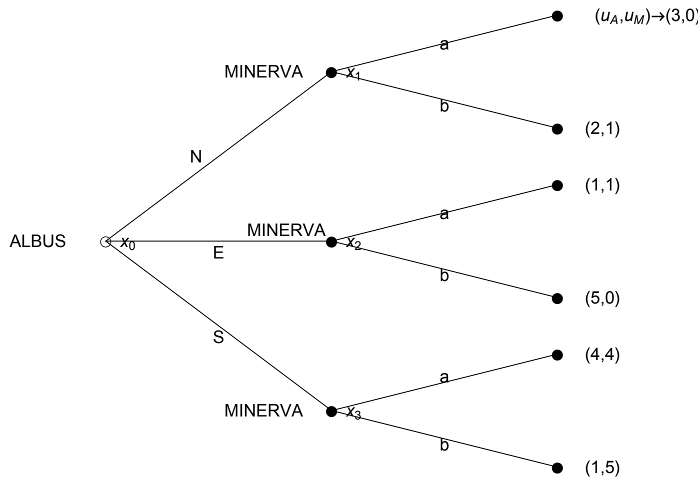
ECO4170A Game Theory with Applications in Corporate Finance
Professor: N. V. Quyen
Assignment 1: Solution
Due Date: 3 February 2017

3U2. In each of the following games, how many pure strategies (complete plans of action, contingent plans) are available to each player? List all the pure strategies of each player.

(a)



Albus makes his move at node x_0, where he has three possible choices: N, E, and S. Thus, he has three contingent plans:

s_{A1} = *play N at* x_0,
s_{A2} = *play E at* x_0,
s_{A3} = *play S at* x_0.

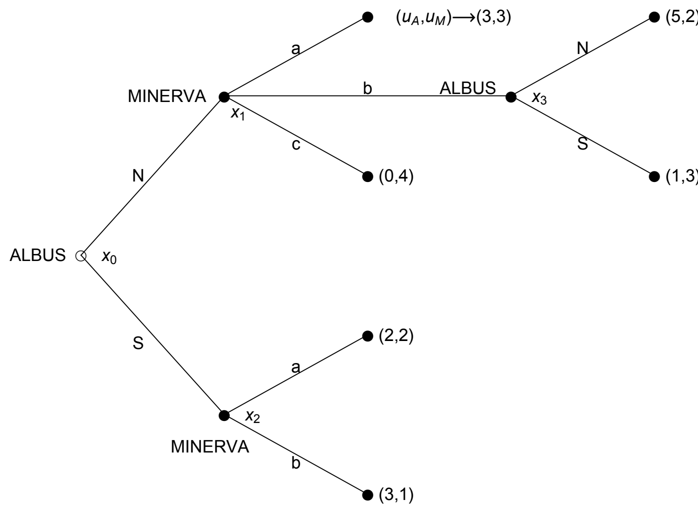
Minerva moves at node x_1, node x_2, and node x_3.

Node	x_1	x_2	x_3
Possible choices	*a*	*a*	*a*
	b	*b*	*b*

She has 8 contingent plans (or 8 pure strategies), and they are listed as follows:

s_{M1} = (a, a, a) = *play a at x_1, a at x_2, and a at x_3,*
s_{M2} = (a, a, b) = *play a at x_1, a at x_2, and b at x_3,*
s_{M3} = (a, b, a) = *play a at x_1, b at x_2, and a at x_3,*
s_{M4} = (a, b, b) = *play a at x_1, b at x_2, and b at x_3,*
s_{M5} = (b, a, a) = *play b at x_1, a at x_2, and a at x_3,*
s_{M6} = (b, a, b) = *play b at x_1, a at x_2, and b at x_3,*
s_{M7} = (b, b, a) = *play b at x_1, b at x_2, and a at x_3,*
s_{M8} = (b, b, b) = *play b at x_1, b at x_2, and b at x_3.*

(b)



Albus makes his move at node x_0 and at node x_3. At node x_0, he has two possible choices: N and S. At node x_3, he has two possible choices N and S. Thus, he has four contingent plans:

$s_{A1} = (N, N) =$ *play N at x_0 and N at x_3,*

$s_{A2} = (N, S) =$ *play N at x_0 and S at x_3,*

$s_{A3} = (S, N) =$ *play S at x_0 and N at x_3,*

$s_{A4} = (S, S) =$ *play S at x_0 and S at x_3.*

Minerva moves at node x_1 and at node x_2.

Node	x_1	x_2
Possible choices	*a*	*a*
	b	*b*
	c	

She has 6 contingent plans (or 6 pure strategies), and they are listed as follows:

$s_{M1} = (a, a) =$ *play a at x_1 and a at x_2,*

$s_{M2} = (a, b) =$ *play a at x_1 and b at x_2,*

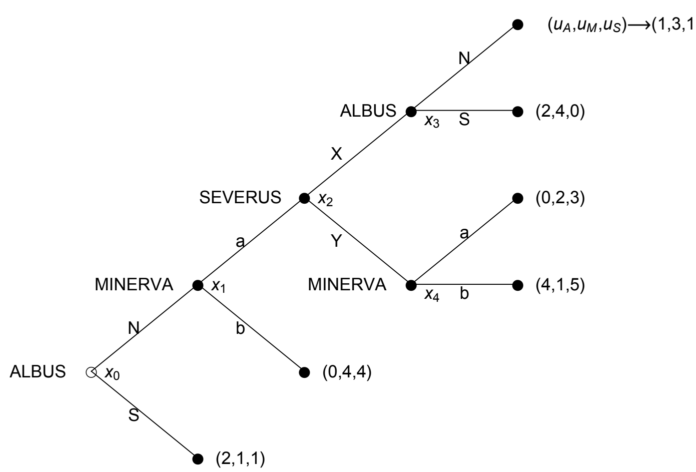
$s_{M3} = (b, a) =$ *play b at x_1 and a at x_2,*

$s_{M4} = (b, b) =$ *play b at x_1 and b at x_2,*

$s_{M5} = (c, a) =$ *play c at x_1 and a at x_2,*

$s_{M6} = (c, b) =$ *play c at x_1 and b at x_2.*

(c)



Albus moves at x_0 and x_3.

x_0 x_3

N	*N*
S	*S*

He has 4 contingent plans:

$s_{A1} = (N, N) =$ *play N at* x_0 *and N at* x_3,
$s_{A2} = (N, S) =$ *play N at* x_0 *and S at* x_3,
$s_{A3} = (S, N) =$ *play S at* x_0 *and N at* x_3,
$s_{A4} = (S, S) =$ *play S at* x_0 *and S at* x_3.

Minerva moves at x_1 *and* x_4.

x_1	x_2
a	*a*
b	*b*

She has 4 contingent plans:

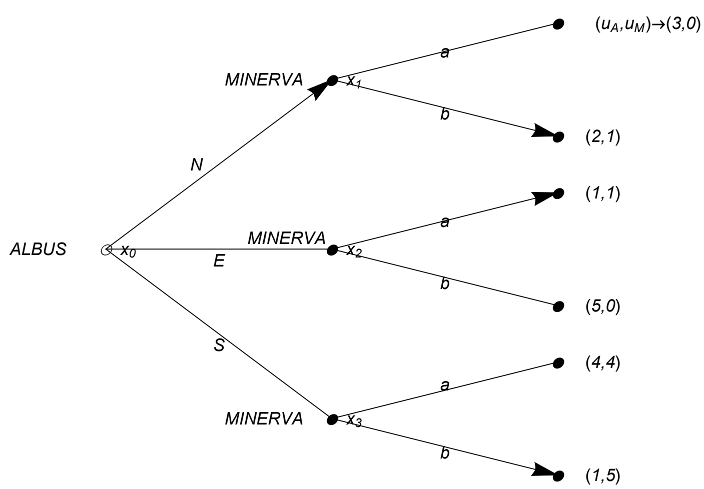
$s_{M1} = (a, a) =$ *play a at* x_1 *and a at* x_4,
$s_{M2} = (a, b) =$ *play a at* x_1 *and b at* x_4,
$s_{M3} = (b, a) =$ *play a at* x_1 *and b at* x_4,
$s_{M4} = (b, b) =$ *play b at* x_1 *and b at* x_4.

Severus moves at x_3. *He has 2 contingent plans:*

$s_{S1} = X =$ *play X at* x_3,
$s_{S2} = Y =$ *play Y at* x_3.

3U3. For each of the games illustrated in Exercise 3U2, identify the rollback equilibrium (sub-game perfect Nash equilibrium) outcome and the complete equilibrium strategy for each player.

(a)



To find the sub-game perfect Nash equilibrium, we use backward induction and begin at the end.

First, consider the sub-game that begins at node x_1. *In this sub-game, Minerva obtains a payoff of 0 if she plays a and a payoff of 1 if she plays b. Thus, she will play b. Her optimal choice is depicted by the arrow.*

Second, consider the sub-game that begins at node x_2. *In this sub-game, Minerva obtains a payoff of 1 if she plays a and a payoff of 0 if she plays b. Thus, she will play a. Her optimal choice is depicted by the arrow.*

Third, consider the sub-game that begins at node x_3. *In this sub-game, Minerva obtains a payoff of 4 if she plays a and a payoff of 5 if she plays b. Thus, she will play b. Her optimal choice is depicted by the* *arrow.*

The contingent plan chosen by Minerva is thus $s_{M6} = (b, a, b)$.

