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Department of Economics

MACROECONOMIC THEORY II
ECO2143B
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HOMEWORK #2

A. Effort in R&D in one country model

Consider a country described as following:

$$Y = AL_Y$$

$$L = L_Y + L_A$$

$$\gamma_A = \frac{L_A}{L}$$

1. Suppose that the country temporarily raises its level γ_A. Draw graphs showing how the time paths of output per worker (y) and productivity (A) will compare under this scenario with what would have happended if there had been no change in γ_A.

With no change in the fraction of workers devoted to research and development, productivity and output per worker would continue to grow indefinitely at the previous rate. That is,

$$g_y = g_A = \frac{\gamma L}{\mu}$$

where γ denotes the fraction of workers devoted to R&D. With an increase in γ to γ', we know from the following equation that,

$$\gamma < \gamma' \text{ implies } g_y = g_A = \frac{\gamma L}{\mu} < g_y' = g_A' = \frac{\gamma' L}{\mu}$$

Therefore, at the time of change, denoted as t_1 in the graph below, the growth rate of output per worker and productivity, g'_y and g'_A respectively, will be greater than before. However, the increase in the rate of growth will be accompanied by a decrease in the *level* of output per worker. Simply put,

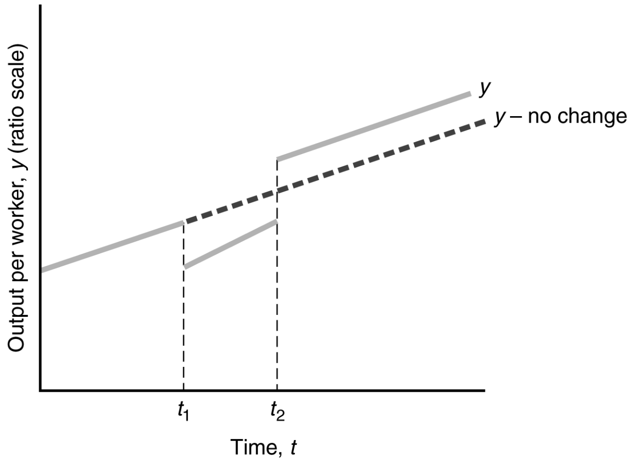
$$y = A(1-\gamma) < y' = A(1-\gamma').$$

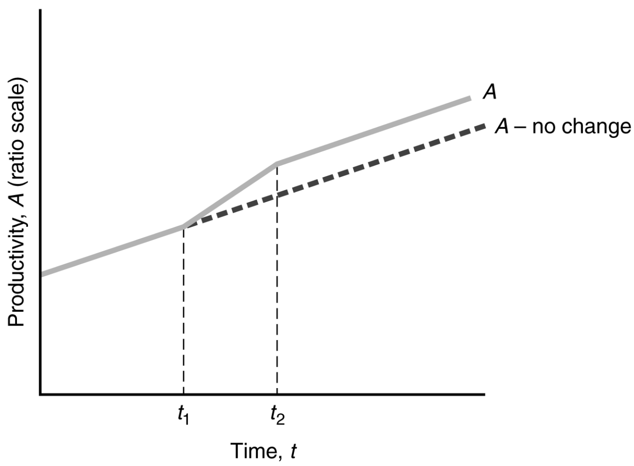
This amounts to an increase in the slope of, and a drop in, the level of output per worker. For productivity, there will not be a drop in the level of A, but the slope will rise at the time of change.

Because the change in γ to γ' is temporary, at the time t_2, that γ' returns to the original fraction γ, the process will reverse itself. Output per worker will jump up to a new level as workers move out of the R&D sector. But the jump up is larger in magnitude than the jump down. Intuitively, the level of productivity has risen during the temporary increase in the number of workers devoted to R&D. Thus, with the same number of people moving to the non-R&D sector as that moving to the R&D sector from before, the same number of workers can now be more productive. Mathematically, the first and second jumps are given as

$$\textbf{At } t_1\textbf{:}\quad \Delta y = y - y' = A(1-\gamma) - A(1-\gamma') = A(\gamma' - \gamma) = A(\Delta\gamma),$$
$$\textbf{At } t_2\textbf{:}\quad \Delta y = y'' - y' = A'(1-\gamma) - A'(1-\gamma') = A'(\gamma' - \gamma) = A'(\Delta\gamma).$$

Because the change in γ is the same and $A' > A$, we know that the absolute difference in the jump up is greater than the jump down. The level of A' is dependent on the length of the temporary increase in the fraction of workers devoted to R&D. Regardless of the level, the new growth path of output per worker will be the same rate before the temporary change, but it will start out at a higher level than if no change had occurred. As for productivity, the change in γ will return the growth rate of productivity to its original level, without any jumps. The figures are given below.





2. Suppose that *L=1*, ***μ*** *=5* and ***γ**_A_* *=0.5*. Calculate the the growth rate of output per worker.

The given parameters of the model are, $L = 1$, $\mu = 5$, and $\gamma = 0.5$. To calculate the growth rate of output per worker, we substitute in these values into the following equation and solve to get:

$$g = \frac{\gamma L}{\mu} = \frac{0.5}{5} = 0.1.$$

The growth rate of output per worker is 10% per year.

