Chapter 23

- Electric Fields

Michael Wong – PHY 1322 Spring/summer 2017

Electricity and Magnetism

- Originally thought to be two separate forces - eventually unified by Maxwell (1873).
- Branch of physics involving the study of electromagnetic force.
- History of EM in text, read if you're interested!
- Key points for this chapter:
  Charge, electric force, electric field, electric field lines, charge distributions

Electromagnetic Force

- In mechanics: concept of force – pushing, pulling, kicking, punching (**contact forces**)

- We also have **field forces** – no contact, represented by the 4 fundamental forces of nature:
  - 1) gravitational force
  - 2) **electromagnetic force** (between *charged* particles)
  - 3) strong force
  - 4) weak force

Some concepts:

- Electric charge (q) is **quantized**.
  - (eg. Money is quantized, time is not).
- Objects can have **negative** or **positive** charge or be **neutral**.
  - **Magnitude** of charge is important.
- "**Like** charges **repel**, opposite charges attract".
- Electric charge is **conserved**.

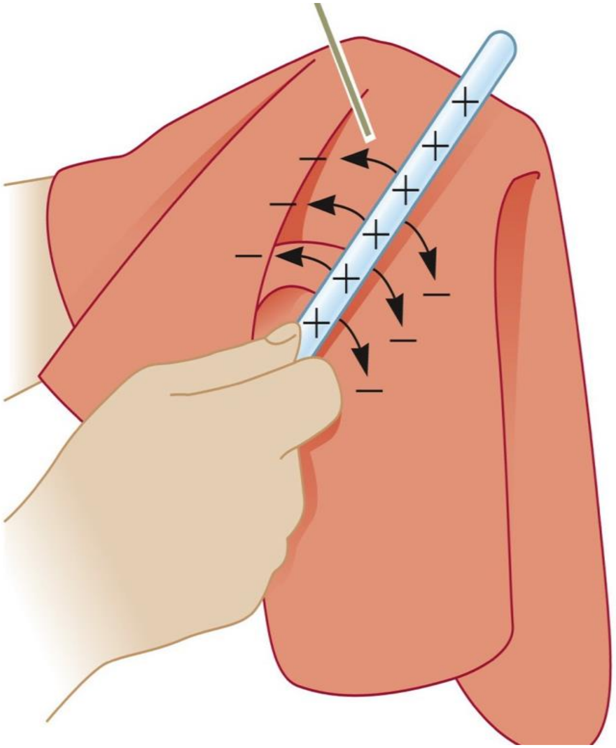
Conservation of Electric Charges

In an isolated system:

- Electric charge is conserved, not created!

- Rubbing of silk and glass:
  - Silk becomes negatively charged.
  - Glass becomes positively charged.

- Only electrons move!



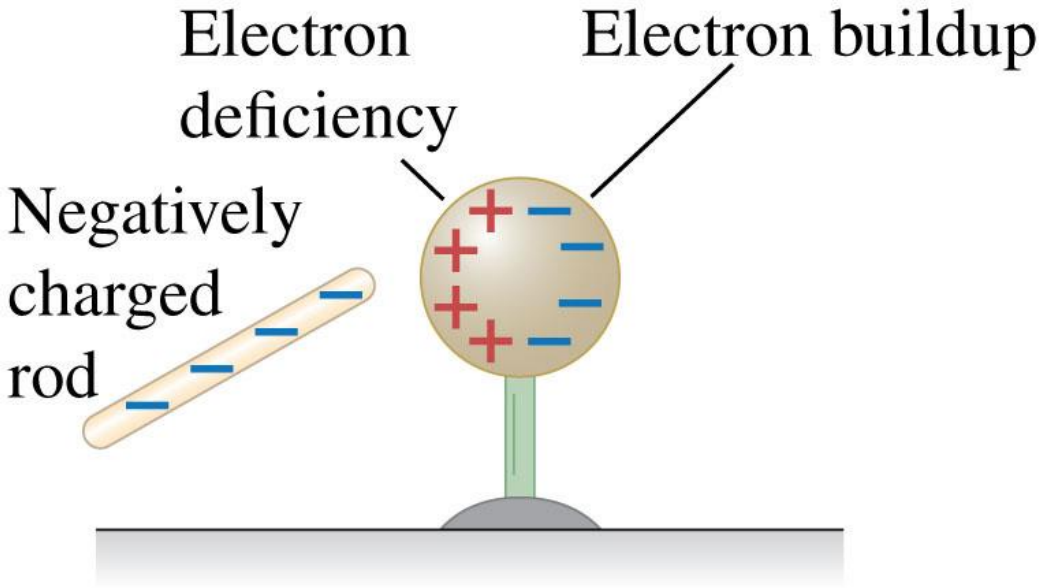
Classification of Materials for Charging

Three types of materials:

- Conductors: contains free electrons, can move around.
  - Examples include copper and aluminum.
- Insulators: all electrons are bound to an atom
  - Examples include glass, rubber, wood.
- Semiconductors: properties lie between conductors and insulators.
  - Examples include silicon and germanium.
  - Electrical properties can be easily changed.

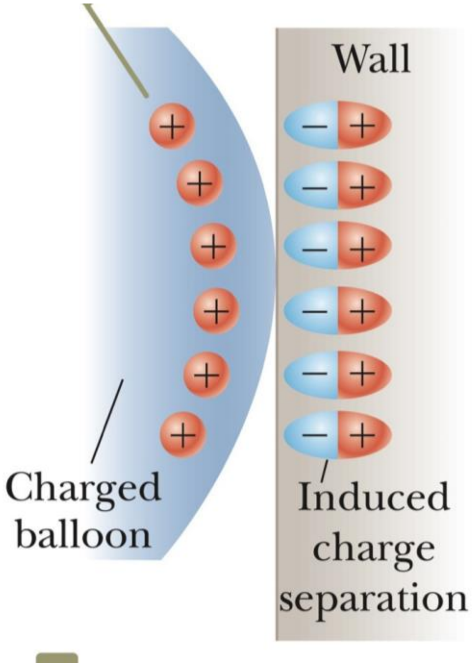
Induced Charges

- Charging by **induction** is different than conduction.
  - No touch!



