MAT 1348A – Assignment 1 solutions

① Propositional variables:

l: "The file system is locked."

f: "The system is functioning normally."

b: "Messages are sent to the message buffer."

q: "Messages are being queued."

Compound propositions from text:

(a) $(l \vee \neg f) \rightarrow \neg b$

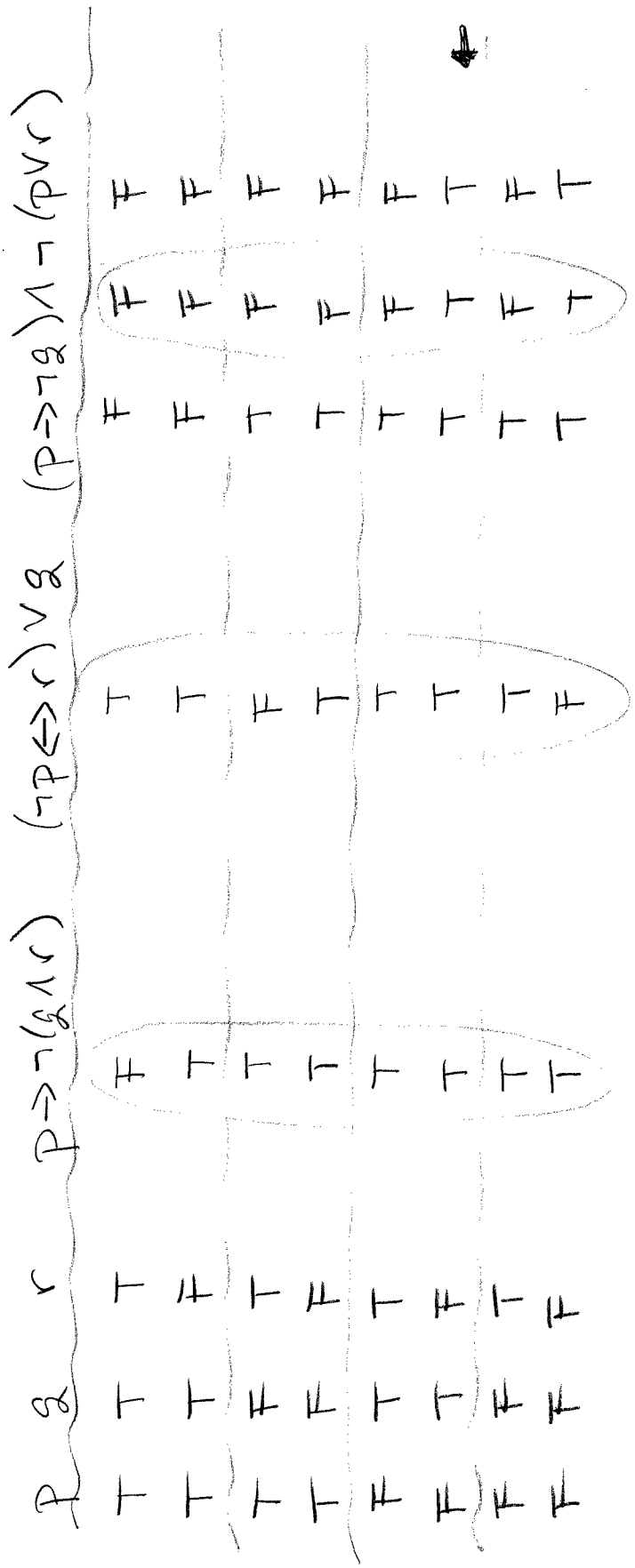
(b) $(\neg l \wedge q) \leftrightarrow f$

(c) $\neg q \rightarrow l$

(d) $\neg b \rightarrow \neg f$

(e) $f \rightarrow q$ (or equivalently, $q \vee \neg f$)

(2) Truth table:



when $p=F, q=T, r=F$, all 3 compound propositions are T (indicated row). Hence the set is consistent.