3. Now suppose that γ_A is raised to 0.75. How many years will it take before output per worker returns to the level it would have reached if γ_A had remained constant?

Similarly, with $\gamma' = 0.75$, we get:

$$g' = \frac{\gamma' L}{\mu} = \frac{0.75}{5} = 0.15.$$

In this case, output per worker grows at 15 percent a year. However, the level of output per worker has dropped. The level before the drop can be found by substituting the previous parameter values into the production function to get:

$$y = A(1-\gamma) = A(1-0.5) = A(0.5).$$

Similarly, we can find the new level of output per worker to be:

$$y' = A(1-\gamma') = A(1-0.75) = A(0.25).$$

Therefore, the original level of output per worker is $A(0.5)$, and we need to find how long it will take to reach this level, starting from a level of $A(0.25)$ with a growth rate of 15 percent. We use the standard growth equation, substitute and solve.

$$y'(1+g)^t = y,$$
$$A(0.25)(1+0.15)^t = A(0.5).$$

Dividing both sides by $A(0.25)$ and taking logs, we get:

$$t \ln(1.15) = \ln(2).$$

Solving for t, we get:

$$t = \frac{\ln(2)}{\ln(1.15)} = 4.96 \approx 5.$$

That is, it will take approximately five years for the level of output per worker to return to its previous level before the change in γ.

We can accept this:

$$y'(1+g')^t = y(1+g)^t,$$
$$A(0.25)(1+0.15)^t = A(0.5)(1+0.10)^t.$$

Solving for t, we get:

$$t = \frac{\ln(0,5)}{\ln(0.96)} = 16.98 \approx 17.$$

That is, it will take approximately seventeen years for the level of output per worker to be equal to the level it will reach without the change in γ.

B. Effort in R&D in two-country model

Consider a two-country model described as following:

$$y_1 = A_1(1-\gamma_{A,1})$$

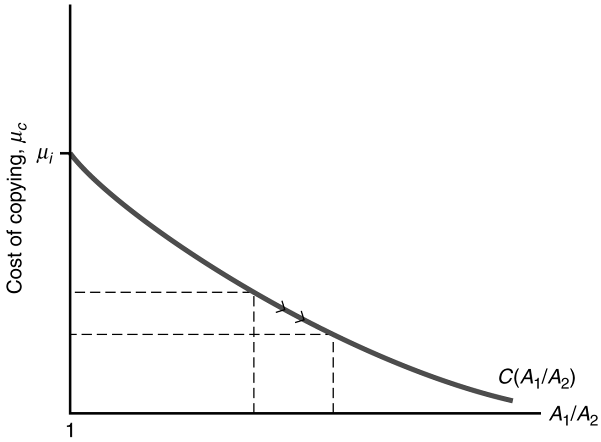
$$y_2 = A_2(1-\gamma_{A,2})$$

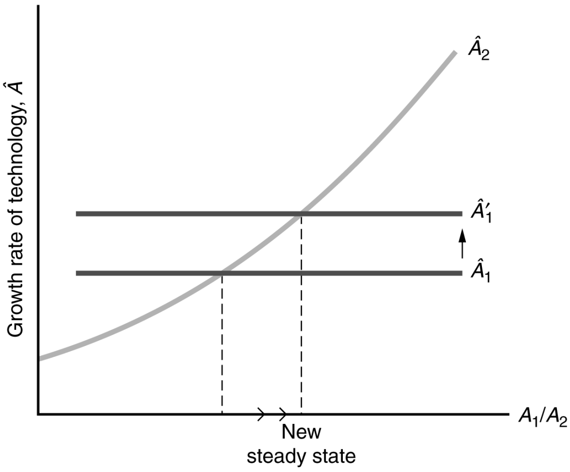
$$\gamma_{A,1} > \gamma_{A,2}$$

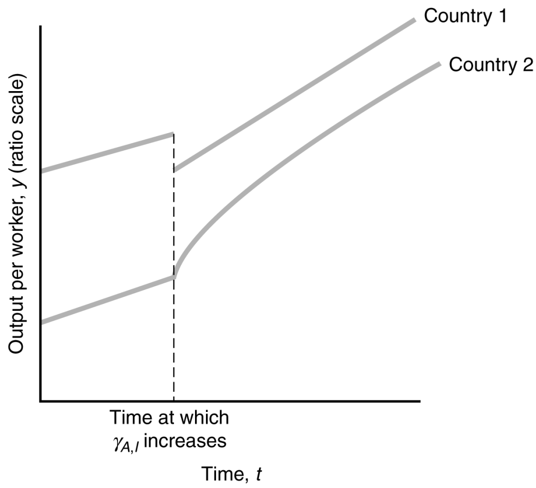
The Country 1 is an innovating country and the Country 2 is a copying country. The two countries are in steady state.

1. Suppose now that the Country 1 raises the fraction of the labor force that is doing R&D. Draw a picture showing how the rates of growth in Countries 1 and 2 will behave over time.