Charge rearrangement in Insulators

- The idea of induction occurs with insulators too.
  - a) A positively charged balloon brought to a wall.
  - b) The charges within the wall's molecules rearrange!
  - c) The proximity of the balloon (+) results in attractive force.



- Demonstrate:
  - Charged plastic comb
  - Glass rod with water stream

Coulomb's Law

- For two point charges, Coulomb's law is represented by the following equation:

$$F_e = k_e \frac{|q_1||q_2|}{r^2} \text{ (in units of N)}$$

  - k_e is the **Coulomb constant**.
    - $k_e = 8.9876 \times 10^9 \ \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$
  - SI unit of charge is the Coulomb (1 C)
  - Charge of an electron is $e = 1.602 \times 10^{-19}\ \mathrm{C}$

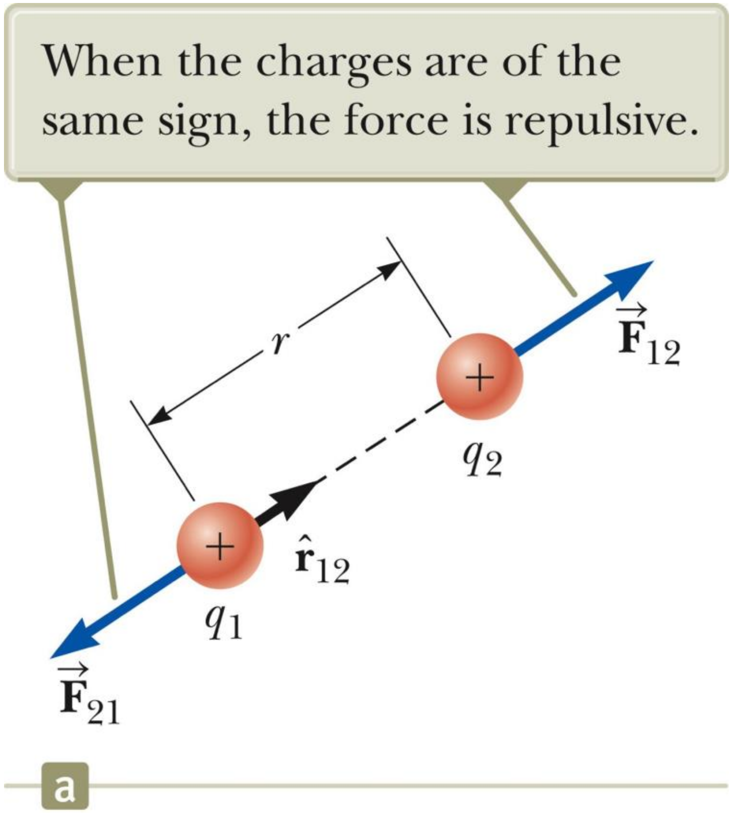
Vector form of Coulomb's Law

- Electric force is a vector quantity:

  $\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12}$ where $\hat{\mathbf{r}}_{12}$ is a unit vector from q_1 to q_2.

- Electrical force obeys Newton's Third Law:
  - $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$

- Note: relative ≠ absolute charge direction!



Multiple Charges (Superposition)

Eg. For 4 charges present, the resultant force on any charge is equal to the vector sum of forces exerted on it by all other charges:

- $\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{41}$

where $\vec{\mathbf{F}}_1$ is the resultant force on the charge q_1

Example 23.2 – Find the Resultant Force

Example 23.3 – Zero Resultant Force

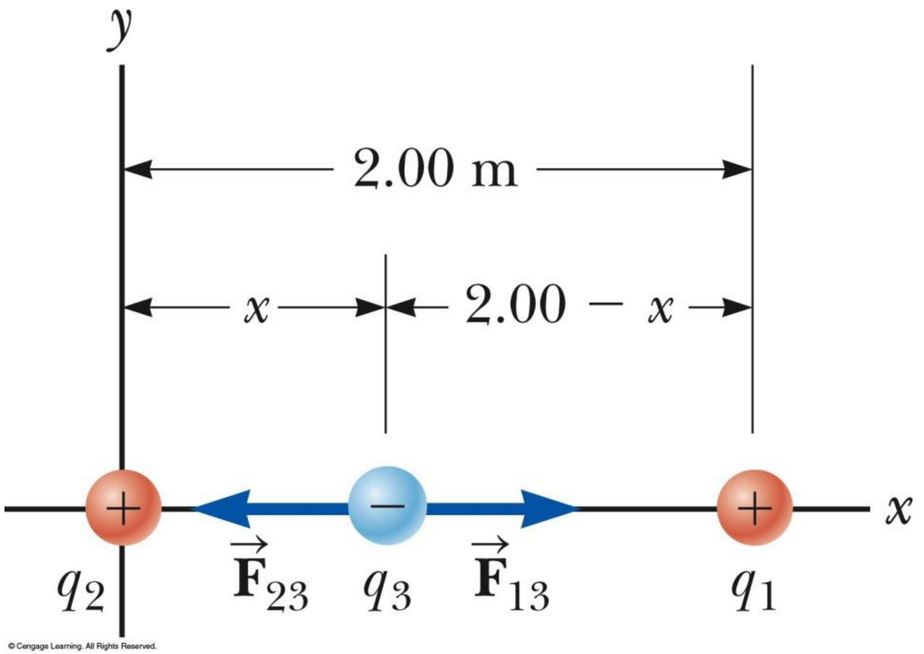
Consider 3 charges on the *x*-axis

$$q_1 = 15.0\ \mu\text{C}$$
$$q_2 = 6.0\ \mu\text{C}$$
$$\vec{\mathbf{F}}_3 = 0$$

Find *x*.

Result is a quadratic.