Having found the solutions for the sub-games after Albus has moved, we now find ourselves at node x_0*, where Albus is supposed to move.*

If Albus plays N at x_0*, the game moves to node* x_1*, and at* x_1 *we know Minerva will play b, resulting in a payoff of 2 for Albus.*

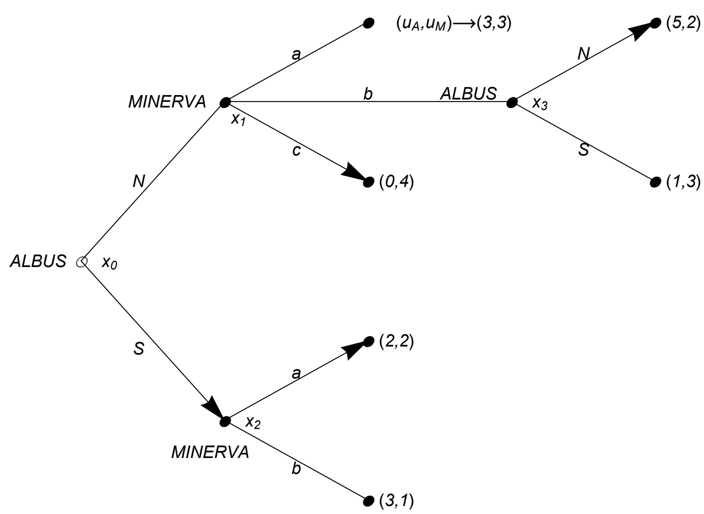
If Albus plays E at x_0*, the game moves to node* x_2*, and at* x_2 *we know Minerva will play a, resulting in a payoff of 1 for Albus.*

If Albus plays S at x_0*, the game moves to node* x_3*, and at* x_3 *we know Minerva will play b, resulting in a payoff of 1 for Albus.*

The optimal choice for Albus at x_0 *is thus* $s_{A3} = N$*, and this choice is depicted by the arrow emanating from* x_0*.*

The pair of contingent plans (or strategies) (s_{A3}, s_{M6}) ***constitutes the sub-game perfect Nash equilibrium of the game 3U2(a).***

(b)



For the sub-game beginning at node x_3*, Albus has two possible choices: N and S. Playing N gives Albus a payoff of 5, while playing S gives Albus a payoff of 1. Thus the optimal choice for Albus is N, as depicted by the arrow.*

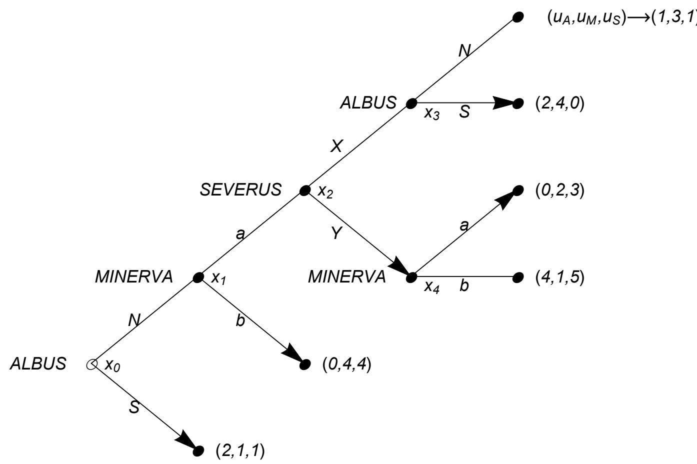
For the sub-game beginning at node x_1*, Minerva has three possible choices: a, b, and c. If she plays a, the game ends after that, and she obtains a payoff of 3. If she plays b, the game evolves to* x_3*, and at* x_3*, we know that Albus will play N, resulting in a payoff of 5 for himself and 2 for Minerva. If she plays c, then the game ends after that, and Minerva gets a payoff of 4. Thus for the sub-game begins at* x_1*, the choice of Minerva is c, as depicted by the arrow, and the choice of Albus (made at node* x_3*) is N.*

For the that sub-game begins at x_2*, the optimal choice of Minerva is a, as depicted by the arrow.*

We are now at node x_0, where Albus has two possible choices: N and S. If he plays N, the game arrives at x_1, and for the sub-game beginning at x_1, we know Minerva will play c, resulting in a zero payoff for Albus. Thus, the best choice for Albus at x_0 is S.

The pair of strategies (s_{A3}, s_{M5}) = ((S, N), (c, a)) constitutes the sub-game perfect Nash equilibrium for the game 3U2(b).

(c)



In the sub-game beginning at node x_3, Albus chooses S. In the sub-game beginning at x_4, Minerva chooses a.

In the sub-game beginning at node x_2, if Severus chooses X, then the game arrives at x_3, and at this node, Albus will choose S, resulting in zero payoff for Severus. If Severus chooses Y at x_2, then the game evolves to x_4, at which Minerva will play a, resulting in a payoff of 3 for Severus. Thus, at x_2 Severus will play Y.

In the sub-game beginning at x_1, if Minerva plays a, the game will evolve to x_2 and then x_4, resulting in a payoff of 2 for Minerva. On the other hand, if she plays b at x_1, then the game ends after that, and Minerva obtains a payoff of 4. Thus, at node x_1, Minerva will play b.

At node x_0, if Albus plays N, the game will arrive in x_1, and then Minerva will choose b, resulting in a zero payoff for Albus. On the other hand, if Albus chooses S at x_0, then the game ends after that, and Albus will obtain a payoff of 2. Thus, Albus will play S at x_0.

The arrows in the preceding game tree depicts the move made by each player when it is his or her turn to move. The list of strategies

(s_{A4}, s_{M3}, S_{S2}) = ((S, S), (b, a), Y)

constitutes the sub-game perfect Nash equilibrium of the game 3U2(c).

The *Mathematica* codes to draw the game tree of 3U2(a)

```
In[1]:= g[0] = Graphics[Text["○", {0, 0}] ];

In[2]:= g[1] = Graphics[Line[{{0, 0}, {1, 3}}]];

In[3]:= g[1 bis] = Graphics[Arrow[{{0, 0}, {1, 3}}]];
```

```
In[4]:= g[2] = Graphics[Text["ALBUS", {-0.3, 0}]];
In[5]:= g[3] = Graphics[Text["MINERVA", {1 - 0.3, 3}]];
In[6]:= g[4] = Graphics[Text["●", {1, 3}]];
In[7]:= g[5] = Graphics[Text["N", {0.5 - 0.1, 1.5}]];
In[8]:= g[6] = Graphics[Line[{{1, 3}, {2, 4}}]];
In[9]:= g[7] = Graphics[Text["●", {2, 4}]];
In[10]:= g[8] = Graphics[Text["(uA,uM)→(3,0)", {2 + 0.4, 4}]];
In[11]:= g[9] = Graphics[Text["a", {1.5, 3.5}]];
In[12]:= g[10] = Graphics[Text["x0", {0.1, 0}]];
In[13]:= g[11] = Graphics[Text["x1", {1 + 0.1, 3}]];
In[14]:= g[12] = Graphics[Line[{{1, 3}, {2, 2}}]];
In[15]:= g[12 bis] = Graphics[Arrow[{{1, 3}, {2, 2}}]];
In[16]:= g[13] = Graphics[Text["b", {1.5, 2.5 - 0.1}]];
In[17]:= g[14] = Graphics[Text["●", {2, 2}]];
In[18]:= g[15] = Graphics[Text["(2,1)", {2 + 0.2, 2}]];
In[19]:= g[16] = Graphics[Line[{{0, 0}, {1, 0}}]];
In[20]:= g[17] = Graphics[Text["●", {1, 0}]];
In[21]:= g[18] = Graphics[Line[{{1, 0}, {2, 1}}]];
In[22]:= g[18 bis] = Graphics[Arrow[{{1, 0}, {2, 1}}]];
In[23]:= g[19] = Graphics[Text["●", {2, 1}]];
In[24]:= g[20] = Graphics[Text["(1,1)", {2 + 0.2, 1}]];
In[25]:= g[21] = Graphics[Text["a", {1.5, 0.5}]];
In[26]:= g[22] = Graphics[Text["MINERVA", {1 - 0.2, 0.2}]];
In[27]:= g[23] = Graphics[Text["x2", {1 + 0.1, 0}]];
In[28]:= g[24] = Graphics[Line[{{1, 0}, {2, -1}}]];
In[29]:= g[25] = Graphics[Text["b", {1.5, -0.5 - 0.1}]];
In[30]:= g[26] = Graphics[Text["●", {2, -1}]];
In[31]:= g[27] = Graphics[Text["(5,0)", {2 + 0.2, -1}]];
In[32]:= g[28] = Graphics[Line[{{0, 0}, {1, -3}}]];
In[33]:= g[29] = Graphics[Text["●", {1, -3}]];
In[34]:= g[30] = Graphics[Text["E", {0.5, -0.2}]];
```

```
In[35]:= g[31] = Graphics[Text["S", {0.5, -1.5 - 0.2}]];

In[36]:= g[32] = Graphics[Text["x3", {1 + 0.1, -3}]];

In[37]:= g[33] = Graphics[Text["MINERVA", {1 - 0.3, -3}]];

In[38]:= g[34] = Graphics[Line[{{1, -3}, {2, -2}}]];

In[39]:= g[35] = Graphics[Text["●", {2, -2}]];

In[40]:= g[36] = Graphics[Text["(4,4)", {2 + 0.2, -2}]];

In[41]:= g[37] = Graphics[Text["a", {1.5, -2 - 0.5 + 0.1}]];

In[42]:= g[38] = Graphics[Text["b", {1.5, -3 - 0.5 - 0.1}]];

In[43]:= g[39] = Graphics[Line[{{1, -3}, {2, -4}}]];

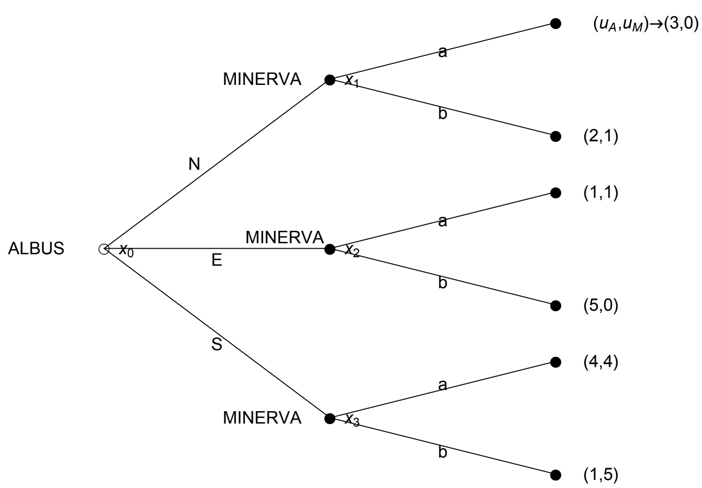
In[44]:= g[39 bis] = Graphics[Arrow[{{1, -3}, {2, -4}}]];

In[45]:= g[40] = Graphics[Text["●", {2, -4}]];

In[46]:= g[41] = Graphics[Text["(1,5)", {2 + 0.2, -4}]];

In[47]:= Show[g[0], g[1], g[2], g[3], g[4], g[5], g[6], g[7], g[8], g[9], g[10], g[11],
 g[12], g[13], g[14], g[15], g[16], g[17], g[18], g[19], g[20], g[21], g[22],
 g[23], g[24], g[25], g[26], g[27], g[28], g[29], g[30], g[31], g[32], g[33],
 g[34], g[35], g[36], g[37], g[38], g[39], g[40], g[41], AspectRatio → 0.75]
```

