Concordia University
Department of Mathematics & Statistics
MATH 209 EE Fundamental Mathematics II
Winter 2017 Midterm solutions

1. (a)

$$\lim_{x\to 0} \frac{x}{\sqrt{x+49}-7}$$
$$= \lim_{x\to 0} \frac{x(\sqrt{x+49}+7)}{(\sqrt{x+49}-7)(\sqrt{x+49}+7)}$$
$$= \lim_{x\to 0} \frac{x(\sqrt{x+49}+7)}{x+49-49}$$
$$= \lim_{x\to 0}(\sqrt{x+49}+7) = \sqrt{0+49}+7 = 14$$

(b) There are many possibilities of the functions $f(x)$ and $g(x)$. Here are the two simplest ones:
$f(x) = 5(5-x)$ and $g(x) = -25(x-5)$ giving

$$\lim_{x\to 5} f(x) = 5(5-x) = 0, \qquad \lim_{x\to 5} g(x) = -25(x-5) = 0$$

$$\lim_{x\to 5} \frac{f(x)}{g(x)} = \frac{1}{5}$$

2. Let $h(x) = 6 - x^3$,

$$\lim_{s\to 0} \frac{h(x+s)-h(x)}{s} = \lim_{s\to 0} \frac{(6-(x+s)^3)-(6-x^3)}{s}$$

$$\lim_{s\to 0} \frac{x^3-(x+s)^3}{s} = \lim_{s\to 0} \frac{x^3-(x^3+3x^2s+3xs^2+s^3)}{s}$$

$$\lim_{s\to 0} \frac{-3x^2s-3xs^2-s^3}{s} = \lim_{s\to 0}(-3x^2-3xs-s^2) = -3x^2$$

3. (a) $f(x) = -3x^{27} - 25, \qquad f'(x) = -3(27)x^{26}$

   (b) $g(x) = (5x^3 - 4)(\ln x^2 + 2)$,
   $g'(x) = 15x^2(\ln x^2 + 2) + (5x^3 - 4)(\frac{1}{x^2}2x)$

   (c)
$$h(x) = \frac{x^3 - e^x}{e^{3x} + \ln x}$$
$$h'(x) = \frac{(e^{3x} + \ln x)\frac{d}{dx}(x^3 - e^x) - (x^3 - e^x)\frac{d}{dx}(e^{3x} + \ln x)}{(e^{3x} + \ln x)^2}$$
$$h'(x) = \frac{(e^{3x} + \ln x)(3x^2 - e^x) - (x^3 - e^x)(3e^{3x} + \frac{1}{x})}{(e^{3x} + \ln x)^2}$$

   (d) $y = f(x) = x^4 + 2,\ x = 3.$ change in x is 0.2, i.e. $\Delta x = dx = 0.2$
   $dy = f'(x)\Delta x = [4x^3\Delta x]_{\ x=3} = 4(3)^3(0.2) = 21.6$

4. Amount $A = Pe^{rt}$ where P is Principal amount, r is the interest rate with continuous compounding and t is time in years
   Therefore we have, $20 = 10e^{9r}$ which implies $e^{9r} = 2, \ln e^{9r} = \ln 2$,
   $9r = \ln 2, r = \frac{\ln 2}{9} = 0.077$.
   Therefore annual percentage rate $\approx 0.077(100) = 7.7\%$

5. Incorrect, since Marginal average cost is required NOT the average marginal cost. The average cost first,
   $\bar{C}(x) = \frac{8000+7x}{x}$ and then differentiate this with respect to x to get the marginal average cost:
$$\frac{d}{dx}\bar{C}(x) = \bar{C}'(x) = \frac{d}{dx}\Big(\frac{8000}{x} + 7\Big) = -\frac{8000}{x^2}$$

6. Related rates problem: $y^3 = x^2$ at point $(-8, 4)$, $\frac{dy}{dt} = 3$, $\frac{dx}{dt} =?$
   $\frac{d}{dt}(y^3) = \frac{d}{dt}(x^2) \Rightarrow 3y^2\frac{dy}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{3y^2}{2x}\frac{dy}{dt} = \frac{3(4)^2}{2(-8)}(3) = -9$
   $\Rightarrow x$ coordinate is decreasing at 3 units/sec.

7. Implicit differentiation problem: given a curve $y - xy^2 + x^2 + 1 = 0$, need to find the equations of the tangent lines at points whose x coordinate is 1. First the coordinates of the points: substitute $x = 1$ in the equation: $y - (1)y^2 + (1)^2 + 1 = 0 \Rightarrow y^2 - y - 2 = 0 \Rightarrow (y-2)(y+1) = 0$, therefore the coordinates are (1,2) and (1, -1). Now the slopes of the tangents

at these points: differentiate implicitly, $\frac{d}{dx}(y - xy^2 + x^2 + 1 = 0) \Rightarrow \frac{dy}{dx} - 1.y^2 - x(2y\frac{dy}{dx}) + 2x = 0$, solving for $\frac{dy}{dx}$ gives:

$$\frac{dy}{dx} = \frac{y^2 - 2x}{1 - 2xy}$$

Slope at (1,2)=$\frac{2^2-2(1)}{1-2(1)(2)} = -\frac{2}{3}$, the equation is $(y - 2) = -\frac{2}{3}(x - 1)$, i.e.$y = -\frac{2}{3}x + \frac{8}{3}$
Slope at (1,-1)=$\frac{(-1)^2-2(1)}{1-2(1)(-1))} = -\frac{1}{3}$, the equation is $(y + 1) = -\frac{1}{3}(x - 1)$, i.e.$y = -\frac{1}{3}x - \frac{2}{3}$