MAT 1330, Fall 2016 Assignment 7
This assignment is for practice purposes only.
It will not be marked. Do not hand it in.

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QUESTION 1. Calculate the following indefinite integrals.

a) $\int \frac{(x^{2/3} + x^{-1/5})^2}{x^{4/3}} dx = \boxed{}$

$= \int \frac{x^{4/3} + 2x^{7/15} + x^{-2/5}}{x^{4/3}} dx = \int 1 + 2x^{-13/15} + x^{-26/15} dx = x - 15x^{2/15} - \frac{15}{11} x^{-11/15} + c$

b) $\int \frac{e^{\sqrt{x}+1}}{\sqrt{x}} dx = \boxed{}$

$y = \sqrt{x}$, $dy = \frac{1}{2\sqrt{x}} dx$

$= \int 2e^{y+1} dy = 2e\int e^y dy = 2e e^y + c = 2e^{\sqrt{x}+1} + c$

c) $\int \frac{\ln(x)}{x} dx = \boxed{}$

$u' = \frac{1}{x}$ $\quad v = \ln(x)$

$u = \ln x$ $\quad v' = \frac{1}{x}$

$= [\ln(x)]^2 - \int \ln(x) \cdot \frac{1}{x} dx$

$\Rightarrow \int \frac{\ln(x)}{x} dx = \frac{1}{2} (\ln(x))^2 + c$

d) $\int \cos^3(x)\sin^2(x)\,dx = \boxed{}$

$y = \sin(x) \Rightarrow dy = \cos(x)\,dx$

$= \int \cos^2(x)\, y^2\,dy = \int (1-y^2)\,y^2\,dy = \int y^2 - y^4\,dy = \frac{1}{3}y^3 - \frac{1}{5}y^5 + c$

$= \frac{1}{3}\sin^3(x) - \frac{1}{5}\sin^5(x) + c$

e) $\int (x+2)\sin(x)\,dx = \boxed{-(x+2)\cos(x) + \sin(x) + c}$

$\int 2\sin(x)\,dx = -2\cos(x) + c$

$\int x\sin(x)\,dx = -x\cos(x) + \int \cos(x)\,dx = -x\cos(x) + \sin(x) + c$

$u = x \quad v' = \sin(x)$

$u' = 1 \quad v = -\cos(x)$

f) $\int x\ln(x+1)^2\,dx = \boxed{}$

$= 2\int x\ln(x+1)\,dx = 2\,\frac{x^2}{2}\ln(x+1) - \int \frac{x^2}{x+1}\,dx = x^2\ln(x+1) - \int \frac{(y-1)^2}{y}\,dy$

$u' = x \quad v = \ln(x+1)$

$u = x^2/2 \quad v' = \frac{1}{x+1}$

$y = x+1$

$dy = dx$

$x = y-1$

$= x^2\ln(x+1) - \int \left(y - 2 + \frac{1}{y}\right) dy = x^2\ln(x+1) - \frac{y^2}{2} + 2y - \ln|y| + c$

$= x^2\ln(x+1) - \frac{1}{2}(x+1)^2 + 2(x+1) - \ln|x+1| + c$

QUESTION 2. Find the value of $F(1)$ if we know that $F(0) = 1$ and $F'(t) = (t^2 + t^{1/3} + 3)^2$.

$$F(t) = \int F'(t)\,dt = \int (t^2 + t^{1/3} + 3)^2\,dt$$

$$= \int \left(t^4 + 2t^{7/3} + 6t^2 + t^{2/3} + 6t^{1/3} + 9\right) dt$$

$$= \frac{1}{5}t^5 + \frac{6}{10}t^{10/3} + 2t^3 + \frac{3}{5}t^{5/3} + \frac{9}{2}t^{4/3} + 9t + c$$

Find c by setting $F(0) = 0 \Rightarrow c = 0$

Now evaluate $F(1) = \frac{1}{5} + \frac{6}{10} + 2 + \frac{3}{5} + \frac{9}{2} + 9 + 1$

$$= 12 + \frac{2 + 6 + 6 + 45}{10}$$

$$= 12 + \frac{59}{10}$$

QUESTION 3. Suppose $Q(t)$ is the quantity of drug (in mg) in the blood t minutes after the drug is initially ingested. For $t \geq 0$, suppose the rate of $Q(t)$ can be modelled by

$$\frac{dQ}{dt} = 5e^{-0.1t} - 2e^{-0.05t}.$$

a) If initially $Q(0) = 0$, find $Q(t)$.

b) Calculate $\lim_{t\to\infty} Q(t)$ ~~and interpret the result~~.

$$Q(t) = \int Q'(t)\,dt = \int 5e^{-0.1t} - 2e^{-0.05t}\,dt$$

$$= \frac{5}{-0.1}e^{-0.1t} - \frac{2}{-0.05}e^{-0.05t} + C$$

$$= -50e^{-0.1t} + 40e^{-0.05t} + C$$

$$Q(0) = -50 + 40 + C = 0 \Rightarrow C = 10$$

$$Q(t) = 10 - 50e^{-0.1t} + 40e^{-0.05t}$$

$$\lim_{t\to\infty} Q(t) = 10$$

Interpret: In the long run, the process stabilizes at $Q(t) = 10$.

Ingestion and assimilation are in balance.