$x = 0.775$ m **or** $x = -3.44$ m

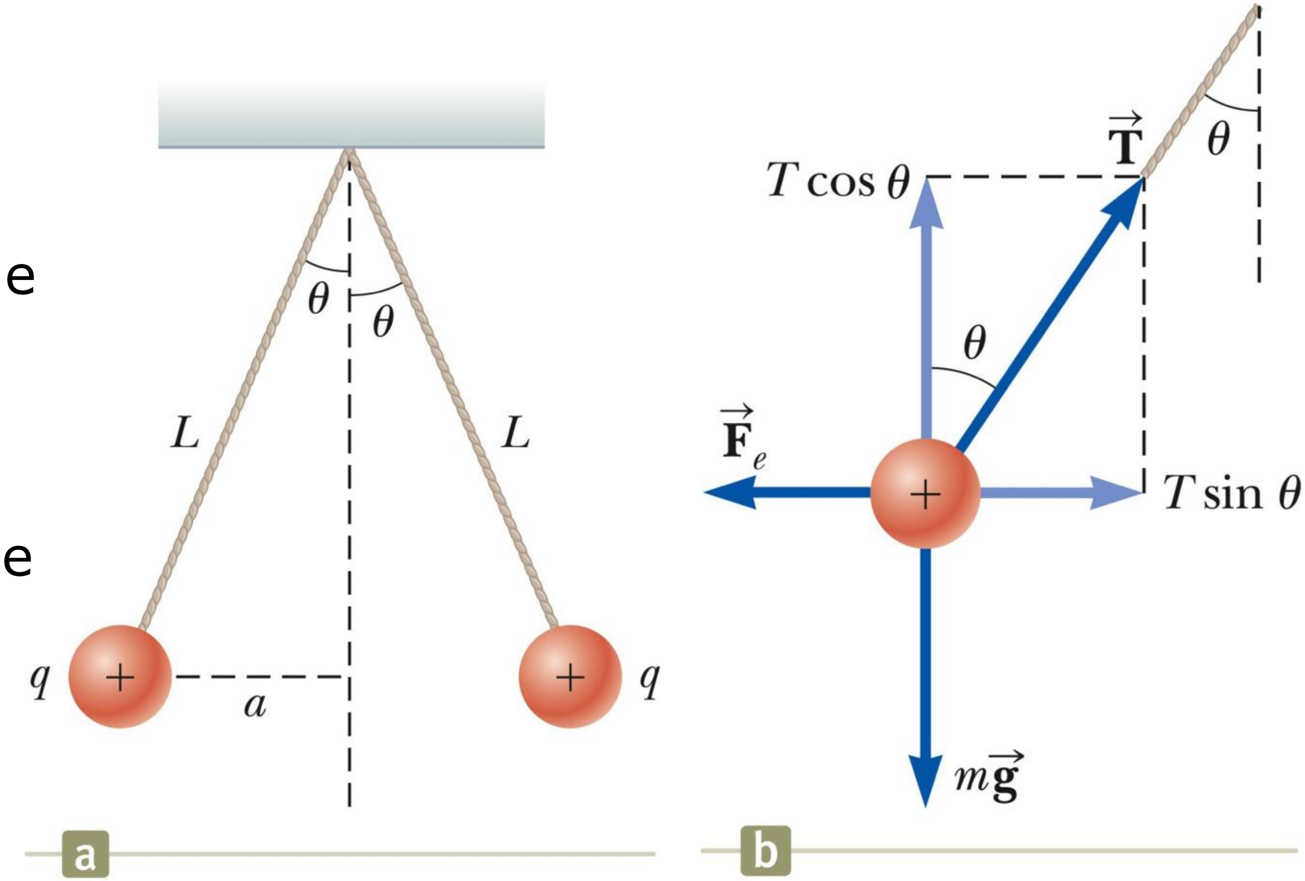


Example 23.4 – Charges on Spheres

Consider the 2 hanging spheres.

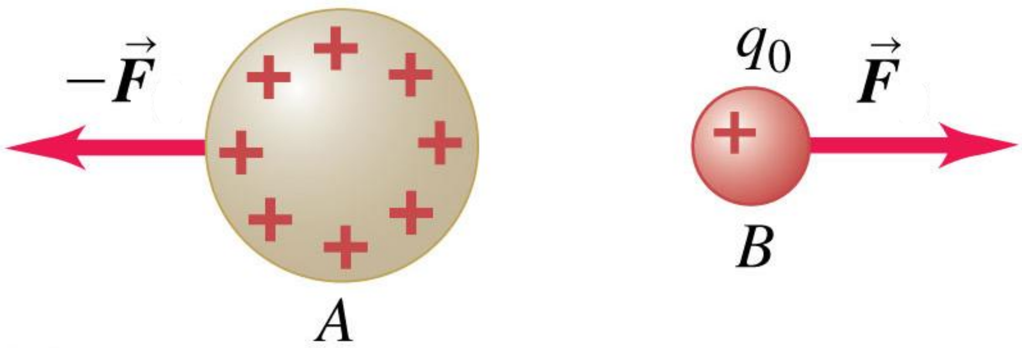
Find the magnitude of the charge on each spheres.

Follow the example in Serway and Jewitt as an exercise.



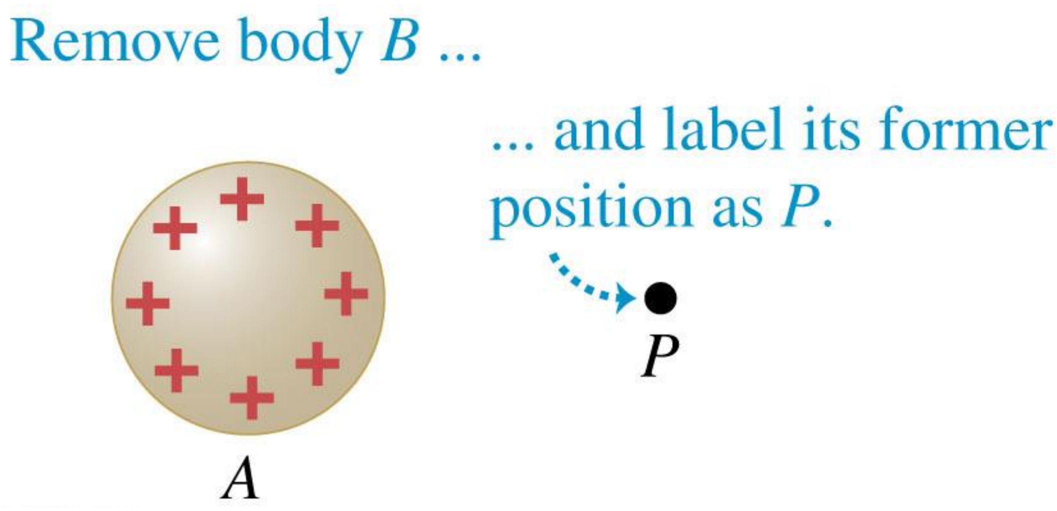
Introduction to Electric Field

- To introduce this section, consider the mutual repulsion of bodies *A* and *B*.



