UNIVERSITY OF CALGARY

Faculty of Science
Department of Mathematics & Statistics

SOLUTIONS

MATH 211 – LINEAR METHODS I – FALL 2012
MIDTERM 1
October 9, 2012
Exam version: 0

COURSE INFORMATION AND STUDENT IDENTIFICATION

INSTRUCTOR NAME	SECTION NUMBER	LAB NUMBER	STUDENT I.D. NUMBER	FIRST NAME	LAST NAME

EXAMINATION RULES

1. No Calculators, electronic equipment, or paper material other than this examination and scantron sheet allowed.
2. Use the back of the previous page for rough work or calculations.
3. Students arriving late will not normally be admitted after one-half hour of the examination time has passed.
4. No candidate will be permitted to leave the examination room until one hour has elapsed after the opening of the examination.
5. All enquiries and requests must be addressed to supervisors only.
6. Candidates are strictly cautioned against:
   (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
   (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
   (c) making use of calculators and/or portable computing machines not authorized by the instructor;
   (d) leaving answer papers exposed to view;
   (e) attempting to read other students' examination papers.

   The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.
7. Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
8. Candidates are cautioned against writing in their answer books any matter extraneous to the actual answering of the question set.
9. A candidate must report to a supervisor before leaving the examination room.
10. Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.

Part A: Mark your answers on the scantron sheet provided. **Exam version: 0**

Each question is worth 2 points.

1. What can you say about the following system of linear equations?

$$\begin{array}{rcrcrcl} x & + & 2y & + & 3z & = & 0 \\ x & + & y & + & z & = & 2 \end{array}$$

   (a) It has no solution.
   (b) It has one solution.
   (c) It has three solutions.
   (d) It has infinitely many solutions.
   (e) None of the above.

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 2 \end{bmatrix} \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 2 & -2 \end{array}\right]$$

2. What can you say about the following system of linear equations?

$$\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & + & 3x_3 & + & x_4 & = & 0 \\ x_1 & + & 2x_2 & + & 4x_3 & + & x_4 & = & 0 \end{array}$$

   (a) It has no solution.
   (b) It has one solution.
   (c) The general solution involves one parameter.
   (d) The general solution involves two parameters.
   (e) None of the above.

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 1 & 2 & 4 & 1 & 0 \end{bmatrix} \xrightarrow{\text{ref}} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

3. Which of the following options constitute the general solution of the following system of linear equations?

$$\begin{array}{rcrcrcl} x & - & y & - & z & = & 0 \\ x & + & y & + & z & = & 4 \end{array}$$

   (a) $x = 2,\ y = 0,\ z = 2$
   (b) $x = 2,\ y = 1,\ z = 1$
   (c) $x = 2,\ y = 2 - s,\ z = s$
   (d) $x = 2,\ y = 2 + s,\ z = s$
   (e) None of the above.

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 4 \end{bmatrix} \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 2 \end{array}\right]$$

$x = 2$
$y = 2 - S$
$z = S$

Exam version: 0

4. Consider the following homogeneous system of linear equations:

$$\begin{array}{rcrcrcrcrcr} x_1 & + & 2x_2 & + & 4x_3 & + & x_4 & - & 3x_5 & = & 0 \\ x_1 & + & 3x_2 & + & 4x_3 & + & x_4 & - & 3x_5 & = & 0 \\ x_1 & + & 2x_2 & + & 4x_3 & + & 2x_4 & - & 6x_5 & = & 0 \end{array}$$

Any solution of the above system $AX = 0$ is a linear combination of the following matrices:

(a) $[-4\ 0\ 1\ -6\ -2]^T$.
(b) $[8\ 0\ -2\ 3\ 1]^T$.
(c) $[-4\ 0\ 1\ 0\ 0]^T$ and $[-8\ 0\ 2\ 0\ 0]^T$.
(d) $[4\ 0\ -1\ 0\ 0]^T$ and $[0\ 0\ 0\ 3\ 1]^T$.
(e) None of the above.

rref $\left[\begin{array}{ccccc|c} 1 & 0 & 4 & 0 & 6 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 0 \end{array}\right]$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -4s \\ 0 \\ s \\ 3t \\ t \end{bmatrix} = s\begin{bmatrix} -4 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

5. Consider a system of m linear equations in n variables. If the system is inconsistent, then the rank of its augmented matrix is:

(a) 0.
(b) At least 1.
(c) At least 2.
(d) n+1.
(e) None of the above.