In the diagram below, the increase in the fraction of labor devoted to R&D in Country 1 will create a drop in the level of output per worker but an increase in the growth rate of productivity as well as output per worker. Country 1 behaves in accordance with the one-country model. However, the speed of growth in productivity in Country 1 raises the steady state A_1/A_2 ratio (the second diagram). Consequently, μ_c ,the cost of copying falls for Country 2. The fall in the cost of copying will raise productivity in Country 2, and so, the growth rate of output per worker in Country 2 will also rise. There will be no jump, down or up, in output for Country 2, but the fall in the cost of copying will place Country 2 on a higher growth path, as illustrated in the figure below. In the long run, the growth rates of output for both Country 1 and Country 2 will be equal, with Country 1 at a higher level.

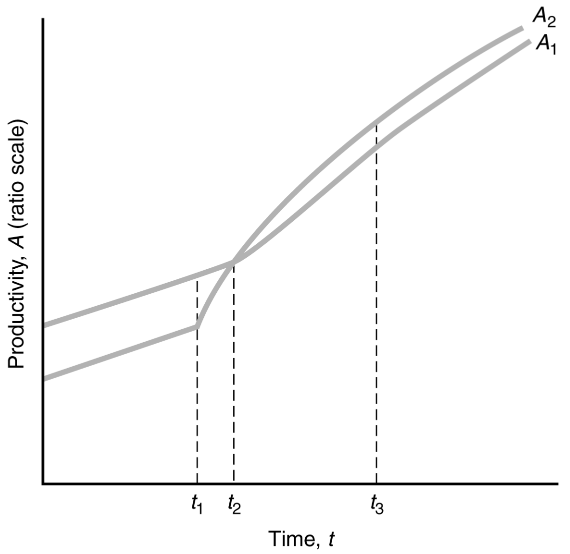


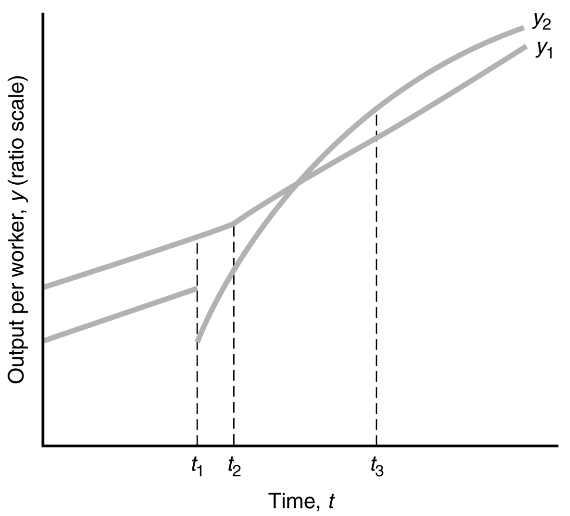




2. Now suppose that Country 2 raises the fraction of the labor force that is doing R&D so much that $\gamma_{A,1} < \gamma_{A,2}$. Draw a picture showing how the rates of growth in Countries 1 and 2 will behave over time.

We are given a scenario where the follower, Country 2, raises the fraction of the labor force devoted to R&D beyond the fraction in the leader, Country 1. The implication in the long run is for Country 2 to become the technological leader. The process by which this occurs is illustrated in the diagrams of the growth rates of productivity and output per worker below. The initial rise in the fraction of workers devoted to R&D in Country 2 will raise the productivity of Country 2. In accordance, Country 2 will experience a drop in the level of output per worker but also achieve a faster rate of growth in output per worker. The timing of these events are denoted as t_1. As productivity growth in Country 2 continues, the technological gap will lessen. That is A_1/A_2 will approach one, and the cost of copying will approach the cost of invention. Because the fraction devoted to R&D in Country 2 exceeds that of Country 1, A_2 will eventually equal A_1. This point in time is denoted as t_2. A_2 will eventually overtake A_1 and Country 2 will then become the technological leader and Country 1 will become the follower. Therefore, Country 1 is no longer faced with the cost of invention. Instead, Country 1 faces a declining cost of copying, thereby raising the growth rate of output per worker and the growth rate of productivity. The technology gap between both countries will increase until the steady-state value of A_2/A_1 is reached. This point in time is denoted as t_3. At this time, both output per worker and productivity of Country 1 and Country 2 will be growing at an equal and constant rate. This is the final long run equilibrium.





Suppose that the cost-of-copying function is

$$\mu_c = \mu_i \left(\frac{A_1}{A_2}\right)^{-\beta}$$

$$with\ 0 < \beta < 1$$

Assume that the two countries have labor forces of equal size.

3. Using this function, solve for the steady-state ratio of technology in the leading country to technology in the follower country (*A₁/A₂*) as a function of the values of $\boldsymbol{\gamma_A}$ in the two countries. Show how this depends on the value of **β**, and explain what is going on.