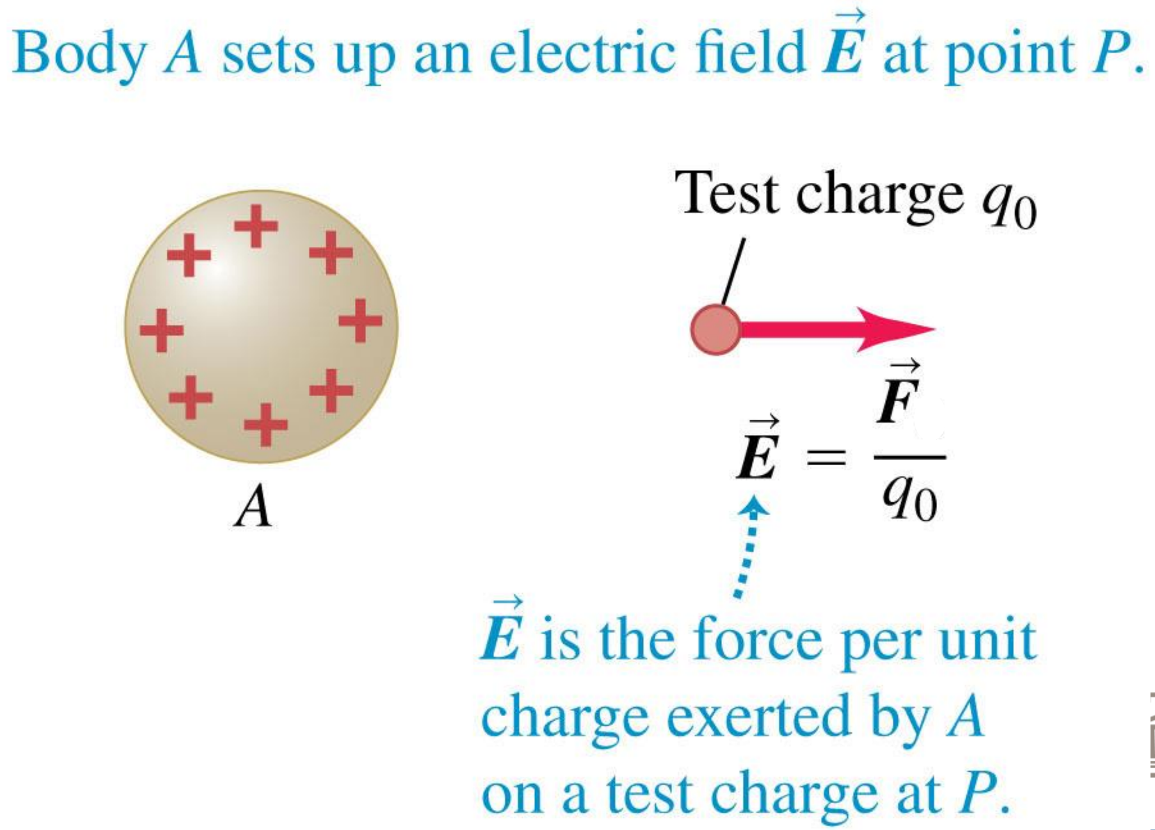
Introduction to Electric Field

- Next, remove body B and label it's position as P.



Introduction to Electric Field

- Because A has a charge, it produces an **electric field**.
- We measure the field using a test charge.



Electric Field Direction

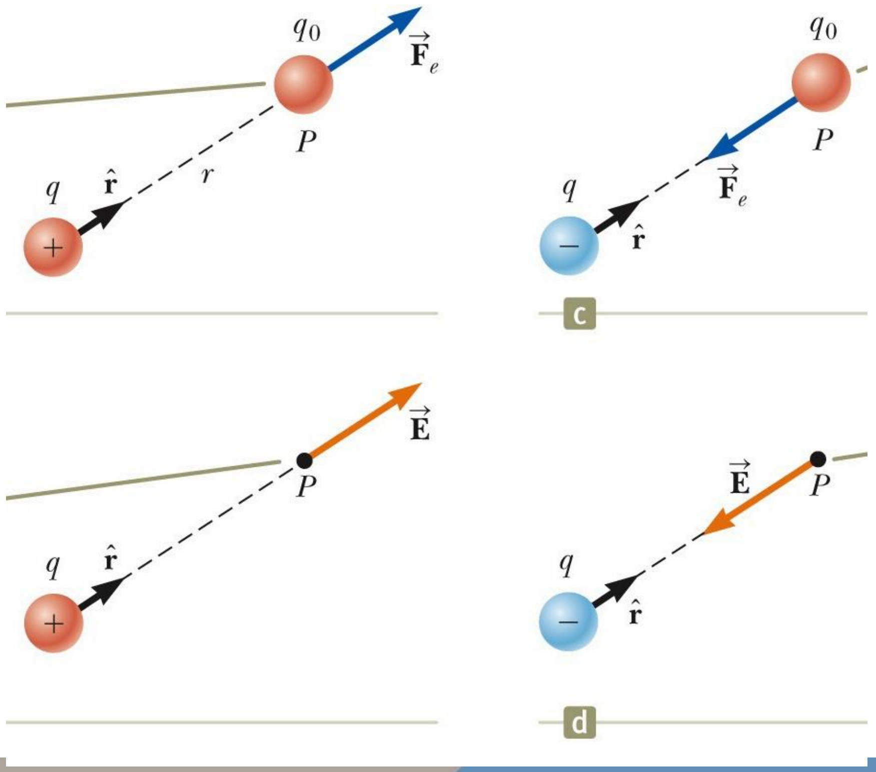
- For point charge q and test charge q_0, for force of q on q_0 is:

$$\vec{\mathbf{F}}_e = k_e \frac{q q_0}{r^2} \hat{\mathbf{r}}$$

- Therefore the electric field at q_0 is:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_0} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

- Notice that $\vec{\mathbf{E}}$ and $\vec{\mathbf{F}}_e$ can be in opposite directions depending on the sign of the two charges.



