Consolidated Notes : Math 202 Midterm stuff

Factor Theorem

$(x-c)$, using c in Synthetic Div
$f(c)$ = remainder
$f(c)=0$, c is a root
& $(x-c)$ is a factor

Multiplicity

i.e. -5 is zero, multiplicity 4
$\downarrow$
$(x+5)^4$

Even/Odd

even : $f(-x)=f(x)$ odd : $f(-x)=-f(x)$
symmetry : y-axis symmetry : w/t origin

even $\pm$ odd $\rightarrow$ neither CONST : even
$\frac{\text{odd}}{\text{even}}$ or $\frac{\text{even}}{\text{odd}}$ $\rightarrow$ odd $\hookrightarrow \sim x^0$

Real Zeros Theorem

in polynomial $A_n x^n + A_1 x^{n-1} \ldots + A_0 x^0$

possible zeros : $\frac{\text{Factors of } a_0}{\text{Factors of } a_n}$

Descartes' rules of Sign

$f(x) \rightarrow$ # positive zeros = # of alternating signs − even integer

$f(-x) \rightarrow$ # negative zeros = # of alternating signs − even integer

Theorem on bounds

from synth division
for $c>0$, all # ≥ 0, c: upper bound
for $c<0$, all # alternating, c: lower bound.

→ synthetic division works
→ $f(x)^n$, $n \geq 1$, exactly n complex zeros
real

Intermediate Value Theorem

from synthetic division

$f(a)>0$, $f(b)<0$
there is a $f(c)=0$
on $c \in [a,b]$

Complex numbers $i=\sqrt{-1}$

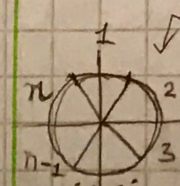
Trig form : $\times$ / $\div$
$[r \times \text{ or } \div] \cdot [\text{cis}(\theta_A \pm \theta_B)]$

De Moivre's Theorem : $[r(\text{cis}\,\theta_A)]^s$

$[r^s] \cdot [\text{cis}(s \cdot \theta_A)]$

Roots

unity : $z^n = 1$ $n \geq 1$
$\text{cis}\left(\frac{2k\pi}{n}\right)$ for $k=0,1,2\ldots$ to $n-1$



n^{th} roots : $z = r(\text{cis}\,\theta)$
$(r^{1/n})\left[\text{cis}\left(\frac{\theta + 2k\pi}{n}\right)\right]$
for $k=0,1,2,3,\ldots,n-1$

Horizontal Asymptotes

in $\frac{A \cdot p(x)^n}{B \cdot q(x)^m}$

$n<m \approx \frac{1}{x}$

$n=m \approx \frac{A}{B}$

$n-m = 1 \approx x$
$\downarrow$
slanted asymptote around $x \rightarrow \infty$

Conjugate Zero Theorem : $(a+bi) \rightarrow$ zero, $(a-bi)$ also

After Midterm stuff

Mathematical Induction

1. Basis (Prove P_1 true)
2. Induction hypothesis (Assume P_K is true)
3. Induction step (Prove P_{K+1} is true)

LHS = RHS

$$\underbrace{P_K}_{P_1+P_2+P_3+\cdots+P_K} + P_{K+1} = RHS$$

step ② RHS

Sigma Notation

Σ: greek symbol for summation

$$\sum_{i=\text{lower index}}^{\text{upper index}} f(i) = f(i)_{\text{lower index}} + f(i)_{+1} + f(i)_{+2} + \cdots f(i)_{\text{upper index}}.$$

$$\sum_{i=0}^{n} (-1)^i \cdot f(i) \rightarrow \text{alternating}$$

Pascal triangle.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

n	Σ
0	1
1	2
2	4
3	8
4	16
n ...	2^n

Sequences

Arithmetic

$a_2 - a_1 = d$

$a_n = a_1 + d(n-1)$

$S_n = \frac{n}{2}(a_1 + a_n)$

Geometric

$r = \frac{a_2}{a_1}$

$a_n = a_1 \cdot r^{(n-1)}$

$S_n = \frac{a_1 \cdot (r^n - 1)}{r-1}$

$S_\infty = \frac{a_1}{1-r}$

Annuities

$$FV = R\left[\frac{(1+i)^n - 1}{\frac{i}{m}}\right]$$

if $i \Rightarrow$ yearly else $m = 1$

n equal period in year

$m = n$

Binomial Theorem

$$(a+b)^n = \sum_{K=0}^{n} \binom{n}{K} a^{n-K} \cdot b^K$$

Factorials

$K! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot K-1 \cdot K$ $\quad 0! = 1$

Permutations and Combinations

Order Matters : Permutations $P(n,r) = \frac{n!}{(n-r)!}$

$P(n,n) = n!$

Order $\neq$ Matters : Combinations $C(n,r) = \frac{n!}{r!(n-r)!}$

↓

based on pascal Δ

↳ $C(n,r) = C(n, n-r)$

↳ pascal Δ symmetry

Basic Counting principle.

H/T then H/T = $2 \cdot 2 = 4$

AND → multiply

OR → add.

Complements Principle.

fringe way = # Total ways − # opposite of fringe way