(3) (a) $A = (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

$B = (p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

(b) $A = (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

$\equiv \neg(\neg p \vee q) \vee \neg(p \vee q)$

$B = (p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

$\equiv \neg(\neg p \vee \neg q) \vee \neg(p \vee \neg q) \vee \neg(p \vee q)$

(c) Recall $x \to y \equiv \neg x \vee y$

$A \equiv \neg(\neg p \vee q) \vee \neg(p \vee q)$

$\equiv \neg(p \to q) \vee \neg(\neg p \to q)$

$\equiv (p \to q) \to \neg(\neg p \to q)$

$B \equiv \neg(\neg p \vee \neg q) \vee \neg(p \vee \neg q) \vee \neg(p \vee q)$

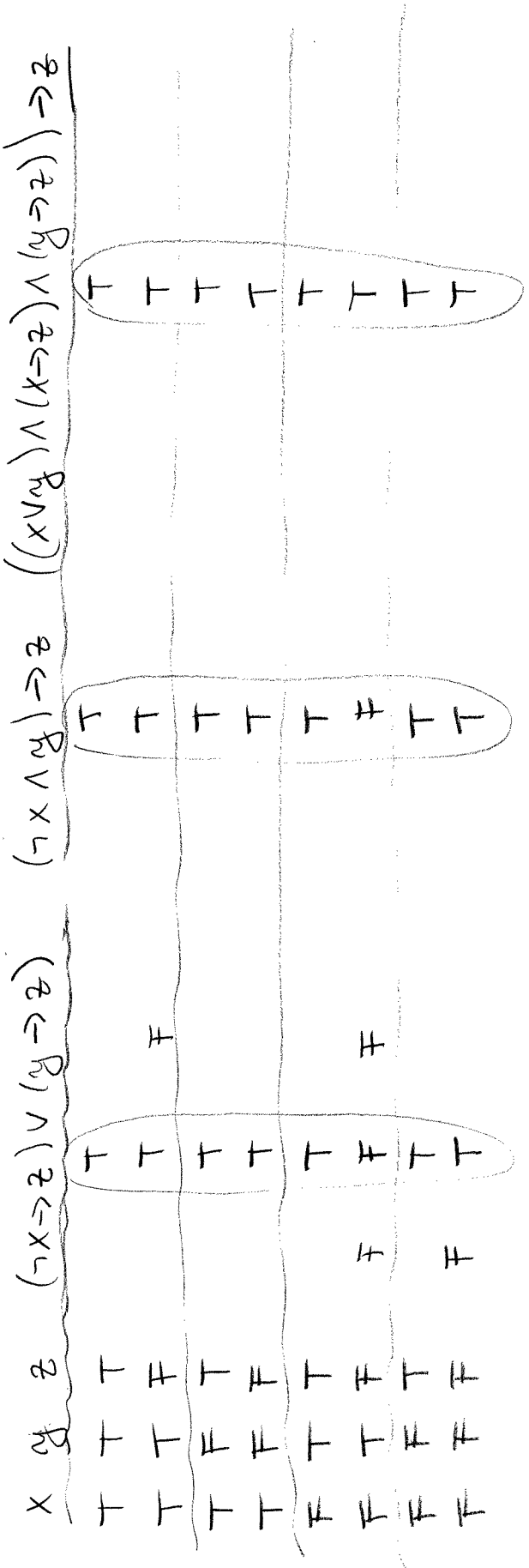
$\equiv \neg(p \to \neg q) \vee \neg(q \to p) \vee \neg(\neg p \to q)$

$\equiv \Big((p \to \neg q) \to \neg(\neg p \to \neg q)\Big) \vee \neg(\neg p \to q)$

$\equiv (\neg p \to q) \to \Big((p \to \neg q) \to \neg(\neg p \to \neg q)\Big)$

Note: other (equivalent) solutions are possible.

(4) Truth table:



(a) (i) contingency

(ii) contingency

(iii) tautology

(b) (i) is F for

$x = F,\ y = T,\ z = F$

(ii) is F for

$x = F,\ y = T,\ z = F$

(c) (i) and (ii) are logically equivalent

(5) Define propositional variables

a : "A is a knight."

b : "B is a knight."

(a) A said $a \to \neg b$

Then a and $a \to \neg b$ must have the same truth value.

a	b	$a \to \neg b$	
T	T	F	
T	F	T	←
F	T	T	
F	F	T	

This happens only for $a = T$, $b = F$.

Hence, A must be a knight and B a knave.

(b) A said $a \vee b$

B said $\neg a \oplus \neg b$

Now a and $a \vee b$, as well as b and $\neg a \oplus \neg b$, must have the same truth value.

a	b	$a \vee b$		$\neg a \oplus \neg b$		
T	T	T	←	F		
T	F	T	←	T		
F	T	T		T	←	
F	F	F	←	F	←	←

This happens only for $a = F$, $b = F$.

Hence A and B must both be knaves.