Multiple or Continuous Charges

- If we have multiple point charges creating the electric field, we calculate all E-fields individually then add them (**superposition of fields**):

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{{r_i}^2} \hat{\mathbf{r}}_i$$

- For many closely spaced point charges, we model the system of charges as a **continuous charge distribution**.

- Examples would be a continuous charge along a line, a surface, or throughout some volume.

Electric Field due to Continuous Charge

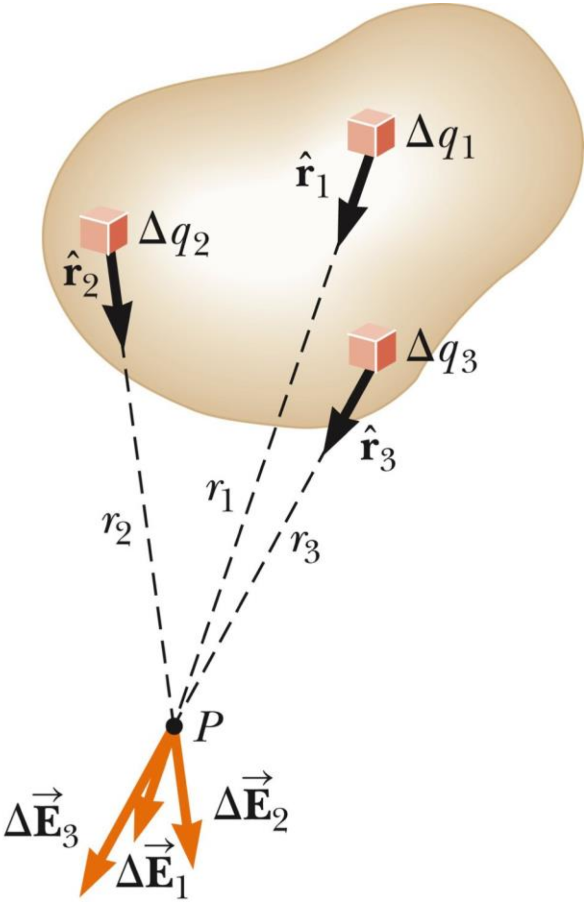
- Divide this charge distribution into small elements that each contain Δq.

- At the P, calculate the electric field due to one of the elements.

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

- Evaluate the total E-Feld by summing up the contribution from all elements:

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{{r_i}^2}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$



Different Charge Densities

- Different types of charge distributions have different charge densities. For uniform continuous charges with charge Q:
  - Volume charge density throughout volume V:
    - $\rho \equiv \frac{Q}{V}$ with units C/m^3
  - Surface charge density throughout area A:
    - $\sigma \equiv \frac{Q}{A}$ with units C/m^2
  - Linear charge density throughout line l:
    - $\lambda \equiv \frac{Q}{l}$ with units C/m

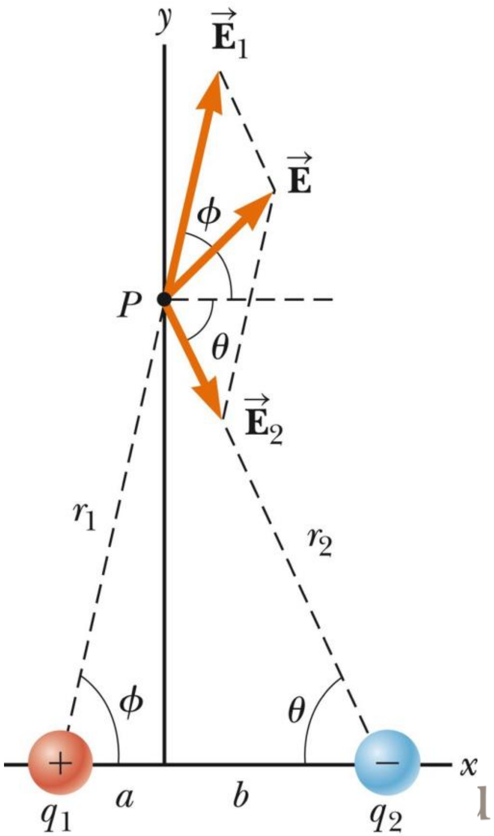
Calculating Electric Fields: Strategies

- Follow these steps:
  - 1) Conceptualize (mental picture of the field(s))
  - 2) Categorize
    - Individual charge? Groups of charges? Continuous?
  - 3) Analyze
    - Use the superposition of charges for groups of charges
    - Use integrals for continuous charges
  - 4) Finalize (check with your mental picture)

Ex. 23.6 – E-Field due to 2 charges*

- Charges $q_1 = 5.0\ \mu\text{C}$ and $q_2 = -2.0\ \mu\text{C}$ are located on the x axis at distances $a = -2$ and $b = 4$ from the origin.