In the steady state, $g_1 = g_2$. Therefore,

$$\frac{\gamma_{A,1}L}{\mu_i} = \frac{\gamma_{A,2}L}{\mu_c}.$$

Rearranging and solving for μ_c, we get,

$$\mu_c = \left(\frac{\gamma_{A,2}}{\gamma_{A,1}}\right)\mu_i.$$

Setting the above steady-state condition equation to the specified cost-of-copying function,

$$\mu_c = \left(\frac{\gamma_{A,2}}{\gamma_{A,1}}\right)\mu_i = \left(\frac{A_1}{A_2}\right)^{-\beta}\mu_i.$$

Rearranging, we find out solution to be:

$$\left(\frac{A_1}{A_2}\right) = \left(\frac{\gamma_{A,1}}{\gamma_{A,2}}\right)^{\frac{1}{\beta}}.$$

Without the exponent, the ratio of technology in Country 1 to Country 2 would be determined proportionally by the ratio of the fraction of the labor force employed in R&D. This is the case when $\beta = 1$. Because we assume $0 < \beta < 1$, with the exponent, the ratios will not be proportional. That is, as the value of β falls to zero, the proportional difference in the level of technology between the two countries grows extremely large, and as the value of β rises to one, the proportional difference in the level of technology between the two countries matches the proportional difference in the fraction of worker devoted to R&D.

4. Assume that $\beta=0.5$, $\mu_i=10$, $\gamma_{A,1}=0.2$, and $\gamma_{A,2}=0.1$. Calculate the steady-state ratio of technology in Country 1 to technology in Country 2.

If we assume $\beta = 1/2$, $\mu_i = 10$, $\gamma_{A,1} = 0.2$, and $\gamma_{A,2} = 0.1$, we can solve the previous equation to get:

$$\left(\frac{A_1}{A_2}\right) = \left(\frac{\gamma_{A,1}}{\gamma_{A,2}}\right)^{\frac{1}{\beta}} = \left(\frac{0.2}{0.1}\right)^2 = 4.$$

That is, the steady-state ratio of technology in Country 1 to technology in Country 2 is 4.

C. Growth Accounting

1. Over the period from B.C. 10,000 to A.D. 1, the world population is estimated to have increased from 4 million to 170 million, while the level of income per capita was constant over time. Assuming that the quantities of human and physical capital per worker did not change, and that the exponent on land in the production function is one-third, calculate the growth rate in productivity over this period. What was the annual growth rate of productivity, A?

The annual growth rate of productivity is given by the following equation (equ 9.3 in the textbook, p262):

$$g_A = g_y + \beta n$$

We are given a value of 1/3 for β and 0% for g_y leaving the growth rate of the population, n, as the only unknown. To solve for n we use the standard growth equation with the initial population as 4 million and the final population after 10,000 years as 170 million. The equation is:

$$4\text{m}.(1+n)^{10,000} = 170m.$$
$$n = (170/4)^{(1/10,000)} - 1$$
$$= 0.000375.$$

Now we substitute to find our growth rate of productivity over this period:

$$g_A = 0 + \left(\frac{1}{3}\right)(0.000375) = 0.000125$$

That is, the growth rate of productivity over this period was roughly 0.0125 percent per year.

2. Over the period from A.D. 1 to A.D. 2000, the world population is estimated to have increased from 170 million to 6 billion, while the level of income per capita increased by 0.13 percent annually. Assuming that the quantities of human and physical capital per worker did not change, and that the exponent on land in the production function is one-third, calculate the growth rate in productivity over this period. What was the annual growth rate of productivity, A?

The annual growth rate of productivity is given by the following equation (equ 9.3 in the textbook, p262):

$$g_A = g_y + \beta n$$

We are given a value of 1/3 for β and 0.13% for g_y leaving the growth rate of the population, n, as the only unknown. To solve for n we use the standard growth equation with the initial population as 170 million and the final population after 2,000 years as 6 billion. The equation is:

$$170\text{m}.(1+n)^{2,000} = 6b.$$
$$n = (6\text{b.}/170\text{m.})^{(1/2,000)} - 1$$
$$= 0.00178345.$$

Now we substitute to find our growth rate of productivity over this period:

$$g_A = 0.0013 + \left(\frac{1}{3}\right)(0.00178345) = 0.00189448$$

That is, the growth rate of productivity over this period was roughly 0.189448 percent per year.

D. Technology and industries

Suppose that people consume only two goods: cheese and bread. They consume these two goods in a fixed ratio: one slice of bread is always eaten with one slice of cheese. Both cheese and bread are produced using only labor as an input. Their production functions are:

$$Y_b = A_b L_b,$$

$$Y_c = A_c L_c,$$

Where Y_b is the quantity of bread, Y_c is the quantity of cheese, L_b is the amount of labor devoted to producing bread, and L_c is the amount of labor devoted to producing cheese.

The total quantity of labor in the economy, L, is constant, and $L_b + L_c = L$.

In the year 2000, $A_b = A_c = 1$. But technological progress takes place at different speeds in the two industries. Specifically, $g_{Ab} = 2\%$ and $g_{Ac} = 1\%$, where these growths rates of technology are exogenous.

1. What quantities of labor will be devoted to producing bread and cheese in 2000?

In any given year, the production of bread must equal the production of cheese in this economy. That is, $Y_b = Y_c$, always. Knowing that the productivity of each good is equal at this point in time, we can solve for the quantity of labor devoted to each sector as follows.

$$Y_b = Y_c,$$
$$A_b L_b = A_c L_c,$$
$$L_b = L_c.$$

Since $L_b + L_c = L$,

$$L_b = L_c = L/2.$$

The labor force will be equally split between the two sectors.