Need a row of the form

$$\left[\begin{array}{ccccc|c} & & * & & & \\ 0 & 0 & \cdots & & 0 & 1 \\ & & * & & & \end{array}\right]$$

6. The system of linear equations

$$\begin{array}{rcrcrcrcr} 2x_1 & - & 2x_2 & + & 7x_3 & + & 5x_4 & = & -9 \\ -2x_1 & + & 2x_2 & - & 9x_3 & - & 3x_4 & = & 3 \\ x_1 & - & x_2 & + & 4x_3 & + & 2x_4 & = & -3 \\ -3x_1 & + & 3x_2 & - & 12x_3 & - & 6x_4 & = & 9 \end{array}$$

(a) has a unique solution.
(b) has infinitely many solutions with one parameter.
(c) has infinitely many solutions with two parameter.
(d) is inconsistent.
(e) None of the above.

rref $\left[\begin{array}{cccc|c} 1 & -1 & 4 & 2 & 3 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

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7. A system of five linear equations in four variables has coefficient matrix C and augmented matrix A. If rank $C = 4$, then which of the following statements about this system must be true?

   (a) It has a unique solution or infinitely many solutions.

   (b) It has no solution or infinitely many solutions.

   (c) It has no solution or a unique solution.

   (d) It has no solution, a unique solution, or infinitely many solutions.

   (e) None of the above.

rref. $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & 1 & 0 & z \\ 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & \end{array}\right]$

8. Let $X_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $X_2 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, $X_3 = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$, $Y = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, and $Z = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

   (a) both Y and Z are linear combinations of X_1, X_2, and X_3.

   (b) neither Y nor Z is a linear combination of X_1, X_2, and X_3.

   (c) Y is a linear combination of X_1, X_2, and X_3, but Z is not.

   (d) Z is a linear combination of X_1, X_2, and X_3, but Y is not.

   (e) None of the above.

$$\begin{array}{ccc|cc} & & & y & z \end{array}$$
$$\left[\begin{array}{ccc|cc} 1 & 1 & -1 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \\ -1 & -2 & 3 & 0 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|cc} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & -2 & 2 & -1 \\ 0 & 0 & 0 & 3 & 0 \end{array}\right]$$
$$\begin{array}{ccc|cc} & & & y & z \end{array}$$

9. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection through the line $y = -x$. The matrix of T is:

   (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

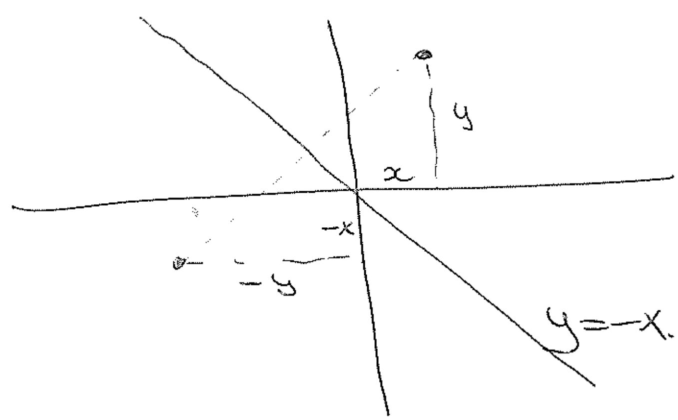
   (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

   (e) None of the above.

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ -x \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$



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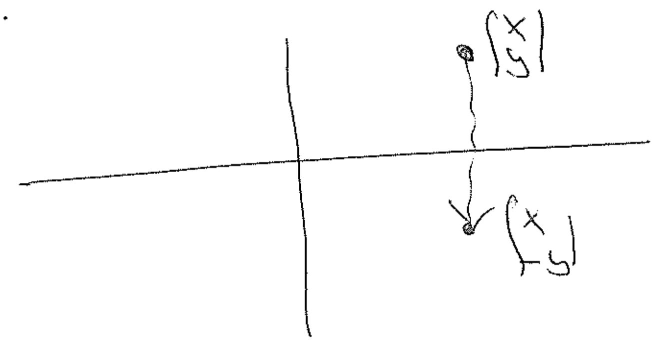
10. Consider the matrix transformation T induced by the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Then T is:

   (a) reflection in the x axis.
   (b) reflection in the y axis.
   (c) rotation through $-\pi/2$ (clockwise).
   (d) rotation through $\pi/2$ (counterclockwise).
   (e) None of the above.