Out[47]=



The sub-game perfect Nash equilibrium of the game 3U2(a)

In[48]:=
```
Show[g[0], g[1 bis], g[2], g[3], g[4], g[5], g[6], g[7], g[8], g[9], g[10], g[11],
 g[12 bis], g[13], g[14], g[15], g[16], g[17], g[18 bis], g[19], g[20], g[21],
 g[22], g[23], g[24], g[25], g[26], g[27], g[28], g[29], g[30], g[31], g[32], g[33],
 g[34], g[35], g[36], g[37], g[38], g[39 bis], g[40], g[41], AspectRatio → 0.75]
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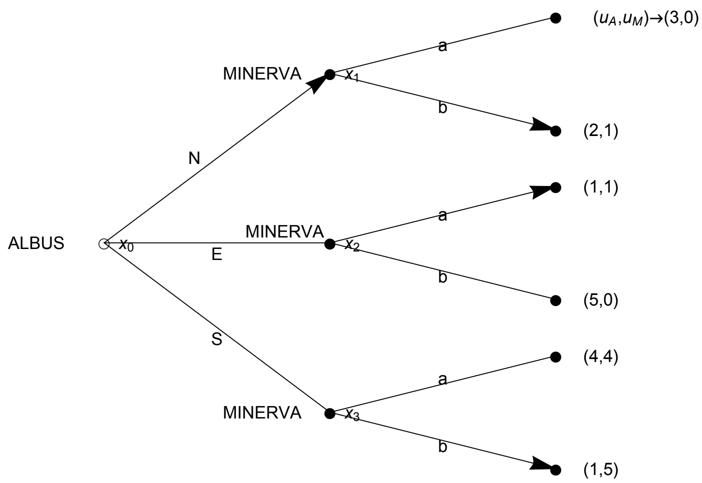
Out[48]=


In[49]:=
```
ClearAll[g];
```

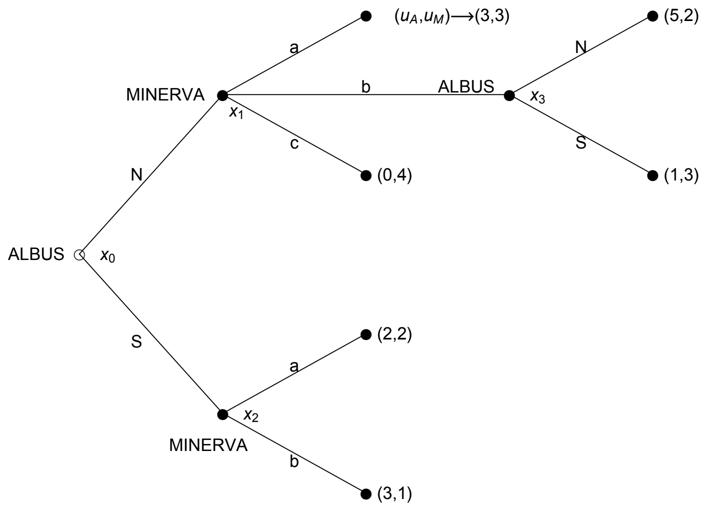
The *Mathematica* codes to draw the game tree of 3U2(b)

```
In[50]:= g[0] = Graphics[Text["○", {0, 0}] ];
In[51]:= g[1] = Graphics[Line[{{0, 0}, {1, 2}}]];
In[52]:= g[2] = Graphics[Text["ALBUS", {-0.3, 0}]];
In[53]:= g[3] = Graphics[Text["MINERVA", {1 - 0.4, 2}]];
In[54]:= g[4] = Graphics[Text["●", {1, 2}]];
In[55]:= g[5] = Graphics[Text["N", {0.5 - 0.1, 1}]];
In[56]:= g[6] = Graphics[Line[{{1, 2}, {2, 3}}]];
In[57]:= g[7] = Graphics[Text["●", {2, 3}]];
In[58]:= g[8] = Graphics[Text["(uA,uM)⟶(3,3)", {2 + 0.6, 3}]];
In[59]:= g[9] = Graphics[Text["a", {1.5, 2.5 + 0.1}]];
In[60]:= g[10] = Graphics[Line[{{1, 2}, {3, 2}}]];
In[61]:= g[11] = Graphics[Text["●", {3, 2}]];
In[62]:= g[12] = Graphics[Text["b", {2, 2 + 0.1}]];
In[63]:= g[13] = Graphics[Text["ALBUS", {3 - 0.3, 2 + 0.1}]];
In[64]:= g[14] = Graphics[Line[{{1, 2}, {2, 1}}]];
```