2. What will the growth rate of total output be in 2000?

To calculate the growth rate of total output, we first calculate the growth rates of each sector by taking the natural log of both sides and differentiating with respect to time. For the bread sector:

$$\ln(Y_b) = \ln(A_b) + \ln(L_b),$$
$$\frac{d}{dt}\ln(Y_b) = \frac{d}{dt}\ln(A_b) + \frac{d}{dt}\ln(L_b),$$
$$gY_b = gA_b + n_b.$$

Similarly, for the cheese sector, we get,

$$gY_c = gA_c + n_c.$$

We know the value for the growth rate of productivity in both sectors. Furthermore, in our answer to Part (a), we found that labor is currently equally divided among the two sectors. Thus, the growth of labor in one sector must be offset by the growth of labor in the other— $n_b = (-n_c)$.
We substitute in these values and get,

$$gY_b = 2\% + n_b.$$

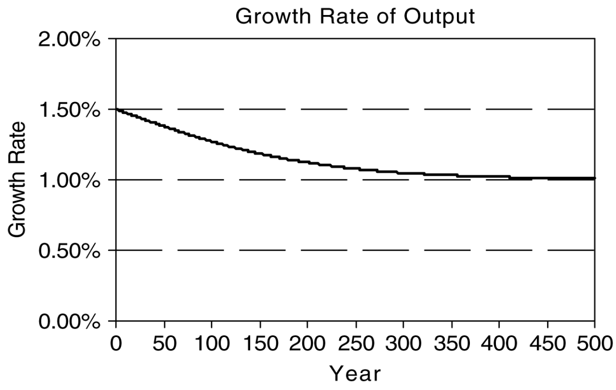
And,

$$gY_c = 1\% + n_c = 1\% - n_b.$$

Setting the growth rate of output in the bread sector to the growth rate of output in the cheese sector, we find that $n_b = -0.5\%$ and $n_c = 0.5\%$. So, $gY_b = gY_c = gY = 1.5\%$.

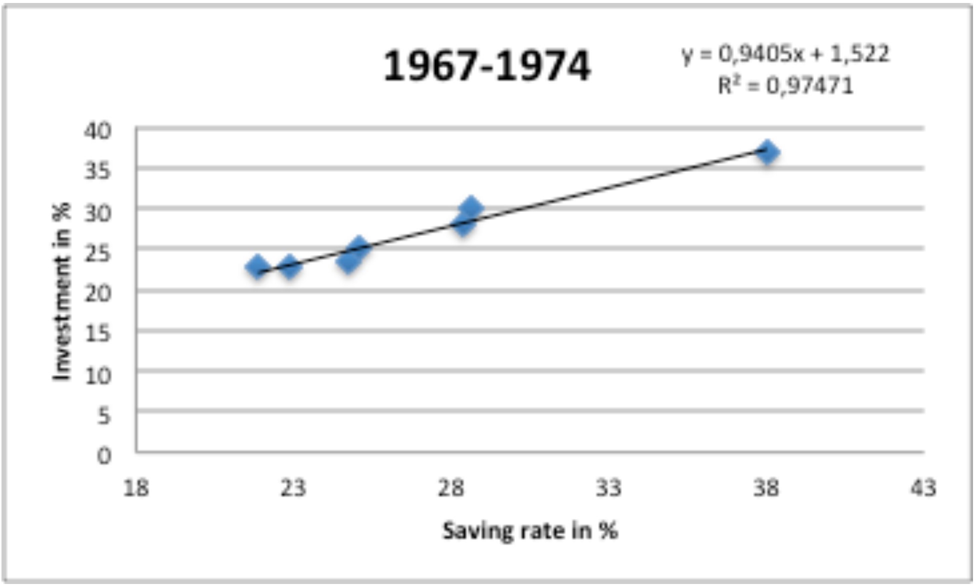
3. Draw a graph showing the growth rate of output from the year 2000 onward. Show whether growth rises, falls, or stays constant, and explain why it does so. What will the growth rate of output be in the long run?