$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$



11. Let A and B be $m \times n$ matrices, and suppose U is an invertible matrix so that $B = UA$. Then

   (a) $A = BU^{-1}$.
   (b) $A = BU$.
   (c) $A = U^{-1}B$.
   (d) $A = UB$.
   (e) None of the above.

$B = UA$

$U^{-1}B = U^{-1}(UA)$

$U^{-1}B = (U^{-1}U)A$

$U^{-1}B = A$

12. Find values of a so that the matrix $A = \begin{bmatrix} 1 & 2 \\ a & 1 \end{bmatrix}$ is invertible.

   (a) $a = 0$.
   (b) $a \neq 0$.
   (c) $a = \frac{1}{2}$.
   (d) $a \neq \frac{1}{2}$.
   (e) None of the above.

$\det A = 1 \cdot 1 - 2 \cdot a = 1 - 2a$

$\det A = 0$ for $a = 1/2$

So A is invertible

$\Leftrightarrow \det A \neq 0$

$\Leftrightarrow a \neq 1/2$.

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13. Find A if

$$(A-2I)^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

$$A - 2I = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

(a) $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

(b) $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.

(c) $A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix}$.

(d) $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

(e) None of the above.

14. Compute and simplify the following matrix product:

$$\left(\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ a & -b \end{bmatrix} \right) \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 2a & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -2a + 0 = -2a$$

(a) $a+b$

(b) $2a$

(c) $-2a$

(d) $2a+b$

(e) None of the above.

15. Consider the x-shear transformation T given by

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+ay \\ y \end{bmatrix}. = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= AX$$

Then T^{-1} is induced by the following matrix:

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix}$$

T^{-1} is induced by A^{-1}!

(a) $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$.

(b) $\begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix}$.

(c) $\begin{bmatrix} -1 & a \\ 0 & -1 \end{bmatrix}$.

(d) $\begin{bmatrix} 1 & \frac{1}{a} \\ 0 & 1 \end{bmatrix}$.

(e) None of the above.

SOLUTION

Part B: Provide your full solution on these pages. **Exam version: 0**

Question 1 (5 points) With full explanation, find (if possible) conditions on a and b such that the system has no solution, one solution, or infinitely many solutions.

$$\begin{array}{ccccccc} x & & & + & z & = & 2 \\ & & y & & & = & -3 \\ x & + & y & + & a & = & b-1 \end{array}$$

Write a clear and complete solution, and provide your final answer in the box below:

Aug. Matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -3 \\ 1 & 1 & a & b-1 \end{array}\right]$$

$-R_1+R_3$ $\downarrow$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 1 & a-1 & b-3 \end{array}\right]$$

$-R_2+R_3$ $\downarrow$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & a-1 & b \end{array}\right]$$

ANSWER:
- No solution: $a=1 \quad b\neq 0$
- One solution: $a-1\neq 0 \quad (a\neq 1)$
- Infinitely many solutions: $a=1 \quad b=0$

Part B: Provide your full solution on these pages. **Exam version: 1**

Question 1 (5 points) With full explanation, find (if possible) conditions on a and b such that the system has no solution, one solution, or infinitely many solutions.

$$\begin{array}{rcrcrcr} x & & & + & z & = & 2 \\ & & y & & & = & -3 \\ x & + & y & + & a & = & b-1 \end{array}$$

Write a clear and complete solution, and provide your final answer in the box below:

Aug matrix
$$\left[\begin{array}{cccc} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -3 \\ 1 & 1 & a & b-1 \end{array}\right]$$

$-R_1+R_3$ $\downarrow$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 1 & a-1 & b-3 \end{array}\right]$$

$-R_2+R_3$ $\downarrow$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & \underline{\underline{a-1}} & \underline{\underline{b}} \end{array}\right]$$

ANSWER:
- No solution: $a=1$ $b\neq 0$
- One solution: $a-1\neq 0$ $(a\neq 1)$
- Infinitely many solutions: $a=1$ $b=0$

Part B: Provide your full solution on these pages. **Exam version: 2**

Question 1 (5 points) With full explanation, find (if possible) conditions on a and b such that the system has no solution, one solution, or infinitely many solutions.

