin{aligned}y_1' &= 59y_1 - 99y_2 \\ y_2' &= 18y_1 + 232y_2\end{aligned}$$

a) Find the general solution.

(0.5 marks for each eigenvalue, 1 for each eigenvector, 1 for the general solution.

Subtract 1 mark if $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is offered as an eigenvector.)

[4]

We have

$$\det(A - \lambda I) = \lambda^2 - 291\lambda + 15470 = (\lambda - 70)(\lambda - 221)$$

so the eigenvalues are $\lambda_1 = 70$ and $\lambda_2 = 221$.

Corresponding to $\lambda_1 = 70$, we have

$$\left[\begin{array}{cc|c} -11 & -99 & 0 \\ 18 & 232 & 0 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 9 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

It follows that the eigenvectors are in the form

$$\begin{pmatrix} -9 \\ 1 \end{pmatrix} r, r \in \mathbb{R}, r \neq 0$$

[You lose half a mark if you don't specify the conditions on r .]

Corresponding to $\lambda_2 = 221$, we have

$$\left[\begin{array}{cc|c} -162 & -99 & 0 \\ 18 & 11 & 0 \end{array} \right] \sim \left[\begin{array}{cc|c} 18 & 11 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

so eigenvectors are in the form

$$\begin{pmatrix} -11 \\ 18 \end{pmatrix} s, s \in \mathbb{R}, s \neq 0$$

[You lose half a mark if you don't specify the conditions on s .]

Hence the general solution is

$$\vec{Y}(t) = c_1 e^{70t} \begin{pmatrix} -9 \\ 1 \end{pmatrix} + c_2 e^{221t} \begin{pmatrix} -11 \\ 18 \end{pmatrix}$$

or $y_1(t) = -9c_1 e^{70t} - 11c_2 e^{221t}$ and $y_2(t) = c_1 e^{70t} + 18c_2 e^{221t}$.

b) Find the solution satisfying $y_1(0) = -16$ and $y_2(0) = -15$.

[2]

We have $y_1(0) = -9c_1 - 11c_2 = -16$ and $y_2(0) = c_1 + 18c_2 = -15$. Solving, we have $c_1 = 3$ and $c_2 = -1$. Hence the particular solution is

$$\vec{Y}(t) = 3e^{70t} \begin{pmatrix} -9 \\ 1 \end{pmatrix} - e^{-10t} \begin{pmatrix} -11 \\ 18 \end{pmatrix}$$

(1 mark for finding each of the constants)

QUESTION 3. Find the general solution of the system

$$\begin{aligned}x' &= -2x + 7y + 3z \\y' &= -2x + 7y - z \\z' &= -13z\end{aligned}$$

[6]

We have

$$\det(A - \lambda I) = -\lambda(\lambda + 13)(\lambda - 5) = 0$$

so the eigenvalues are $\lambda_1 = 0$ and $\lambda_2 = -13$ and $\lambda = 5$.

Corresponding to $\lambda_1 = 0$, we have

$$\left[\begin{array}{ccc|c} -2 & 7 & 3 & 0 \\ -2 & 7 & -1 & 0 \\ 0 & 0 & -6 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|c} -2 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

It follows that the eigenvectors are $\begin{pmatrix} 7 \\ 2 \\ 0 \end{pmatrix} r, r \in \mathbb{R}, r \neq 0$.

Corresponding to $\lambda_2 = -13$, we have

$$\begin{aligned}\left[\begin{array}{ccc|c} 11 & 7 & 3 & 0 \\ -2 & 20 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] &\sim \left[\begin{array}{ccc|c} 11 & 7 & 3 & 0 \\ -1 & 10 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \\ &\sim \left[\begin{array}{ccc|c} 0 & 117 & -5/2 & 0 \\ -1 & 10 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]\end{aligned}$$

so the eigenvectors are $\begin{pmatrix} -67 \\ 5 \\ 234 \end{pmatrix} s, s \in \mathbb{R}, s \neq 0$

Corresponding to $\lambda_3 = 5$, we have

$$\left[\begin{array}{ccc|c} -7 & 7 & 3 & 0 \\ -7 & 7 & -1 & 0 \\ 0 & 0 & -13 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

so the eigenvectors are $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} w, w \in \mathbb{R}, w \neq 0$

It follows that the general solution is

$$X(t) = c_1 \begin{pmatrix} 7 \\ 2 \\ 0 \end{pmatrix} + c_2 e^{-6t} \begin{pmatrix} -67 \\ 5 \\ 234 \end{pmatrix} + c_3 e^{4t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

(0.5 for each eigenvalue, 1 for each eigenvector, 1.5 for the general solution. Subtract 0.5 if conditions on eigenvectors are not present.)