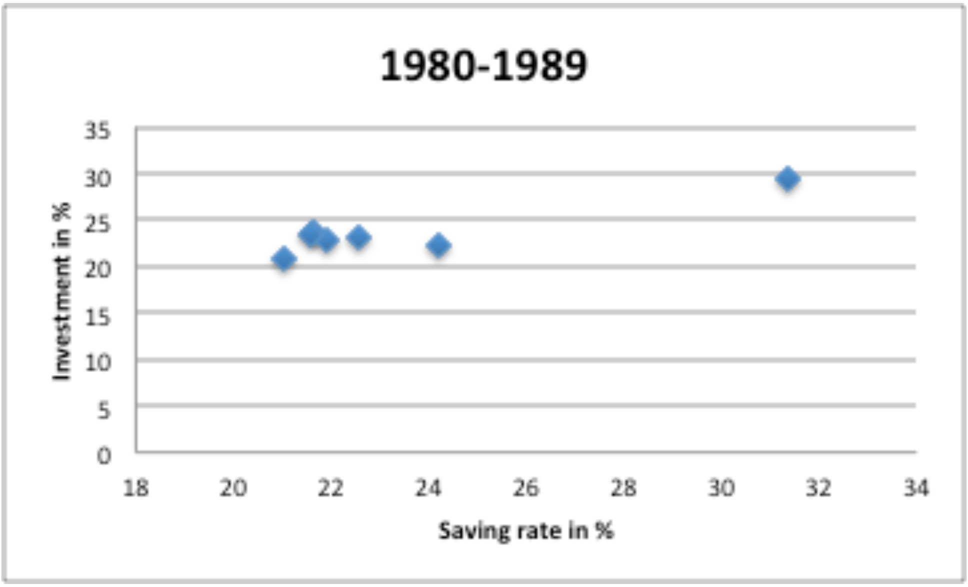
The figure below depicts the growth rate of output over time. From Part (2), we know that the growth rate of output is equal to 1.5 percent. But, productivity in the bread sector rises by a greater percent than in the cheese sector. Bread production would rise faster than cheese production and because one piece of bread is consumed with one piece of cheese, labor resources are continually shifted into the cheese sector. Over time, nearly the whole of the economy's resources will be shifted to the cheese sector with a minimal amount devoted to the high productivity bread sector. Therefore, growth will be limited by the growth and productivity of the cheese sector. As the figure shows, the economy's growth rate nears 1 percent, the growth rate of productivity in the cheese sector.

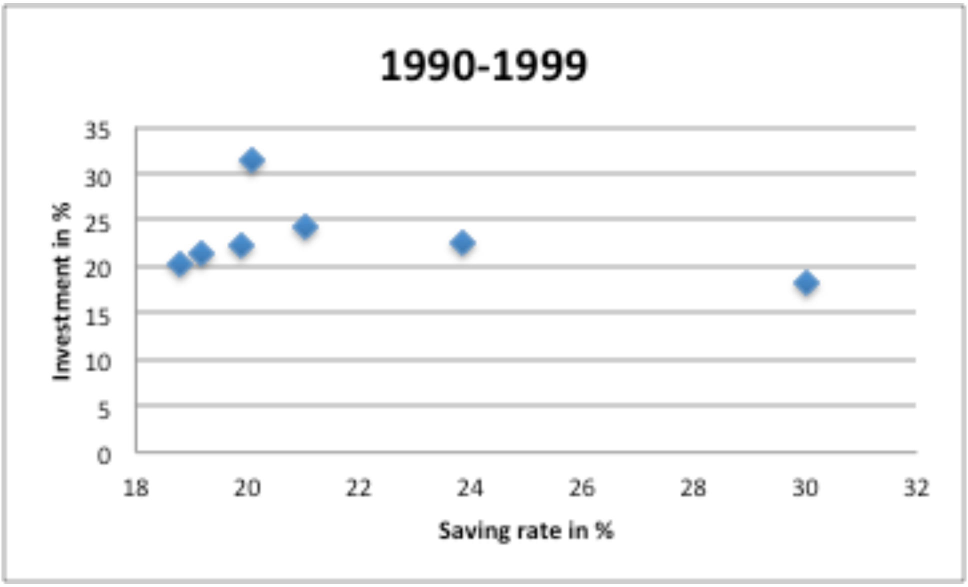


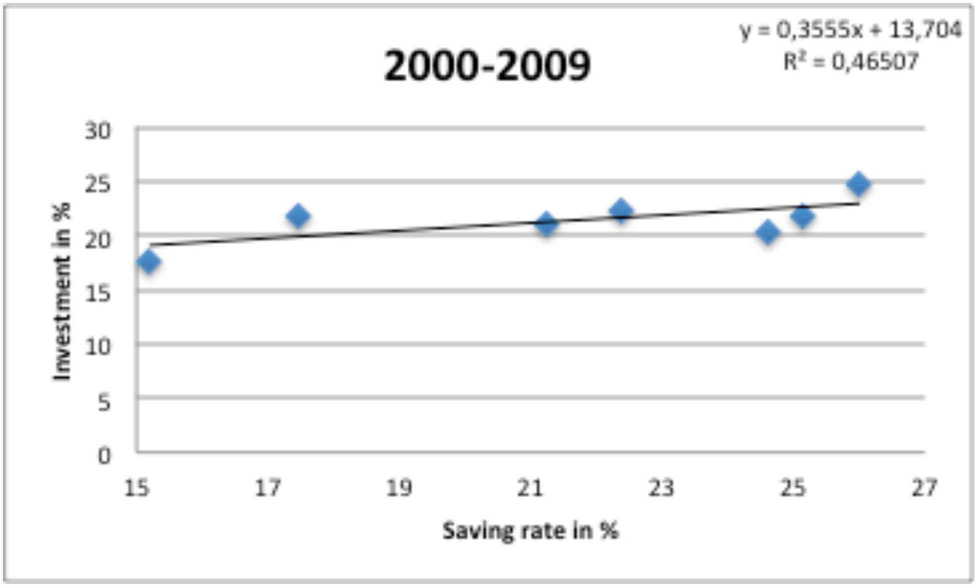
E. Puzzle

Using data from the World Bank database (WDI, here), replicate the Figure 11.5 (of the textbook page 315). You have to use the gross domestic savings in % of the GDP and Gross capital formation in % of the GDP data for UK, USA, Canada, Italy, France, West Germany and Japan). Use a scatter plot for the period 1967-1974 and do the same job for periods 1980-1989, 1990-1999, 2000-2009. Interpret your results.

