MAT 1330, Fall 2016 Assignment 3
Due Thursday October 20 by 8:00pm.
Late assignments will not be accepted; nor will unstapled assignments. Professors in the math department will not lend you a stapler; do not ask for one.

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QUESTION 1. Consider the function

$$f(x) = \begin{cases} \frac{x-9}{\sqrt{x}-3}, & x \neq 9 \\ a, & x = 9. \end{cases}$$

Can you choose a value for a such that the function is continuous at $x = 9$? Justify your answer and give the value of a if it exists.

For continuity, we need $\lim_{x \to 9^-} f(x) = \lim_{x \to 9^+} f(x) = f(9) = a$

$$\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} = \lim_{x \to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3} = \lim_{x \to 9} (\sqrt{x}+3) = 6$$

$\Rightarrow$ The function is continuous at $x = 9$ if $a = 6$.

QUESTION 2. Find the following limits, using the rules from class. Then use your calculator to check how large (for (a) and (b)) or how close to zero (for (c)) x needs to be in order to be 3 digits accurate.

(a) $\lim_{x\to\infty} [3\ln(x+2) - \ln(4x^3 - 2x + 1)] =$ $\boxed{}$

$x = 20000$ gives -1.386

$$\lim_{x\to\infty}\left(3\ln(x+2) - \ln(4x^3-2x+1)\right) = \lim_{x\to\infty} \ln\left(\frac{(x+2)^3}{4x^3-2x+1}\right)$$

$$= \ln\left(\lim_{x\to\infty} \frac{x^3+6x^2+12x+8}{4x^3-2x+1}\right) = \ln\left(\lim_{x\to\infty} \frac{1+6/x+12/x^2+8/x^3}{4-2/x^2+\frac{1}{x^3}}\right) = \ln\left(\tfrac{1}{4}\right)$$

$$= -1.3863$$

(b) $\lim_{x\to-\infty} \dfrac{7x^4}{\sqrt[3]{2x^{12}+3x^8+4x^6}} =$ $\boxed{}$

$$\lim_{x\to-\infty} \frac{7x^4}{\sqrt[3]{2x^{12}+3x^8+4x^6}} = \lim_{x\to-\infty} \frac{7x^4\cdot\frac{1}{x^4}}{\frac{1}{x^4}\sqrt[3]{2x^{12}+3x^8+4x^6}} = \lim_{x\to-\infty}\frac{7}{\sqrt[3]{2+\frac{3}{x^4}+\frac{4}{x^6}}}$$

$$= \frac{7}{\sqrt[3]{2}} \approx 5.5559$$

$x = 8$ gives 5.5552

(c) $\lim_{x\to 0} \dfrac{3x^8}{\sqrt{x^8+49}-7} =$ $\boxed{}$

$$\lim_{x\to 0} \frac{3x^8}{\sqrt{x^8+49}-7} = \lim_{x\to 0} \frac{3x^8\left(\sqrt{x^8+49}+7\right)}{\left(\sqrt{x^8+49}-7\right)\left(\sqrt{x^8+49}+7\right)} = \lim_{x\to 0}\frac{3x^8\left(\sqrt{x^8+49}+7\right)}{x^8+49-49}$$

$$= \lim_{x\to 0} 3\left(\sqrt{x^8+49}+7\right) = 42$$

$x = 0.1$ gives the exact value, but $x = 0.01$ gives ∞ or NaN.

QUESTION 3. Use the definition of the derivative to find the derivative of

$$f(x) = \frac{x+3}{x^2+2}.$$

See back →

QUESTION 4. When cells grow in a culture of two different sugars, there can be two distinct growth phases. Cells first consumer their preferred sugar and grow relatively fast. Once the preferred sugar is depleted, they switch to the second sugar and grow slower. This phenomenon is called 'diauxic growth'.

Suppose that the mass $m(t)$ of a cell population at time t undergoing diauxic growth is given by the function

$$m(t) = \begin{cases} t^3 + a, & t \leq 1 \\ t^2 + b, & t > 1. \end{cases}$$

(a) Can you find a condition on a, b such that m is a continuous function?

$\lim_{t \to 1^-} m(t) = 1+a$, $\lim_{t \to 1^+} m(t) = 1+b$, $m(1) = 1+a$.

$\Rightarrow$ If $a = b$, then m is continuous.

(b) Examine the definition of $m'(1)$ to show that the function m cannot be made differentiable at $t = 1$, no matter of how one chooses a, b.

$$\lim_{h \to 0^+} \frac{1}{h}\left(m(1+h) - m(1)\right) = \lim_{h \to 0^+} \frac{1}{h}\left((1+h)^2 + b - (1+b)\right) = \lim_{h \to 0^+} \frac{1}{h}\left(2h + h^2\right) = 2$$

$$\lim_{h \to 0^-} \frac{1}{h}\left(m(1+h) - m(1)\right) = \lim_{h \to 0^-} \frac{1}{h}\left((1+h)^3 + a - (1+a)\right) = \lim_{h \to 0^-} \frac{1}{h}\left(3h + 3h^2 + h^3\right) = 3$$

Since $2 \neq 3$, the limit $\lim_{h \to 0} \frac{m(1+h) - m(1)}{h}$ does not exist.

Q3:

$$f'(x) = \lim_{h \to 0} \frac{1}{h}\left(f(x+h) - f(x)\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{x+h+3}{(x+h)^2+2} - \frac{x+3}{x^2+2}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{(x+h+3)(x^2+2) - (x+3)((x+h)^2+2)}{((x+h)^2+2)(x^2+2)}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{(x+3)(x^2+2) + h(x^2+2) - [(x+3)(x^2+2) + (x+3)(2hx+h^2)]}{((x+h)^2+2)(x^2+2)}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{h(x^2+2) - h(x+3)(2x+h)}{((x+h)^2+2)(x^2+2)}\right)$$

$$= \lim_{h \to 0} \frac{x^2+2 - (x+3)(2x+h)}{((x+h)^2+2)(x^2+2)}$$

$$= \frac{-x^2 - 6x + 2}{(x^2+2)^2}$$