```
In[65]:= g[14 bis] = Graphics[Arrow[{{1, 2}, {2, 1}}]];
In[66]:= g[15] = Graphics[Text["●", {2, 1}]];
In[67]:= g[16] = Graphics[Text["c", {1.5, 1.5 - 0.1}]];
In[68]:= g[17] = Graphics[Text["(0,4)", {2 + 0.2, 1}]];
In[69]:= g[18] = Graphics[Line[{{3, 2}, {4, 3}}]];
In[70]:= g[18 bis] = Graphics[Arrow[{{3, 2}, {4, 3}}]];
In[71]:= g[19] = Graphics[Text["●", {4, 3}]];
In[72]:= g[20] = Graphics[Text["(5,2)", {4 + 0.2, 3}]];
In[73]:= g[21] = Graphics[Line[{{3, 2}, {4, 1}}]];
In[74]:= g[22] = Graphics[Text["●", {4, 1}]];
In[75]:= g[23] = Graphics[Text["(1,3)", {4 + 0.2, 1}]];
In[76]:= g[24] = Graphics[Line[{{0, 0}, {1, -2}}]];
In[77]:= g[24 bis] = Graphics[Arrow[{{0, 0}, {1, -2}}]];
In[78]:= g[25] = Graphics[Text["●", {1, -2}]];
In[79]:= g[26] = Graphics[Text["S", {0.5 - 0.1, -1 - 0.1}]];
In[80]:= g[27] = Graphics[Line[{{1, -2}, {2, -1}}]];
In[81]:= g[27 bis] = Graphics[Arrow[{{1, -2}, {2, -1}}]];
In[82]:= g[28] = Graphics[Text["●", {2, -1}]];
In[83]:= g[29] = Graphics[Text["(2,2)", {2 + 0.2, -1}]];
In[84]:= g[30] = Graphics[Line[{{1, -2}, {2, -3}}]];
In[85]:= g[31] = Graphics[Text["●", {2, -3}]];
In[86]:= g[32] = Graphics[Text["(3,1)", {2 + 0.2, -3}]];
In[87]:= g[33] = Graphics[Text["MINERVA", {1 - 0.1, -2 - 0.4}]];
In[88]:= g[34] = Graphics[Text["a", {1.5, -1.5 + 0.1}]];
In[89]:= g[35] = Graphics[Text["b", {1.5, -2.5 - 0.1}]];
In[90]:= g[36] = Graphics[Text["x0", {0.2, 0}]];
In[91]:= g[37] = Graphics[Text["x1", {1 + 0.1, 2 - 0.2}]];
In[92]:= g[38] = Graphics[Text["x2", {1 + 0.2, -2}]];
In[93]:= g[39] = Graphics[Text["x3", {3 + 0.2, 2}]];
In[94]:= g[40] = Graphics[Text["N", {3.5, 2.5 + 0.1}]];
In[95]:= g[41] = Graphics[Text["S", {3.5, 1.5 - 0.1}]];
```

In[96]:=
```
Show[g[0], g[1], g[2], g[3], g[4], g[5], g[6], g[7], g[8], g[9], g[10], g[11],
 g[12], g[13], g[14], g[15], g[16], g[17], g[18], g[19], g[20], g[21], g[22],
 g[23], g[24], g[25], g[26], g[27], g[28], g[29], g[30], g[31], g[32], g[33],
 g[34], g[35], g[36], g[37], g[38], g[39], g[40], g[41], AspectRatio → 0.75]
```

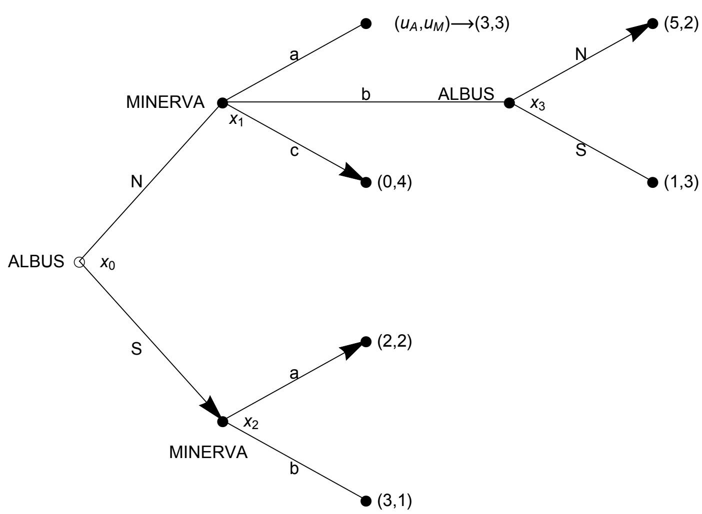
Out[96]=



The sub-game perfect Nash equilibrium of the game 3U2(b)

In[97]:=
```
Show[g[0], g[1], g[2], g[3], g[4], g[5], g[6], g[7], g[8], g[9], g[10], g[11], g[12],
 g[13], g[14 bis], g[15], g[16], g[17], g[18 bis], g[19], g[20], g[21], g[22],
 g[23], g[24 bis], g[25], g[26], g[27 bis], g[28], g[29], g[30], g[31], g[32],
 g[33], g[34], g[35], g[36], g[37], g[38], g[39], g[40], g[41], AspectRatio → 0.75]
```

Out[97]=



In[98]:=
```
ClearAll[g];
```

The *Mathematica* codes to draw the game tree of 3U2(c)

In[99]:=
```
g[0] = Graphics[Text["○", {0, 0}] ];
```

In[100]:=
```
g[1] = Graphics[Line[{{0, 0}, {1, 1}}]];
```

```
In[101]:= g[2] = Graphics[Text["ALBUS", {-0.5, 0}]];
In[102]:= g[3] = Graphics[Text["MINERVA", {1 - 0.6, 1}]];
In[103]:= g[4] = Graphics[Text["●", {1, 1}]];
In[104]:= g[5] = Graphics[Line[{{1, 1}, {2, 2}}]];
In[105]:= g[6] = Graphics[Text["●", {2, 2}]];
In[106]:= g[7] = Graphics[Text["SEVERUS", {2 - 0.6, 2}]];
In[107]:= g[8] = Graphics[Line[{{2, 2}, {3, 3}}]];
In[108]:= g[9] = Graphics[Text["●", {3, 3}]];
In[109]:= g[10] = Graphics[Text["ALBUS", {3 - 0.4, 3}]];
In[110]:= g[11] = Graphics[Line[{{3, 3}, {4, 4}}]];
In[111]:= g[12] = Graphics[Text["●", {4, 4}]];
In[112]:= g[13] = Graphics[Text["(uA,uM,uS)⟶(1,3,1)", {4 + 1, 4}]];
In[113]:= g[14] = Graphics[Line[{{3, 3}, {4, 3}}]];
In[114]:= g[14 bis] = Graphics[Arrow[{{3, 3}, {4, 3}}]];
In[115]:= g[15] = Graphics[Text["●", {4, 3}]];
In[116]:= g[16] = Graphics[Text["(2,4,0)", {4 + 0.4, 3}]];
In[117]:= g[17] = Graphics[Text["N", {3.5, 3.5 + 0.1}]];
In[118]:= g[18] = Graphics[Text["S", {3.5, 3 - 0.1}]];
In[119]:= g[19] = Graphics[Text["X", {2.5 - 0.2, 2.5}]];
In[120]:= g[20] = Graphics[Line[{{2, 2}, {3, 1}}]];
In[121]:= g[20 bis] = Graphics[Arrow[{{2, 2}, {3, 1}}]];
In[122]:= g[21] = Graphics[Text["●", {3, 1}]];
In[123]:= g[22] = Graphics[Text["Y", {2.5 - 0.2, 1.5}]];
In[124]:= g[23] = Graphics[Line[{{3, 1}, {4, 2}}]];
In[125]:= g[23 bis] = Graphics[Arrow[{{3, 1}, {4, 2}}]];
In[126]:= g[24] = Graphics[Text["●", {4, 2}]];
In[127]:= g[25] = Graphics[Text["(0,2,3)", {4 + 0.4, 2}]];
In[128]:= g[26] = Graphics[Line[{{3, 1}, {4, 1}}]];
In[129]:= g[27] = Graphics[Text["●", {4, 1}]];
In[130]:= g[28] = Graphics[Text["(4,1,5)", {4 + 0.4, 1}]];
In[131]:= g[29] = Graphics[Text["a", {3.5, 1.5 + 0.1}]];
```

```
In[132]:= g[30] = Graphics[Text["b", {3.5, 1 - 0.1}]];

In[133]:= g[31] = Graphics[Text["MINERVA", {3 - 0.6, 1}]];

In[134]:= g[32] = Graphics[Line[{{1, 1}, {2, 0}}]];

In[135]:= g[32 bis] = Graphics[Arrow[{{1, 1}, {2, 0}}]];

In[136]:= g[33] = Graphics[Text["●", {2, 0}]];

In[137]:= g[34] = Graphics[Text["(0,4,4)", {2 + 0.4, 0}]];

In[138]:= g[35] = Graphics[Text["a", {1.5 - 0.1, 1.5}]];

In[139]:= g[36] = Graphics[Text["b", {1.5 - 0.1, 0.5}]];

In[140]:= g[37] = Graphics[Line[{{0, 0}, {1, -1}}]];

In[141]:= g[37 bis] = Graphics[Arrow[{{0, 0}, {1, -1}}]];

In[142]:= g[38] = Graphics[Text["●", {1, -1}]];

In[143]:= g[39] = Graphics[Text["(2,1,1)", {1 + 0.4, -1}]];

In[144]:= g[40] = Graphics[Text["N", {0.5 - 0.1, 0.5}]];

In[145]:= g[41] = Graphics[Text["S", {0.5 - 0.1, -0.5}]];

In[146]:= g[42] = Graphics[Text["x0", {0.2, 0}]];

In[147]:= g[43] = Graphics[Text["x1", {1 + 0.2, 1}]];

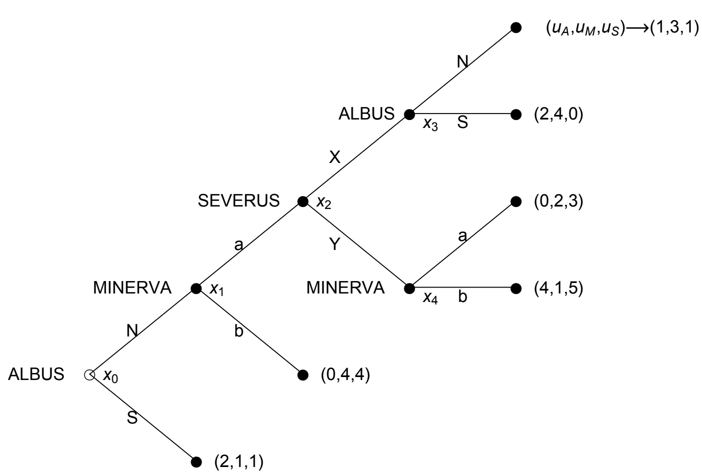
In[148]:= g[44] = Graphics[Text["x2", {2 + 0.2, 2}]];

In[149]:= g[45] = Graphics[Text["x3", {3 + 0.2, 3 - 0.1}]];

In[150]:= g[46] = Graphics[Text["x4", {3 + 0.2, 1 - 0.1}]];