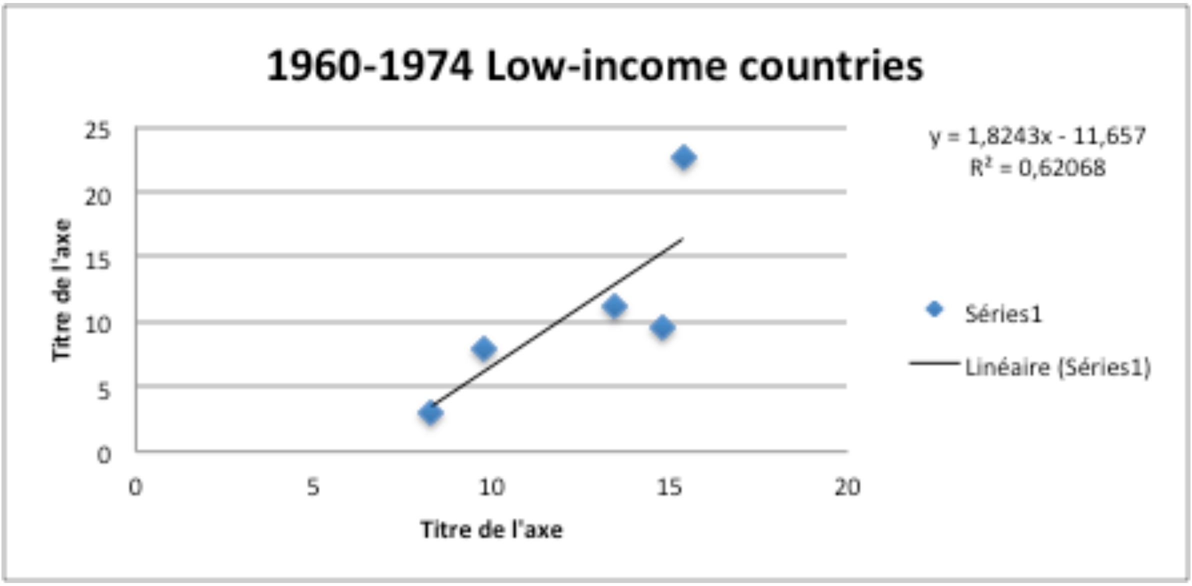


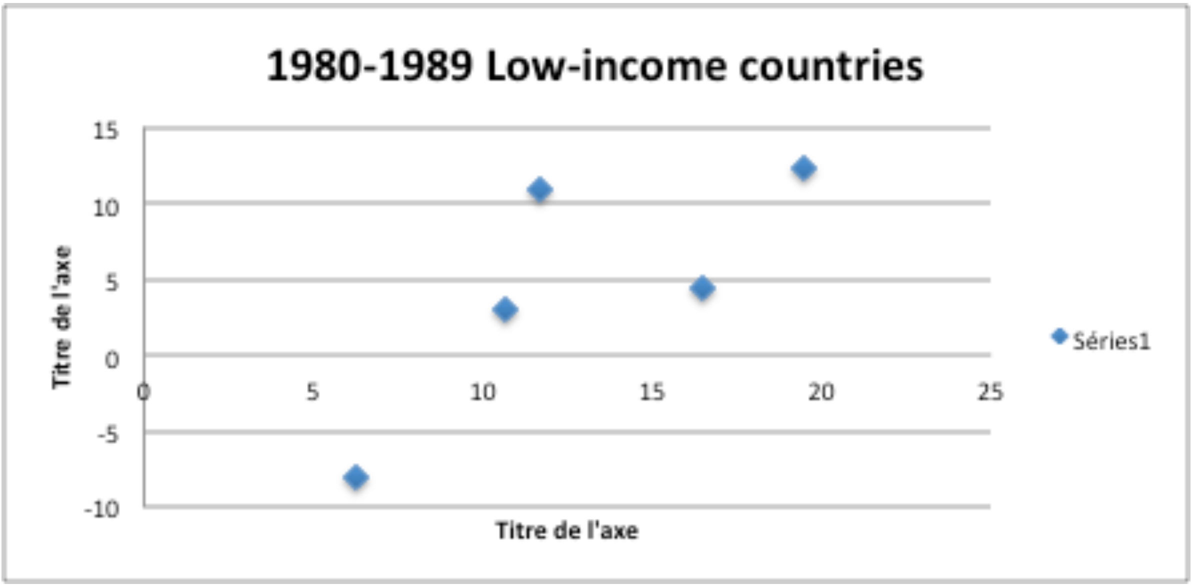

1980-1989
Investment in %
35
30
25
20
15
10
5
0
18
20
22
24
26
28
30
32
34
Saving rate in %

