IMPORTANT Determinants

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Row Operations & How They Affect the Determinant

Given $\det(A) = 2$, if...

$\leftrightarrow$: $\det(-A)$, so $\det(A) = -2$

$cR_{\#}$: $c \cdot \det(A)$, so $\det(A) = c \cdot 2$

$cR_{\#_1} + R_{\#_2}$: no change to $\det(A)$, so $\det(A) = 2$

One row is a multiple of another row: $\det(A) = \emptyset$, so $\det(A) = \emptyset$

These are not the same (nor do they equal each other)

Determinant Properties

Given $\det(A) = 15$ and it is a 7×7 matrix, if...

$\det(A)^T = \det(A)$ ∴ No change

$\det(A)^{-1} = \frac{1}{\det(A)}$ ∴ $\frac{1}{15}$

$\det(cA) = c^n \det(A)$ ∴ $c^7 \cdot 15$ ← # of rows in the matrix from here

number

same number as

$\det(A)^{2,3,\text{etc}\ldots} = \det(A)\det(A)$ for $\det(A)^2$
$\det(A)\det(A)\det(A)$ for $\det(A)^3$
and so on

$\det(\text{Adj } A) = (\det A)^{n-1}$ ∴ $(\det A)^{7-1} = (\det A)^6$

Row Operations & How They Affect the Determinant

Given $\det(A) = 2$, if...

$\leftrightarrow$: $\det(-A)$, so $\det(A) = -2$
(swap a row with another row [example: $R_1 \leftrightarrow R_2$], then the determinant is negated)

$cR_{\#}$: $c \cdot \det(A)$, so $\det(A) = c \cdot 2$
(multiply one row by itself [eg: $\frac{1}{2} R_2$], then the determinant is multiplied by ↑)

$cR_{\#_1} + R_{\#_2}$: no change to $\det(A)$, so $\det(A) = 2$
(multiply one row and add it to another [ex: $3R_2 + R_4$], the determinant does not change)

One row is a multiple of another row: $\det(A) = \emptyset$, so $\det(A) = \emptyset$
(This is not necessarily a row operation, but if the matrix looks something like this:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 8 & 7 & 9 & 2 \end{bmatrix}$$

these two rows are multiples of each other. Meaning, if you took -2 times Row 1 and added it to Row 2 [$-2R_1 + R_2$], then it would look like this:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 8 & 7 & 9 & 2 \end{bmatrix}$$

← and if you see a row of zero's, the determinant is automatically $\emptyset$.