$$\begin{array}{ccccccc} x & & & + & z & = & 2 \\ & & y & & & = & -1 \\ x & + & y & + & a-2 & = & b+2 \end{array}$$

Write a clear and complete solution, and provide your final answer in the box below:

Aug. matrix

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & a-2 & b+2 \end{bmatrix}$$

$-R_1+R_3 \downarrow$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & a-3 & b \end{bmatrix}$$

$-R_2+R_3 \downarrow$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & a-3 & b+1 \end{bmatrix}$$

ANSWER:	No solution:	$a=3 \quad b\neq -1$
	One solution:	$a-3\neq 0 \quad (a\neq 3)$
	Infinitely many solutions:	$a=3 \quad b=-1$

Exam version: 0

Question 2 (5 points) Consider the system of linear equations $AX = B$ where:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 0 \\ -1 & -1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

1. Use the matrix inversion algorithm to find A^{-1}.

2. Use your answer in part 1. to solve for X.

1.
$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-3R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 3 & -3 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 3 & -3 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{2R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 2 \\ 0 & -1 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 & 2 \\ 0 & -1 & -1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 & 2 \\ 0 & -1 & 0 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{-R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & 0 & -1 & -3 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & -1 & -3 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right]$$

So $A^{-1} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & -1 & -3 \\ -1 & 1 & 2 \end{bmatrix}$

2. $\boxed{X = A^{-1}B \quad 1 \text{ MARK}} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & -1 & -3 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & -1 & -3 \\ -1 & 1 & 2 \end{bmatrix}$	$X = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

Exam version: 1

Question 2 (5 points) Consider the system of linear equations $AX = B$ where:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 2 & 1 & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

1. Use the matrix inversion algorithm to find A^{-1}.
2. Use your answer in part 1. to solve for X.

1) $\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ -1 & 2 & 0 & 0 & 1 & 0 \\ 2 & 1 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 1 & 1 & 0 \\ 2 & 1 & -3 & 0 & 0 & 1 \end{array}\right]$

$\xrightarrow{-2R_1+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 1 & 1 & 0 \\ 0 & 1 & -1 & -2 & 0 & 1 \end{array}\right] \xrightarrow{-2R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 & 1 & -2 \\ 0 & 1 & -1 & -2 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & 1 & -2 \\ 0 & 0 & 1 & 5 & 1 & -2 \\ 0 & 1 & -1 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & 1 & -2 \\ 0 & 0 & 1 & 5 & 1 & -2 \\ 0 & 1 & 0 & 3 & 1 & -1 \end{array}\right]$

$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & 1 & -2 \\ 0 & 1 & 0 & 3 & 1 & -1 \\ 0 & 0 & 1 & 5 & 1 & -2 \end{array}\right]$ so $A^{-1} = \begin{bmatrix} 6 & 1 & -2 \\ 3 & 1 & -1 \\ 5 & 1 & -2 \end{bmatrix}$

2) $\boxed{X = A^{-1}B \quad 1 \text{ MARK}} = \begin{bmatrix} 6 & 1 & -2 \\ 3 & 1 & -1 \\ 5 & 1 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 6 & 1 & -2 \\ 3 & 1 & -1 \\ 5 & 1 & -2 \end{bmatrix}$	$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Exam version: 2

Sol

Question 2 (5 points) Consider the system of linear equations $AX = B$ where:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & -3 \\ 2 & 1 & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

1. Use the matrix inversion algorithm to find A^{-1}.
2. Use your answer in part 1. to solve for X.

PART 1

ok

perfect

1-

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & 2 & -3 & 0 & 1 & 0 \\ 2 & 1 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-2R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 1 & 0 \\ 2 & 1 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$-2R_1+R_3 \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 1 & 0 \\ 0 & 1 & -1 & -2 & 0 & 1 \end{array}\right] \xrightarrow{-R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$-R_2+R_3 \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -2 & -1 & 2 \end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 2 & 1 & -2 \end{array}\right]$$

$$R_3+R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & -2 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 2 & 1 & -2 \end{array}\right] \text{ so } A^{-1} = \begin{bmatrix} 3 & 1 & -2 \\ 0 & 1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$$

2- $\boxed{X = A^{-1}B \quad \text{1 mark}}$ $= \begin{bmatrix} 3 & 1 & -2 \\ 0 & 1 & -1 \\ 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ -2 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} 3 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix} \qquad X = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$