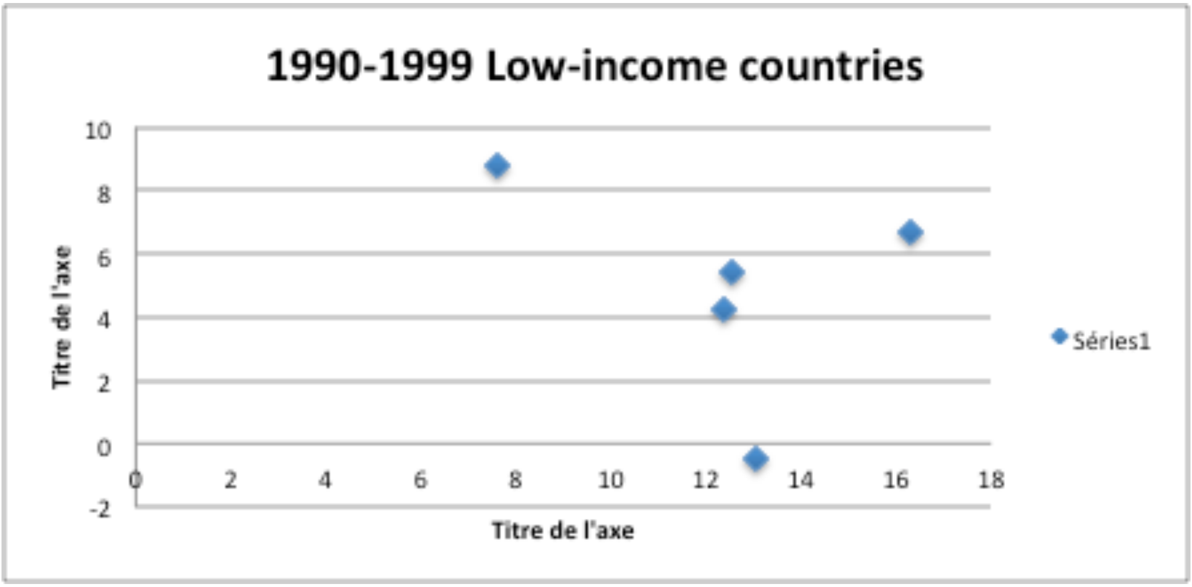

1990-1999
Investment in %
35
30
25
20
15
10
5
0
18
20
22
24
26
28
30
32
Saving rate in %

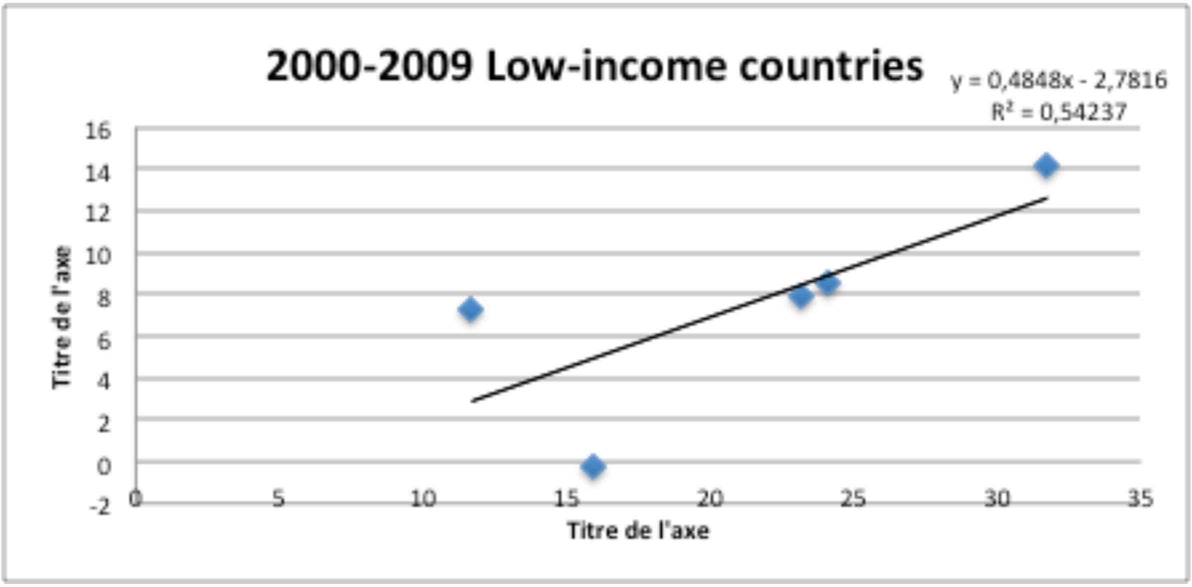


We can see the Feldstein-Horioka puzzle is still true but more the coefficient is lower: from 0.9 to 0.3. So the presumption of free capital movement is not very far to be appropriate.




1980-1989 Low-income countries
15
10
5
0
-5
-10
0
5
10
15
20
25
Titre de l'axe
Titre de l'axe
Séries1


1990-1999 Low-income countries
10
8
6
4
2
0
-2
0
2
4
6
8
10
12
14
16
18
Titre de l'axe
Titre de l'axe
Séries1



The presumption of free capital movement is inappropriate for low-income countries. However, as for the high-income countries, the coefficient is decreasing over time: from 1.8 to 0.5. This puzzle is more important for low-income countries than high-income countries because, given the fact that the national domestic saving rate is low for the low-income countries, the only way to have a high investment rate is to attract foreign investment.