In[151]:= Show[g[0], g[1], g[2], g[3], g[4], g[5], g[6], g[7], g[8], g[9],
  g[10], g[11], g[12], g[13], g[14], g[15], g[16], g[17], g[18], g[19],
  g[20], g[21], g[22], g[23], g[24], g[25], g[26], g[27], g[28], g[29],
  g[30], g[31], g[32], g[33], g[34], g[35], g[36], g[37], g[38], g[39],
  g[40], g[41], g[42], g[43], g[44], g[45], g[46], AspectRatio → 0.75]
```

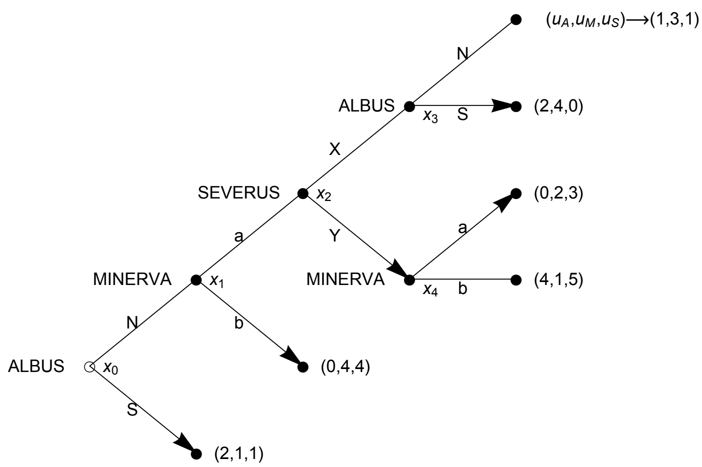
Out[151]=



The sub-game perfect Nash equilibrium for the game 3U2(c)

In[152]:=
```
Show[g[0], g[1], g[2], g[3], g[4], g[5], g[6], g[7], g[8], g[9], g[10],
 g[11], g[12], g[13], g[14 bis], g[15], g[16], g[17], g[18], g[19], g[20 bis],
 g[21], g[22], g[23 bis], g[24], g[25], g[26], g[27], g[28], g[29], g[30],
 g[31], g[32 bis], g[33], g[34], g[35], g[36], g[37 bis], g[38], g[39],
 g[40], g[41], g[42], g[43], g[44], g[45], g[46], AspectRatio → 0.75]
```

Out[152]=



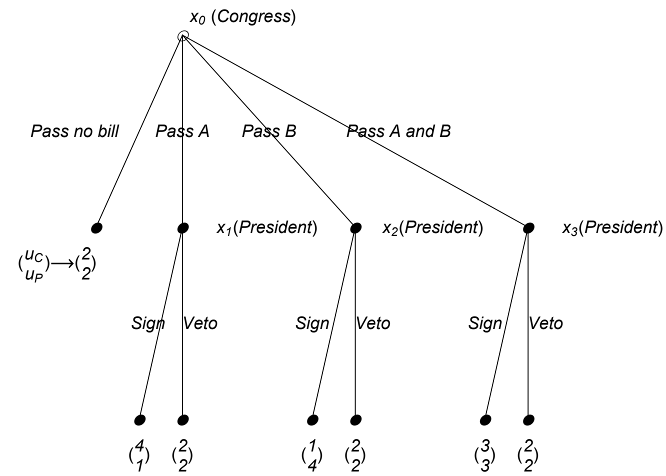
In[153]:=
```
ClearAll[g];
```

3U4. Two distinct proposals, A and B, are being debated in Washington. Congress likes Proposition A, and the president likes Proposition B. The proposals are not mutually exclusive; either or both or neither may become law. Thus, there are 4 possible outcomes, and the ranking of the two sides are as follows:

Outcome	*Congress*	*President*
A becomes law	*4*	*1*
B becomes law	*1*	*4*
Both A and B becomes law	*3*	*3*
Neither (status quo prevails)	*2*	*2*

Here a larger number represents a more favored outcome.

(a) The game tree is

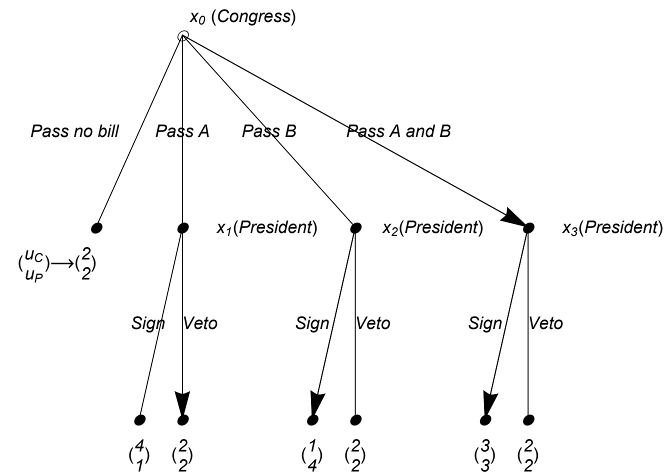


At node x_0, Congress has four possible choices: pass no bill, pass A, pass B, and pass both A&B. Thus, at x_0, Congresds has four contingent plans.

At node x_1, the Preseident has two possible choices: Sign or Veto ther bill. Similarly, at each of the two nodes x_2 and x_3, the President also has two possible choices. Thus, the President has 8 contingent plans.

Sign at x_1, Sign at x_2, and Sign at x_3,
Sign at x_1, Sign at x_2, and Veto at x_3,
Sign at x_1, Veto at x_2, and Sign at x_3,
Sign at x_1, Vetoat x_2, and Veto at x_3,
Veto at x_1, Sign at x_2, and Sign at x_3,
Veto at x_1, Sign at x_2, and Veto at x_3,
Veto at x_1, Veto at x_2, and Sign at x_3,
Veto at x_1, Veto at x_2, and Veto at x_3.

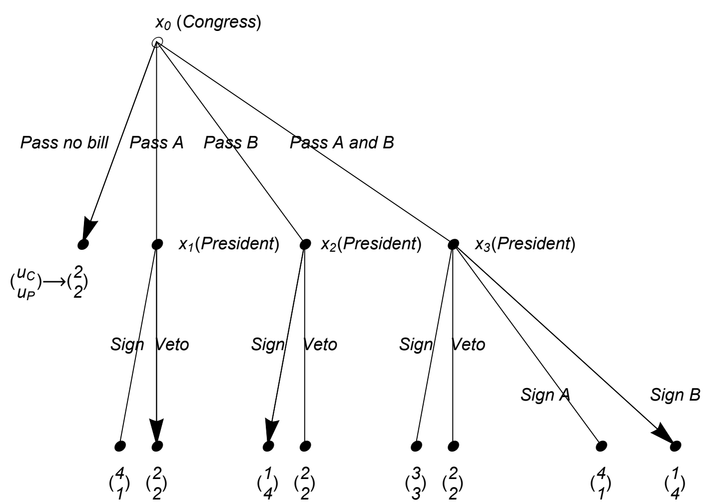
The sub-game perfect Nash equilibrium is the combination of strategies
Congress passses both A and B,
The President vetoes at x_1, signs at x_2, and signs at x_3.
The combination of strategies that constitutes the sub-game perfect Nash equilibrium is depicted by the arrows in the following game tree:



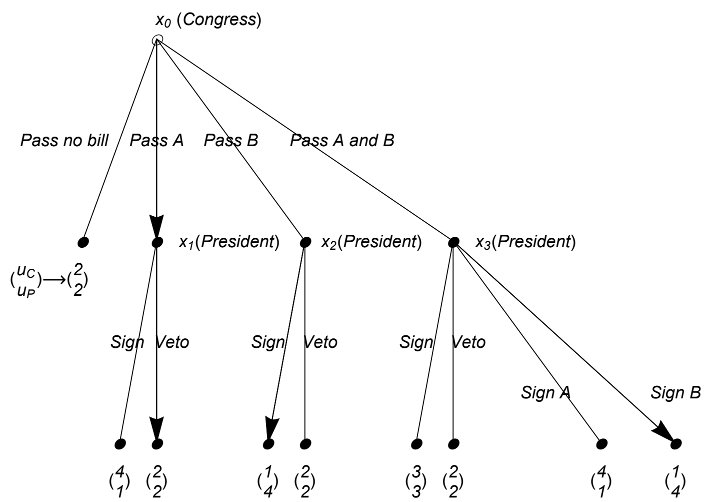
Under the sub-game perfect Nash equilibrium, Congress will pass both A and B. The President will then sign the bill.

(b) If the President may choose not only to sign or veto the bill as a whole, but also to veto just one of the two items, the game has two sub-game perfect Nash equilibria.

The following figure depicts the first sub-game perfect Nash equilibrium:



The second sub-game pefect Nash Equilibrium is depicted in the following figure



(c) When the President can veto one of the two items in the bill, she has more choices than before at node x_3, i.e., she can choose a new strategy that is better than the original strategy. That is why, the new sub-game perfect Nash equilibrium is different from the old one. Because the interest of Congress and the interest of the President are opposite to each other, Congress anticipates what the President will do and feels that it is better-off by not passing the bill or by passing only A. The end result is that both Congress and the President are worse-off: the payoff for both players falls from 3 to 2.

The *Mathematica* codes to draw the game tree of 3U4(a)

