Asymmetric Information

A state government is carrying out a highway construction procurement project. To simplify the exposition, suppose that there is only one construction firm that bids for the project.

The decision variable of the state government is n, the number of lanes of the proposed new highway. The soil quality can be either good or poor. The construction firm, because of its expertise, knows exactly whther the soil quality is good or poor. However, the state government has limited information about the soil condition of the land on which the new highway is to be constructed. It believes that there is a probability of $\frac{1}{3}$ that the soil quality is good and that there is a probabily of $\frac{2}{3}$ that the soil quality is poor.

It is more costly to the firm to construct a highway on poor soil, and the state government has to pay the firm its opportunity cost. Suppose that the opportunity cost for the firm is \$3Billion/lane when the soil qualty is good and \$5Billion/lane when the soil quality is poor.

The gross social welfare when an n – lane highway is constructed is

$$15\,n - \frac{1}{2}\,n^2.$$

If the state government has perfect information, i.e., if it knows exactly the soil quality, and if the soil quality is good, then the opportunity costs is low, and it solves the following maximization problem:

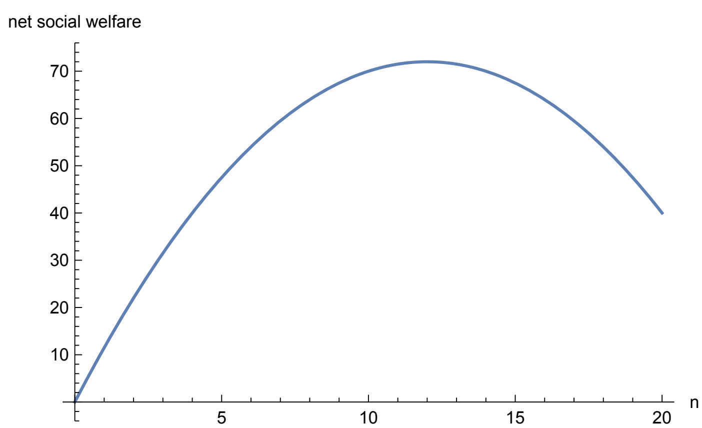
(1) $\max_n 15\,n - \frac{1}{2}\,n^2 - 3\,n.$

In[1]:=
```
m[0] = 15 n - 1/2 n^2 - 3 n
```

Out[1]= $12\,n - \frac{n^2}{2}$

In[2]:=
```
Plot[m[0], {n, 0, 20}, AxesLabel → {"n", "net social welfare"}]
```

Out[2]=



In[3]:=
```
Maximize[m[0], n]
```

Out[3]= {72, {n → 12}}

Thus, if the state government knows that the soil quality is good, it will propose a contract of 12 lanes and pays the construction firm $\$12 \times 3 = 36$ Billion. The social welfare obtained is 72. That is the optimal contract for the case of good soil quality is (12, 36).

If the state government has perfect information, i.e., if it knows exactly the soil quality, and if the soil quality is poor, then the opportunity cost of the construction firm is high, and the state government solves the following maximization problem:

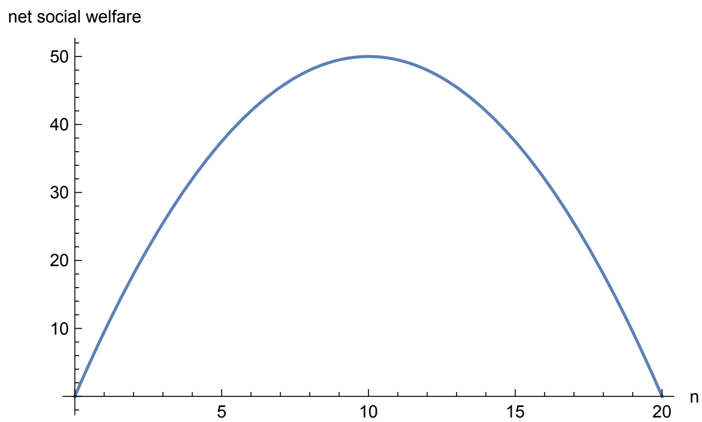
(2) $\max_n 15\,n - \frac{1}{2}\,n^2 - 5\,n.$