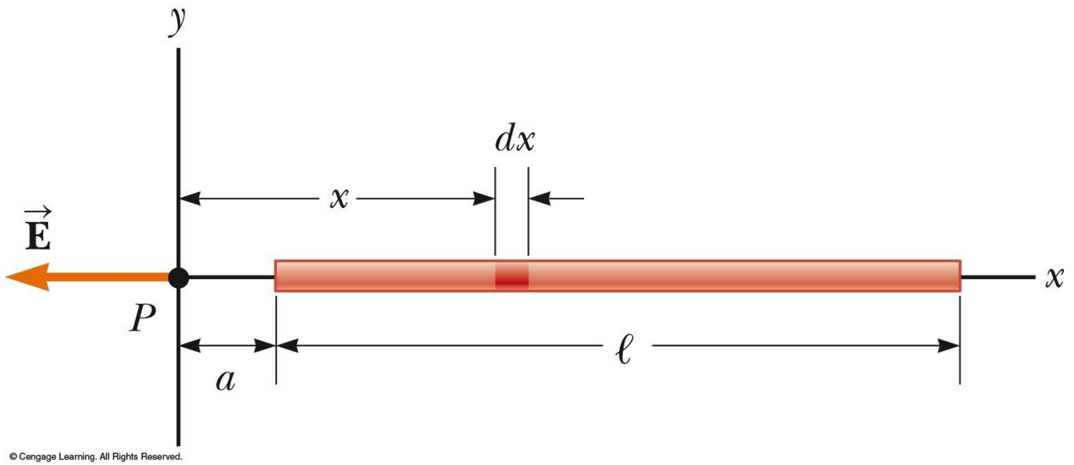
  A) Find the components of the net electric field at the point P which is at position (0, 10).



Ex. 23.7 – E-Field due to charged rod

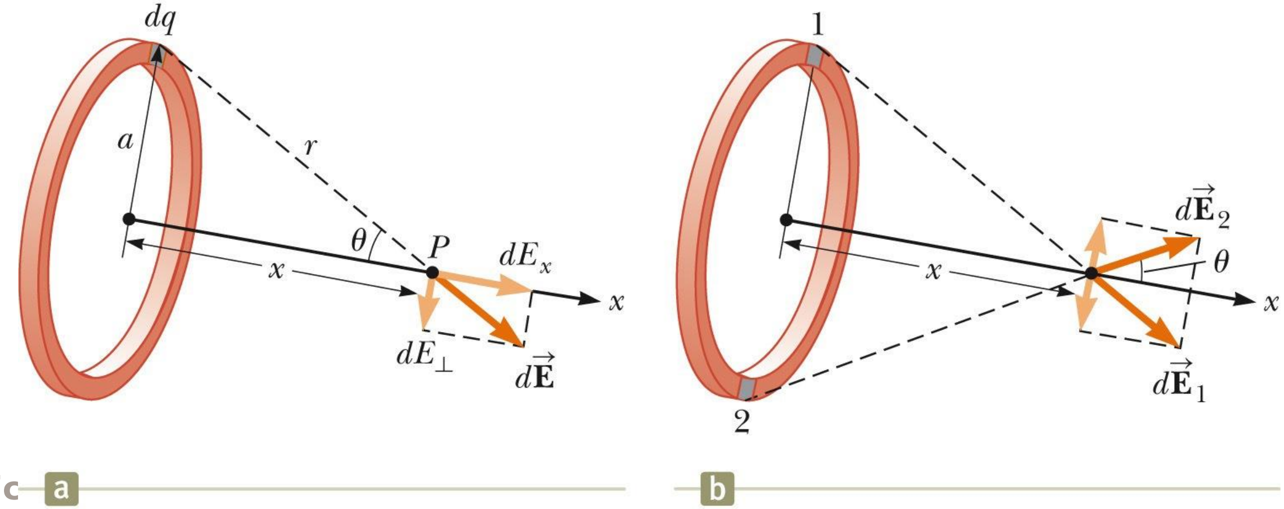
- A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.
- ....
- $E = \dfrac{k_e Q}{a(\ell + a)}$

(profit!)



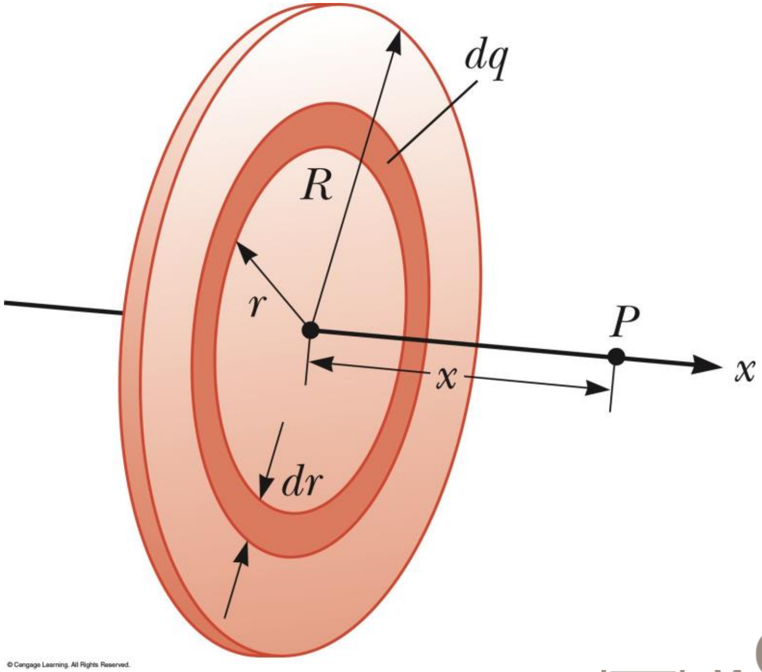
Ex. 23.8 – E-Field of a Uniform Ring

- A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring.
- …
- $E = \frac{k_e x}{(a^2+x^2)^{3/2}} Q$



Ex. 23.9 – E-Field (Uniform Charged Disk)

- A disk of radius R has uniform surface charge density σ. Calculate E-Field at P that lies along the central perpendicular axis of the disk and a distance x from the center.
- …
- $E = 2\pi k_e \sigma \left[1 - \frac{x}{(R^2+x^2)^{1/2}}\right]$