```
In[154]:= g[0] = Graphics[Text["○", {1, 1}] ];
In[155]:= g[0, 0] = Graphics[Text["●", {0, 0}]];
In[156]:= g[1] = Graphics[Line[{{0, 0}, {1, 1}}]];
In[157]:= g[1, 0] = Graphics[Arrow[{{1, 1}, {0, 0}}]];
In[158]:= g[2] = Graphics[Text["x0  (Congress)", {1.7, 1.1}]];
In[159]:= g[3] = Graphics[Text["Pass no bill", {-0.25, 0.5}]];
In[160]:= g[3, 0] = Graphics[Text["●", {0.5, -1}]];
In[161]:= g[4] = Graphics[Text["(uC uP)⟶(2 2)", {-0.45, -0.2}]];
In[162]:= g[5] = Graphics[Line[{{1, 1}, {1, 0}}]];
In[163]:= g[5, 0] = Graphics[Arrow[{{1, 1}, {5, 0}}]];
In[164]:= g[5, 1] = Graphics[Arrow[{{1, 1}, {1, 0}}]];
In[165]:= g[6] = Graphics[Text["●", {1, 0}]];
In[166]:= g[7] = Graphics[Text["x1(President)", {1.98, 0}]];
In[167]:= g[8] = Graphics[Text["Pass A", {1, 0.5}]];
In[168]:= g[9] = Graphics[Line[{{1, 0}, {0.5, -1}}]];
```

```
In[169]:= g[10] = Graphics[Text["Sign", {0.6, -0.5}]];
```

In[170]:= **g[11] = Graphics[Text["$\binom{4}{1}$", {0.5, -1.2}]];**

```
In[171]:= g[12, 0] = Graphics[Arrow[{{1, 0}, {1, -1}}]];

In[172]:= g[12] = Graphics[Line[{{1, 0}, {1, -1}}]];

In[173]:= g[13, 0] = Graphics[Text["●", {1, -1}]];

In[174]:= g[13] = Graphics[Text["Veto", {1.2, -0.5}]];
```

In[175]:= **g[14] = Graphics[Text["$\binom{2}{2}$", {1, -1.2}]];**

```
In[176]:= g[15] = Graphics[Line[{{1, 1}, {3, 0}}]];

In[177]:= g[16] = Graphics[Text["Pass B", {2, 0.5}]];

In[178]:= g[17] = Graphics[Text["●", {3, 0}]];

In[179]:= g[18] = Graphics[Text["x_2(President)", {3 + 0.9, 0}]];

In[180]:= g[19] = Graphics[Line[{{3, 0}, {2.5, -1}}]];

In[181]:= g[19, 0] = Graphics[Arrow[{{3, 0}, {2.5, -1}}]];

In[182]:= g[20] = Graphics[Text["Sign", {2.5, -0.5}]];

In[183]:= g[20, 0] = Graphics[Text["●", {3, -1}]];

In[184]:= g[21] = Graphics[Line[{{3, 0}, {3, -1}}]];

In[185]:= g[21, 0] = Graphics[Text["●", {2.5, -1}]];

In[186]:= g[21, 1] = Graphics[Arrow[{{3, 0}, {2.5, -1}}]];

In[187]:= g[22] = Graphics[Text["Veto", {3.2, -0.5}]];
```

In[188]:= **g[23] = Graphics[Text["$\binom{1}{4}$", {2.5, -1.2}]];**

In[189]:= **g[24] = Graphics[Text["$\binom{2}{2}$", {3, -1.2}]];**

```
In[190]:= g[25] = Graphics[Line[{{1, 1}, {5, 0}}]];

In[191]:= g[25, 0] = Graphics[Arrow[{{1, 1}, {5, 0}}]];

In[192]:= g[26] = Graphics[Text["Pass A and B", {3.5, 0.5}]];

In[193]:= g[27] = Graphics[Text["●", {5, 0}]];

In[194]:= g[28] = Graphics[Text["x_3(President)", {5 + 0.98, 0}]];

In[195]:= g[29] = Graphics[Line[{{5, 0}, {4.5, -1}}]];

In[196]:= g[29, 0] = Graphics[Arrow[{{5, 0}, {4.5, -1}}]];

In[197]:= g[30, 0] = Graphics[Text["●", {4.5, -1}]];

