CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	204	All
Examination	**Date**	**Pages**
Final	April 2014	2
Instructors		**Course Examiner**
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Special Instructions

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.
- ▷ All questions have equal value.

1. Using the Gauss-Jordan method, find all solutions of the following system of equations:

$$\begin{array}{rcrcrcrcl} x_1 & + & x_2 & - & 2x_3 & + & 3x_4 & = & 4 \\ 2x_1 & + & 3x_2 & + & 3x_3 & - & x_4 & = & 3 \\ 5x_1 & + & 7x_2 & + & 4x_3 & + & x_4 & = & 5 \end{array}$$

2. Let $M = \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 4 \\ 3 & 1 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 2 & 3 \\ 1 & 4 \end{pmatrix}$.

   (a) Calculate M^{-1}.

   (b) Find the matrix C such that $MC = B$.

3. (a) Use Cramer's rule to solve the following system:

$$\begin{array}{rcrcrcl} 2x & + & y & - & 3z & = & 1 \\ 5x & + & 2y & - & 6z & = & 5 \\ 3x & - & y & - & 4z & = & 7 \end{array}$$

   (b) Find the determinant of $A = \begin{pmatrix} 1 & 0 & 1 & 4 \\ -2 & 1 & 1 & 7 \\ 3 & 0 & 1 & 2 \\ -4 & 1 & 5 & 6 \end{pmatrix}$.

4. (a) Find the parametric equation of the plane in $\mathbb{R}^3$ that contains the points $P(-2,1,3)$, $R(-1,-1,1)$, $S(3,0,-2)$.
   (b) Find equation of the plane that contains the points $P(-2,1,0)$ and parallel to the plane $-8x+6y-z=4$.

5. Let $P_1(1,1,0)$, $P_2(1,0,1)$, $P_3(0,1,1)$, $P_4(1,1,1)$.
   (a) Find an equation of the plane containing $P_2,\ P_3,\ P_4$.
   (b) Find the volume of the parallelepiped determined by the vector $\overrightarrow{P_1\,P_2}$, $\overrightarrow{P_1\,P_3}$, $\overrightarrow{P_1\,P_4}$.

6. Find vectors w_1 and w_2 so that $v = w_1 + w_2$ where $v = (1,2,-4)$ and such that w_1 is parallel to $u = (-2,0,1)$ and w_2 is orthogonal to u.

7. (a) Express the vector $(1,4,6)$ as a linear combination of the vector $(1,0,1),(0,1,1),(1,1,0)$.
   (b) Prove that $\{(1,1,1),(1,1,0),(1,0,0)\}$ is a basis of $\mathbb{R}^3$.

8. Let $A = \begin{pmatrix} 1 & 3 & 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & 7 & 1 \\ 0 & 0 & 0 & 1 & 3 & 1 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \\ z \\ u \\ v \\ w \end{pmatrix}$. Find a basis for solution space of the homogeneous system of equations $AX = 0$.

9. Let $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & 6 & -4 \end{pmatrix}$. If 4 is an eigenvalue of A, find an invertible matrix P and a diagonal matrix D so that $P^{-1}AP = D$.

10. Let $A = \begin{pmatrix} 5 & 6 \\ 3 & -2 \end{pmatrix}$. Compute A^{100}.