Final test - December 2015

1.(10) (a) $f(x) = \begin{cases} 1+\sqrt{4-x^2} & if \quad -2 \le x \le 2 \\ 3-x & if \quad x > 2 \end{cases}$ $\rightarrow$ draw the graph on $[-2,5]$

and use it to evaluate: $\int\limits_{-2}^{5} f(x)\,dx$.

(b) Calculate $F'(x)$ if $F(x) = \int\limits_{-x^2}^{x} e^{1-t^2}dt$ and determine whether $F(x)$ is increasing, or decreasing.

2.(10) Evaluate:

(a) $\int \frac{\cos^3 x}{\sin^2 x}dx$

(b) $\int (e^x + \ln x)\,dx$

3.(6) Calculate $F(x)$ if $F'(x) = \frac{x^2+4}{x^2-4}$ and $F(-1) = 0$.

4.(18) Evaluate:

(a) $\int\limits_{0}^{\pi/2} \frac{\cos x}{4+\sin^2 x}dx$

(b) $\int\limits_{0}^{\pi/4} \sec^4 x dx$

(c) $\int\limits_{0}^{3} x^2\sqrt{1+x}dx$

5.(8) Evaluate the improper integrals, or show why they are divergent:

(a) $\int\limits_{e}^{\infty} \frac{dx}{x\ln^3 x}$

(b) $\int\limits_{-1}^{1} \frac{1dx}{x^2-1}$

6.(16) (a) Sketch and calculate the area enclosed by $y = 6 - x^2$ & $y = 2 - 3x$.

(b) Evaluate the volume of a solid obtained by rotating the region bounded by $y = \sin x$ and the $x - axis$ on $[0, \pi]$ about $y = -1$.

(c) Calculate f_{ave} of $f(x) = \dfrac{x}{\sqrt{16 + x^2}}$ on $[0, 3]$.

7.(6) Calculate $\lim a_n$ or prove that it does not exist:

(a) $a_n = \dfrac{(3^n + 1)^2}{6^n}$

(b) $a_n = \ln\left(1 + 2n^2\right) - \ln\left(30 + 2n^2\right)$

8.(12) Determine whether the series $\sum_{n=2}^{\infty} a_n$ is divergent, absolutely convergent, or conditionally convergent:

(a) $a_n = \dfrac{(-1)^n \sqrt{1 + n^3}}{n^2}$

(b) $a_n = \dfrac{(-3)^n}{5 + e^n}$

(c) $a_n = \dfrac{1}{n \ln^2 n}$

9.(6) Determine the radius and interval of convergence of $\sum_{n=0}^{\infty} \dfrac{(x+1)^n}{(n+1)\, 2^n}$.

10(8) (a) Calculate the MacLaurin series for $f(x) = x^2 \ln(1 + 2x)$

(b) Calculate $S(x) = \sum_{n=1}^{\infty} n x^{2n-1}$ and determine its radius of convergence.

Bonus(5) Show that for a continuous f : $\int_0^{\pi/2} f(\cos x)\, dx = \int_0^{\pi/2} f(\sin x)\, dx$.