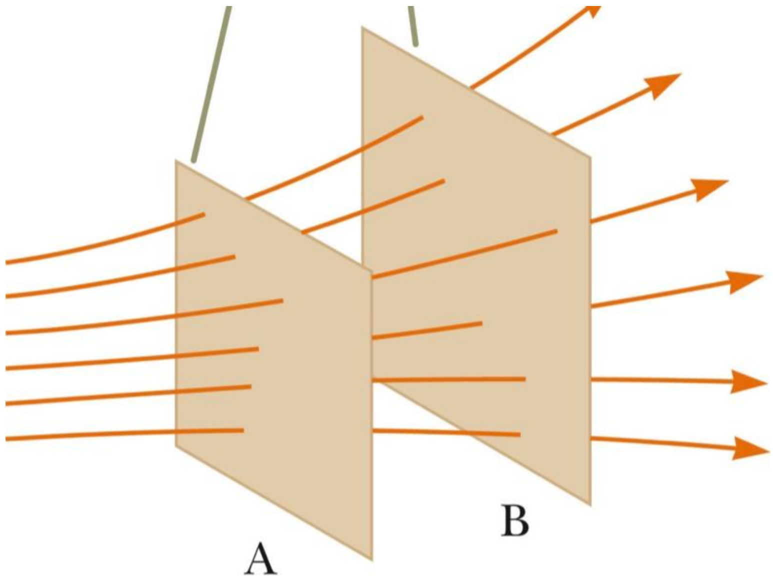


Electric Field Lines

- **Electric field lines** were introduced by Faraday and are a visual representation of the E-Field.

- $\vec{\mathbf{E}}$ is tangent to the field line at every point. The line has the same direction as the vector.

- The density of lines through a surface perpendicular to the lines is proportional to the magnitude of the E-field in that region.

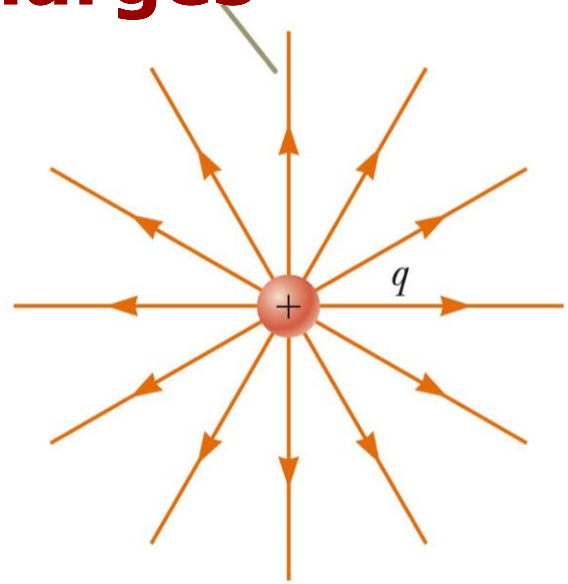


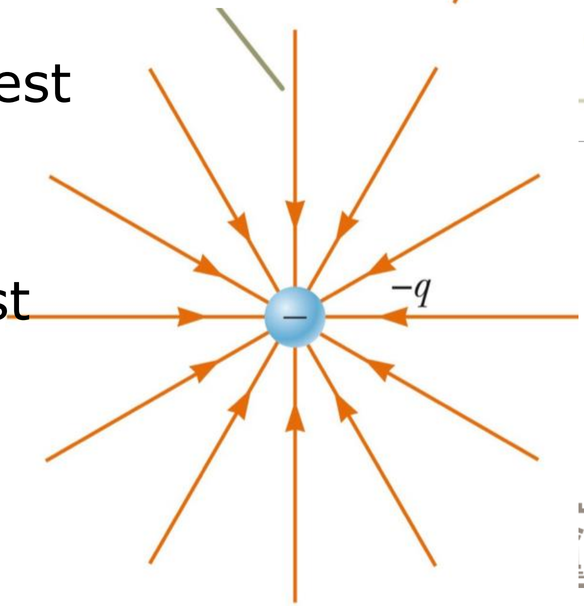
Electric Field Lines - Point Charges

- Field lines radiate outward (or inward) in all directions.
  - In 2-D, the distribution is circularly outwards (inwards).
  - In 3-D, the distribution is spherical.

- Outward lines means a positive test charge would be repelled away.

- Inward lines means a positive test charge would be attracted in.



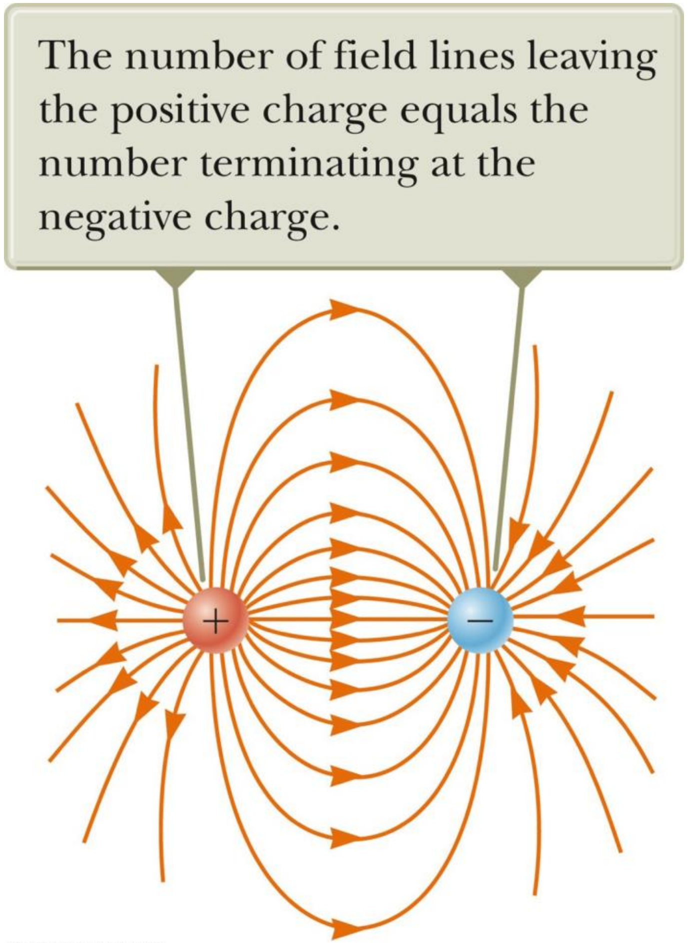


Drawing Electric Field Lines

- Lines begin on positive charges and terminate on negative charges.
- # of lines drawn leaving a + to a — is proportional to the magnitude of the charge.
- **Field lines may not cross.**
- Note! Field lines are not material objects you can manipulate They are just a pictorial representation of the field.