In[198]:= Graphics[Line[{{5, 0}, {4.5, -1}}]];
```

In[199]:=
```
g[30] = Graphics[Text["Sign", {4.5, -0.5}]];
```

In[200]:=
```
g[31] = Graphics[Line[{{5, 0}, {5, -1}}]];
```

In[201]:=
```
g[31, 0] = Graphics[Text["●", {5, -1}]];
```

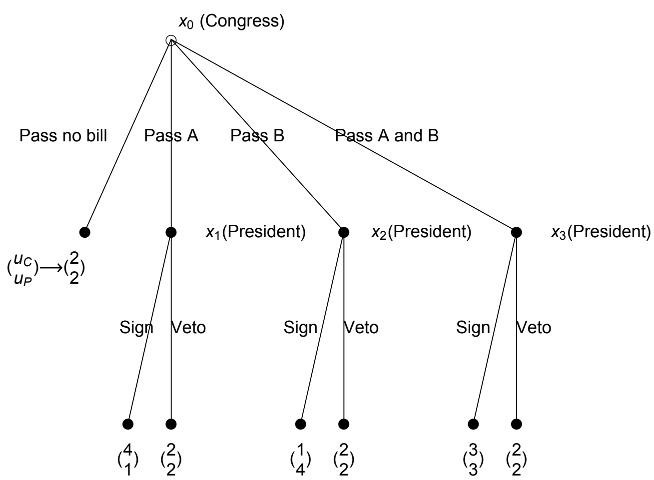
In[202]:=
```
g[32] = Graphics[Text["Veto", {5.2, -0.5}]];
```

In[203]:=
```
g[33] = Graphics[Text["(3/3)", {4.5, -1.2}]];
```

In[204]:=
```
g[34] = Graphics[Text["(2/2)", {5, -1.2}]];
```

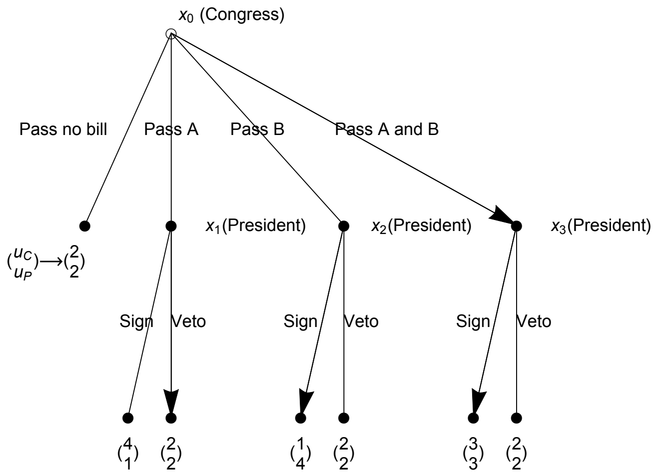
In[205]:=
```
Show[g[0], g[0, 0], g[1], g[2], g[3], g[3, 0], g[4], g[5], g[6], g[7],
 g[8], g[9], g[10], g[11], g[12], g[13], g[13, 0], g[14], g[15], g[16],
 g[17], g[18], g[19], g[20], g[20, 0], g[21], g[21, 0], g[22], g[23], g[24],
 g[25], g[26], g[27], g[28], g[29], g[30], g[30, 0], g[31], g[31, 0], g[32],
 g[33], g[34], PlotRange → {{-1, 7}, {-1.5, 1.2}}, AspectRatio → 0.75]
```

Out[205]=



In[206]:=
```
Show[g[0], g[0, 0], g[1], g[2], g[3], g[3, 0], g[4], g[5], g[5, 0], g[6], g[7], g[8],
 g[9], g[10], g[11], g[12], g[12, 0], g[13], g[13, 0], g[14], g[15], g[16], g[17],
 g[18], g[19], g[20], g[20, 0], g[21], g[21, 0], g[21, 1], g[22], g[23], g[24],
 g[25], g[26], g[27], g[28], g[29], g[29, 0], g[30], g[30, 0], g[31], g[31, 0],
 g[32], g[33], g[34], PlotRange → {{-1, 7}, {-1.5, 1.2}}, AspectRatio → 0.75]
```

Out[206]=



The *Mathematica* codes to draw the game tree of 3U4(b)

In[207]:=
```
g[35] = Graphics[Line[{{5, 0}, {7, -1}}]];
```

In[208]:=
```
g[36] = Graphics[Text["Sign A", {6.25, -0.75}]];
```

In[209]:=
```
g[37, 0] = Graphics[Arrow[{{5, 0}, {8, -1}}]];
```

In[210]:=
```
g[37] = Graphics[Line[{{5, 0}, {8, -1}}]];
```

In[211]:=
```
g[38] = Graphics[Text["Sign B", {8, -0.75}]];
```

In[212]:=
```
g[39] = Graphics[Text["(4 over 1)", {7, -1.2}]];
```

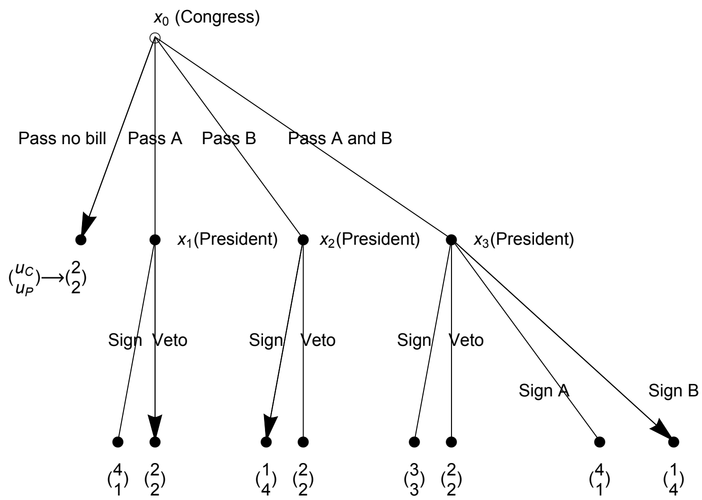
In[213]:=
```
g[40] = Graphics[Text["(1 over 4)", {8, -1.2}]];
```

In[214]:=
```
g[41] = Graphics[Text["●", {7, -1}]];
```

In[215]:=
```
g[42] = Graphics[Text["●", {8, -1}]];
```

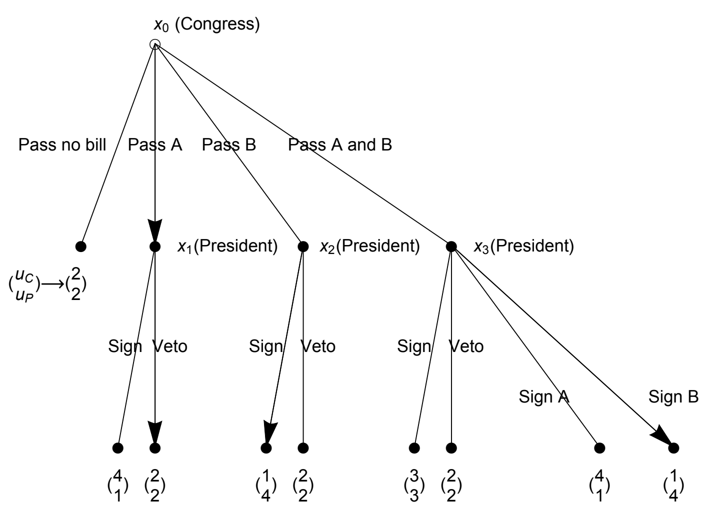
In[216]:=
```
Show[g[0], g[0, 0], g[1, 0], g[1], g[2], g[3], g[3, 0], g[4], g[5], g[6], g[7], g[8],
 g[9], g[10], g[11], g[12], g[12, 0], g[13], g[13, 0], g[14], g[15], g[16], g[17],
 g[18], g[19], g[20], g[20, 0], g[21], g[21, 0], g[21, 1], g[22], g[23], g[24], g[25],
 g[26], g[27], g[28], g[29], g[30], g[30, 0], g[31], g[31, 0], g[32], g[33], g[34],
 g[35], g[36], g[37], g[37, 0], g[38], g[39], g[40], g[41], g[42], AspectRatio → 0.75]
```

Out[216]=



In[217]:=
```
Show[g[0], g[0, 0], g[1], g[2], g[3], g[3, 0], g[4], g[5], g[5, 1], g[6], g[7], g[8],
 g[9], g[10], g[11], g[12], g[12, 0], g[13], g[13, 0], g[14], g[15], g[16], g[17],
 g[18], g[19], g[20], g[20, 0], g[21], g[21, 0], g[21, 1], g[22], g[23], g[24], g[25],
 g[26], g[27], g[28], g[29], g[30], g[30, 0], g[31], g[31, 0], g[32], g[33], g[34],
 g[35], g[36], g[37], g[37, 0], g[38], g[39], g[40], g[41], g[42], AspectRatio → 0.75]
```

Out[217]=



4U1 and 4U2. Find all the Nash equilibria in pure strategies for the following games. First, check for dominated strategies. If there are some, solve the games by using iterated elimination of dominated strategies. Which game is a zero-sum game and which game is a non-zero-sum game?

(a)

		Colin	
		Left	Right
	Up	(5,1)	(4,2)
Rowena			

Down (5,2) (2,3)

For Colin, Left is dominated by Right. Hence he will play Right. For Rowena, she knows that Colin will play Right, regardless of what she does. So, her best response is to play Up. The Nash equilibrium in pure strategies is (Up, Right) that yields (4, 2) as their pay-off vector.

This game is not a constant-sum game because for the combination of strategies (Down, Left), the sum of the payoffs for the two players is 7, which is not equal to 6, the sum of their payoffs under the Nash equilibriumin in pure strategies (Up, Right). Because the textbook calls a constant-sum game a zero-sum game, this is a non-zero-sum game.

(b)

		Colin		
		Left	Middle	Right
	Up	(2,9)	(5,5)	(6,2)
Rowena	Straight	(6,4)	(9,2)	(5,3)
	Down	(4,3)	(2,7)	(7,1)

For Colin, Right is dominated by Left. After Right has been eliminated, the game is reduced to

		Colin	
		Left	*Middle*
	Up	*(2,9)*	*(5,5)*
Rowena	*Straight*	*(6,4)*	*(9,2)*
	Down	*(4,3)*	*(2,7)*

Now for Rowena, Down is dominated by Straight. After eliminating Down from the preceding matrix, we obtain the following more simple game

		Colin	
		Left	*Middle*
	Up	*(2,9)*	*(5,5)*
Rowena	*Straight*	*(6,4)*	*(9,2)*

For Colin, now Middle is dominated by Left, and after Middle has been eliminated, the game is reduced to

		Collin
		Left
	Up	*(2,9)*
Rowena	*Straight*	*(6,4)*

Knowing that Colin now plays Left, Rowena's best response is Straight. Thus, the Nash equilibrium in pure strategies is (Straight, Left). Under this equilibrium, the payoff vector is (6,4). This is a non-zero-sum game because the sum of payoffs are not the same for all the possible combinations of strategies.

(c)

		Colin		
		Left	Middle	Right
	Up	(5,3)	(3,5)	(2,6)
Rowena	Straight	(6,2)	(4,4)	(3,5)
	Down	(1,7)	(6,2)	(2,6)

For Colin, Middle is dominated by Right. After Middle has been eliminated, the game is reduced to

		Colin	
		Left	Right
	Up	(5,3)	(2,6)
Rowena	Straight	(6,2)	(3,5)
	Down	(1,7)	(2,6)

For Rowena, Up is dominated by Straight, and after Up has been eliminated, the game is reduced to

		Colin	
		Left	*Right*
Rowena	Straight	(6,2)	(3,5)
	Down	(1,7)	(2,6)

The Nash equilibrium in pure strategies for the preceding game is (Sraight, Right). The vector of payoffs is (3,5) under the Nash equilibrium in pure strategies. Because the sums of the payoffs for Rowena and Colin are equal to 8 for all possible combinations of strategies, this is a zero-sum game according to the definition of the textbook.

(d)

		Colin			
		North	South	East	West
	Up	(6,4)	(7,3)	(5,5)	(6,4)
Rowena	High	(7,1)	(3,7)	(4,6)	(5,5)
	Low	(6,2)	(4,4)	(3,5)	(2,8)
	Down	(3,7)	(5,5)	(4,6)	(5,5)

Because the sum of payoffs for each entry in the preceding matrix is 10, the game is a zero-sum game.

For Rowena, Low is weakly dominated by Up. If Low is eliminated, then the game is reduced to

		Colin			
		North	South	East	West
	Up	(6,4)	(7,3)	(5,5)	(6,4)
Rowena	High	(7,1)	(3,7)	(4,6)	(5,5)
	Down	(3,7)	(5,5)	(4,6)	(5,5)

For Colin, now West is dominated by East. If West is eliminated, then the game is reduced to

		Colin		
		North	South	East
	Up	(6,4)	(7,3)	(5,5)
Rowena	High	(7,1)	(3,7)	(4,6)
	Down	(3,7)	(5,5)	(4,6)

For Rowena, now Down is dominated by Up. If Down is eliminated, then the game is reduced to

		Colin		
		North	South	East
	Up	(6,4)	(7,3)	(5,5)
Rowena	High	(7,1)	(3,7)	(4,6)

In the preceding game, North is dominated by East. After North has been eliminated, the game is reduced to

		Colin	
		South	East
	Up	(7,3)	(5,5)
Rowena	High	(3,7)	(4,6)

Now for Rowena, High is dominated by Up, and after Up has been eliminated, the game becomes

		Colin	
		South	East
Rowena	Up	(7,3)	(5,5)

The unique Nash equilibrium in pure strategies is (Up, East), which yields the payoff vector (5,5).

4U5. Use successive elimination of dominated strategiesto solve the following game. Explain the steps you followed. Show that yourv solution is a Nash equilibrium.

		Colin		
		Left	Middle	Right
	Up	(4,3)	(2,7)	(0,4)
Rowena	Down	(5,0)	(5,-1)	(-4,-2)

In the preceding game, the unique Nash equilibrium in pure strategies is (Down, Left), which yields the payoff vector (5,0). This Nash equilibrium can be found by successive eliminartions of dominated strategies as follows. First, note that for Colin, Right is dominated by Middle. After Right is eliminated, the game is reduced to

		Colin	
		Left	Middle
	Up	(4,3)	(2,7)
Rowena	Down	(5,0)	(5,-1)

For Rowena, Up is dominated by Down in the preceding game. After Up has been eliminated, the game becomes

		Colin	
		Left	Middle
Rowena	Down	(5,0)	(5,-1)

For Colin, Middle is dominated by Left in the preceding game. After Middle has been eliminated, the game is reduced to the single strategy combination (Down, Left), which is the unique Nash equilibrium in pure strategies of the original game.

4U8. The Battle of Bismarck Sea (named for that part of the southwestern Pacific Ocean separating the Bismarck Archipelago from Papua New Guinea) was a naval engagement played between the United States and Japan during World War II. In 1943, a Japanese admiral was ordered to move a convoy of ships to New Guinea; he had to choose between a rainy northern route and a summer southern route, both of which required three days' sailing time.The Americans knew that the convoy would sail and wanted to send bombers after it, but they did not know which route it would take. The Americans had to send reconnaissance planes to scout for the convoy, but they had only enough reconnaissance planes to explore one route at a time. Both the Japanese and the Americans had to make their decisions with no knowledge of the plans being made by the other side.