In[4]:=
```
m[1] = 15 n - 1/2 n^2 - 5 n
```

Out[4]= $10\,n - \frac{n^2}{2}$

In[5]:=
```
Plot[m[1], {n, 0, 20}, AxesLabel → {"n", "net social welfare"}]
```

Out[5]=



In[6]:=
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Maximize[m[1], n]
```

Out[6]= {50, {n → 10}}

Thus, if the state government knows that the soil quality is poor, it will propose a contract of 10 lanes and pays the construction firm $10 \times 5 = \$50$Billion. The net social welfare is 50. That is, the optimal contract for the case of poor soil quality is (10, 50).

Now in reality the state government does not have perfect information, and if it offers two contract (12, 36) and (10, 50), hoping that the constuction firm will accept the former contract if the soil quality is good and the latter contract if the soil quality is poor, what will happen?

Such a strategy will not work. Indeed, suppose that the soil quality is good. If the construction firm accepts the contract (12, 36) designed for the good soil quality scenario, then its profit is

In[7]:=
```
36 - 12 × 3
```

Out[7]= 0

On the other hand, if the construction firm accepts the contract (10, 50) designed for the poor soil quality scenario, then it builds a 10-lane highway at the cost of 3 Billion/lane, and is paid \$50B. Its profit is then

In[8]:=
```
50 - 10 × 3
```

Out[8]= 20

Thus, if these two contracts are offered, the intended objectives of the state government will not be attained: The construction company will choose the contract intended for the high-cost case when the cost is low.

The problem is to design a procurement policy that induces the firm to reveal its private information. The state government can offer two contracts - one for the good soil quality scenbario and one for the poor soil quality scenario - and stucture these contracts in such a way that the construction firm will select the contract designed for the poor (good) scenario when the soil quality is poor (good).

Let (n_L, r_L) be the contract designed for the low-cost scenario and (n_H, r_H) be the contract designed for the high-cost scenario.

In order for the construction firm to choose the contract (n_L, r_L) when the cost is low, the following incentive compatibility constraint must be satisfied:

(1) $r_L - 3\, n_{\{} \geq r_H - 3\, n_H$.

```
In[9]:= g[1] = r[L] - 3 n[L] ≥ r[H] - 3 n[H]

Out[9]= -3 n[L] + r[L] ≥ -3 n[H] + r[H]
```

In order for the construction firm to choose the contract (n_H, r_H) when the cost is high, the following incentive compatibility constraint must be satisfied:

(2) $r_H - 5\, n_H \geq r_L - 5\, n_L$.

```
In[10]:= g[2] = r[H] - 5 n[H] ≥ r[L] - 5 n[L]

Out[10]= -5 n[H] + r[H] ≥ -5 n[L] + r[L]
```

In order for the construction firm to choose the contract (n_H, r_H) when the cost is high, the following participation constraint must be satisfied:

(3) $r_H - 5\, n_H \geq 0$.

```
In[11]:= g[3] = r[H] - 5 n[H] ≥ 0

Out[11]= -5 n[H] + r[H] ≥ 0
```

In order for the construction firm to choose the contract (n_L, r_L) when the cost is low, the following participation constraint must be satisfied:

(4) $r_L - 3\, n_L \geq 0$.

```
In[12]:= g[4] = r[L] - 3 n[L] ≥ 0

Out[12]= -3 n[L] + r[L] ≥ 0
```

If the constraints (1)-(4) are satified, then the construction firm, when it is offered to choose one of the two contracts (n_L, r_L) and (n_H, r_H), will choose the former contract if the cost is low and the latten contract if the cost is high. The expected payoff for the state government is then given by

In[13]:= $g[0] = \frac{2}{3}\left(15\, n[L] - \frac{1}{2}\, n[L]^2 - r[L]\right) + \frac{1}{3}\left(15\, n[H] - \frac{1}{2}\, n[H]^2 - r[H]\right)$

Out[13]= $\frac{1}{3}\left(15\, n[H] - \frac{n[H]^2}{2} - r[H]\right) + \frac{2}{3}\left(15\, n[L] - \frac{n[L]^2}{2} - r[L]\right)$

The optimal mechanism is found by maximizing *g*[0] subject to the four constraints (1)-(4).

```
In[14]:= Maximize[{g[0], g[1], g[2], g[3], g[4]}, {n[L], r[L], n[H], r[H]}]

Out[14]= {54, {n[L] → 12, r[L] → 48, n[H] → 6, r[H] → 30}}
```

The two contracts offered by the state government are

$(n_L, r_L) = (12, 48)$, $(n_H, r_H) = (6, 30)$.

The expected social welfare is 54.

The following table presents the results for the perfect information case and the case of asymmetric information.

	Low cost		High cost	
	n_L	r_L	n_H	r_H
Perfect information	12	36	10	50
Asymmetric information	12	48	6	30

Compared to the perfect information case, the optimal mechanism under asymmetric information proposes the same number of lanes (12) and pays more ($48 > 36$) under the low-cost scenario. Under the high-cost scenario, the mechanism proposes fewer lanes (6 instead of 10) and pays less (30 instead of 50). To induce the construction company to select the contract designed for the good soil scenario when this is the true state of nature, the mechnism pays more ($48 > 32$) than under the perfect information case. On the other hand, to discourage the construction company from choosing the contract (n_H, r_H) when the opportunity cost is low, this contract requires fewer lanes ($6 < 10$) than that under the perfect information case and pays less ($30 < 50$) than that under the perfect information case when the soil quality is poor.