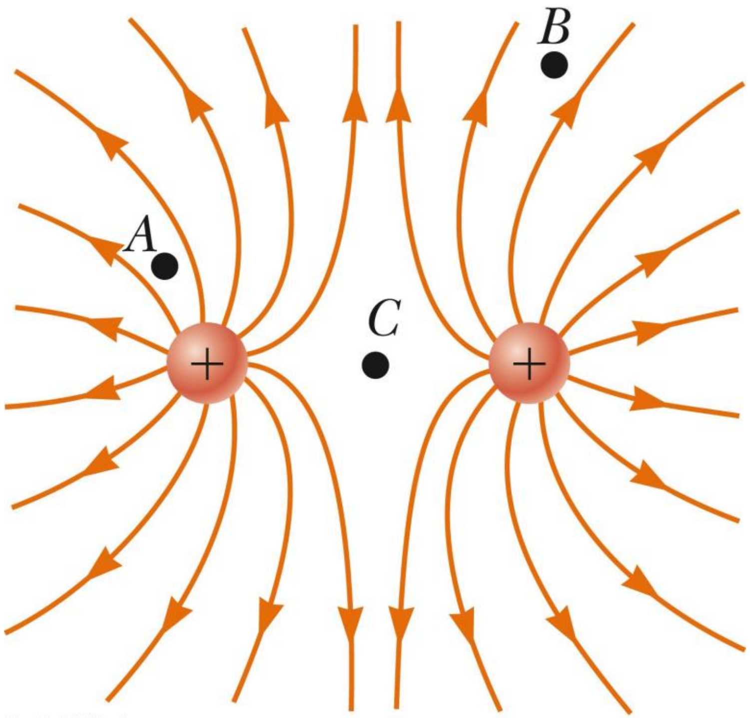
Electric Field Lines of a Dipole

- The charges are equal and opposite.
- The number of field lines leaving the positive charge is equal to the number of field lines terminating on the negative charge.



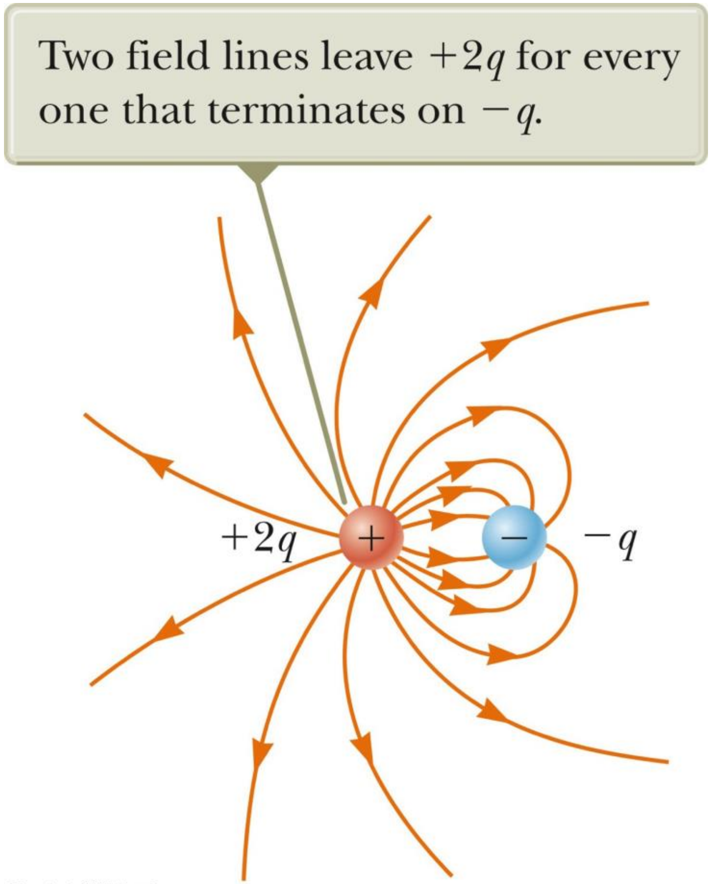
Electric Field Lines of Like Charges

- Charges (q) are equal and positive.
- The **same** number of lines leave each charge.
- At very far away, the E-field is approx. equal to a single charge of $2q$.
- Since there are no negative charges nearby, the field lines are infinitely long.



Electric Field Lines of Unequal Charges

- Positive charge $+2q$ and negative charge $-q$.
- Therefore double the lines leaving $+2q$ that terminate in $-q$.
- At very far away distance, the field is approx. equal to a single charge of $+q$.



Motion of a Charged Particle
(in a uniform electric field)

- When a charged particle is placed in an E-field, it experiences an electrical force.
- If there are no competing forces then the electrical force is the net force.
- By Newton's 2nd law, the particle will accelerate.

Equation of Motion

- Recall our equation for the electric force and add in the well known Newton's 2nd law equation:

$$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}} = m\vec{\mathbf{a}}$$

and acceleration is given by: $\vec{\mathbf{a}} = \frac{q\vec{\mathbf{E}}}{m}$

- If the E-field is uniform then acceleration is constant.

- If the particle has positive charge then its acceleration is in the direction of the field. Opposite if it has negative charge.

Ex. 23.11 - Electron in a Uniform Field

- The electron has initial velocity into the uniform electric field.
- Due to the E-field, the electron experiences a downwards acceleration which will make its trajectory parabolic.
- Equations to remember:

  $F = ma$

  $x_f = x_i + v_x t$

  $y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$