If the convoy was on the route that the Americans explored first, they could send bombers right away; if not, they lost a day of bombing. Poor weather on the northern route would also hamper bombing. If the Americans explored the northern route and found the Japanese right away, they could only expect two (of three) good days of bombings; if they explored the northern route and found that the Japanese had gone south, they could also expect two days of bombings. If the Americans chose to explore the southern route first, they could expect three full days of bombings if they found the Japanese right away, but only one day of bombings if they found the Japanese had gone north.

(a) Illustrate this game in a game table (matrix).
(b) Identify any dominant strategy in the game, and then solve for the Nash equilibrium.

(a) The number of days that the Americans could bomb the Japanese convoy can be taken as the payoffs in the game. The payoff for the Japanese is opposite to that of the Americans. This is a zero-sum game in which the damage inflicted on the Japanese represents the payoff for the Americans and the damage suffered by the Japanese - expressed negatively - represents the payoff for the Japanes. The payoff matrix for the game is

		The United States	
		Explore the northern route	*Explore the southern route*
Japan	*Take the northern route*	*(-2, 2)*	*(-1, 1)*
	Take the southern route	*(-2, 2)*	*(-3, 3)*

(b) Observe that <<Take the northern route>> is a weakly dominant strategy for the Japanese. Hence the Japanese admiral will take the northern route. Knowing that the Japanese will take the northern route, the best response for the Americans is to explore the northern route. The Nash equilibrium is (Take the northern route, Explore the northern route), and under the Nash equilibrium the Americans will have two days of bombings.

7U1. In footbal, the offense can either run or pass the ball, whereas the defense can either anticipate (and prepare for) a run or anticipate (and prepare for) a pass. Assume that the expected payoffs (in yards) for the two teams on any given down are as follows.

	DEFENSE	
	Anticipate Run	Anticipate Pass
Run	(1, -1)	(5, -5)

OFFENSE			
	Pass	(9, -9)	(-3, 3)

(a) Show that the game has no pure-strategy Nash equilibrium.
(b) Find the unique mixed- strategy Nash equilibrium of this game.
(c) Explain why the mixture used by OFFENSE is different from the mixture used by DEFENSE.
(d) How many yards is OFFENSE is expected to gain per down in equilibrium?

(a) Consider the strategy profile (Run, Anticipate Run). For OFFENSE, the best response to <<Anticipate Run>> is <<Pass>>.Thus, this strategy profile is not a Nash equilibrium in pure strategies. In the same manner, (Run, Anticipate Pass), (Pass, Anticipate Run), and (Pass, Anticipate Pass) are not Nash equilibria in pure strategies.

(b) Let q be the probability that DEFENSE will anticipate <<Run>>. If OFFENSE plays <<Run>>, then its expected payoff is $1 \times q + 5(1-q) = 5 - 4q$. *On the other hand, if OFFENSE plays << Pass >>, then* its expected payoff is 9×q+(-3)(1-q)=-3+12q. *Thus, OFFENSE will play <<Run>> with probability 1 if* $5 - 4q > -3 + 12q$, *i.e., if* $q < \frac{1}{2}$. *On the other hand, if* $q > \frac{1}{2}$, *then OFFENSE will play <<Pass>> with probability 1. When* $q = \frac{1}{2}$, *OFFENSE is indifferent between <<Run>> and <<Pass>>.*

Let p be the probability that OFFENSE will play Run. If DEFENSE anticipates <<Run>>, then its expected payoff is $-1 \times p - 9(1-p) = -9 + 8p$. On the other hand, if DEFENSE anticipates <<Pass>>, then its expected payoff is $-5p + 3(1-p) = 3 - 8p$. *Thus, DEFENSE will anticipate <<Run>> with probability 1 if* $-9 + 8p > 3 - 8p$, *i.e., if* $p > \frac{3}{4}$. *On the other hand if* $p < \frac{3}{4}$, *then DEFENSE will anticipate <<Pass>> with probability 1. When* $p = \frac{3}{4}$, *DEFENSE's anticipations can take on any value of q,* $0 \le q \le 1$.

The unique Nash equilibrium in mixed strategies is $(p, q) = \left(\frac{3}{4}, \frac{1}{2}\right)$.

(c) Because the two teams are not symmetric, under the Nash equilibrium in mixed strategies, OFFENSE and DEFENSE plays different mixtures.
(d) In equilibrium, the expected gain per down for OFFENSE is $1q + 5(1-q) = \frac{1}{2} + 5\left(1 - \frac{1}{2}\right) = 3$ *yards.*

7U6. Consider the following variant of <<chicken>> in which James' payoff from being <<tough>> when Dean is <<chicken>> is 2, rather than 1.

		Dean	
		Swerve	Straight
James	Swerve	(0, 0)	(-1, 1)
	Straight	(2, -1)	(-2, -2)

(a) Find the Nash equilibrium in mixed strategies.
(b) Compare the results with those of the original game in Section 4.B of this chapter. Is Dean's probability of playing <<straight>> (being tough) is higher now than before? What about James' probability of playing <<straight>>?
(c) What has happened to the two players' expected payoffs? Are these differences in the equilibrium

outcomes paradoxical in light of the new payoff structure? Explain how your findings can be understood in light of the opponent's indifference principle.

(a)

		Dean	
		Swerve (q)	*Straight (1-q)*
James	*Swerve (p)*	*(0, 0)*	*(-1, 1)*
	Straight (1-p)	*(2, -1)*	*(-2, -2)*

Let q be the probability that Dean plays <<swerve>>. If James plays <<swerve>>, then his expected payoff is $0(q)+(-1)(1-q)=-1+q$. *If James plays <<straight>>, then his expected payoff is* $2q+(-2)(1-q)=-2+4q$. *Thus, if James plays a mixed strategy, then we must have (according to the indifference principle)* $-1+q=-2+4q$, *i.e.,* $q=\frac{1}{3}$.

Let p be the probability that James plays <<swerve>>. If Dean plays <<swerve>>, then his expected payoff is $0(p)+(-1)(1-p)=-1+p$. *If Dean plays <<straight>>, then his expected payoff is* $1\times p+(-2)(1-p)=-2+3p$. *Thus, if Dean plays a mixed strategy, then we must have (according to the indifference principle)* $-1+p=-2+3p$, *i.e.,* $p=\frac{1}{2}$.

(b) The game of <<chicken>> in Section 7.4.B

		Dean	
		Swerve	*Straight*
James	*Swerve*	*(0, 0)*	*(-1, 1)*
	Straight	*(1, -1)*	*(-2, -2)*

In the original game of <<chicken>>, the two players are symmetric, and this structure leads to the Nash equilibrium in mixed strategies that each player plays <<swerve>> with probability $\frac{1}{2}$. *Furthermore, the expected payoff for each player is* $\frac{-1}{2}$.

Under the new payoff structure, James still plays the same mixture as that of the original game, i.e., $p=\frac{1}{2}$ *in the new as well as in the original game. However, for Dean, now he is tougher, and plays <<straight>> with probability* $1-q=\frac{2}{3}$, *which is higher than that in the original game.*

(c) Now under the new payoff structure, James obtains a higher payoff under the strategy combination (Straight, Swerve) $(2>1)$. *This might lead us to believe that James will have a higher expected payoff in equilibrium under the new payoff structure. However, this expectation turns out to be incorrect. More precisely, under the new payoff structure, the expected payoff of James is* $1\times q-2(1-q)=1\times\frac{1}{3}-2\left(1-\frac{1}{3}\right)=\frac{-2}{3}<\frac{-1}{2}$. *That is James is worse-off under the new payoff structure. As for Dean, his payoff remains the same because James continues to use the same mixtuture and Dean's payoff structure does not change, The apparent paradox can be explained as followd.*

If Dean still uses the same mixture as before, the expected payoff for James when he swerves is $0\times q+(-1)(1-q)=-\left(1-\frac{1}{2}\right)=\frac{-1}{2}$. *On the other hand, if James plays <<straight>>, then his expected payoff is now* $2q-2(1-q)=2\left(\frac{1}{2}\right)-2\left(1-\frac{1}{2}\right)=0>\frac{-1}{2}$. *In order for James to play a mixed strategy, the principle of indifference suggests that the expected payoff differential between <<swerve>> and <<straight>> must be eliminatedf, and this means that the probability with which Dean plays*

<<straight>> must rise. We have computed this new probability as $\frac{2}{3}$. When Dean plays <<straight>> with probability $\frac{2}{3}$, James will be indifferent between playing <<swerve>> and playing <<straight>>. In particular, using the same mixture as before is also James' best response to $q = \frac{1}{3}$. Because the payoff structure for Dean does not change, and because James plays the same mixture as before, Dean is also indifferent between playing <<swerve>> and playing <<straight>>. Thus, $p = \frac{1}{2}$, $q = \frac{1}{3}$ constitute the new Nash equilibrium in mixed strategies.