Solutions
Attached

Math 202, Class test, October 21, 2012

1. **(4 points)** Use the remainder theorem and synthetic division to show that $\frac{1}{2}$ and $\frac{1}{3}$ are solutions of the equation

$$6x^4 - 23x^3 + 28x^2 - 13x + 2 = 0$$

2. **(3 points)** Factor the polynomial

$$6x^4 - 23x^3 + 28x^2 - 13x + 2$$

from the previous problem completely.

3. **(3 points)** Discuss and draw the graph of

$$f(x) = (1 - 9x^2)(x^2 - 9)$$

4. **(3 points)** Discuss and draw the graph of

$$f(x) = \frac{1 - x}{x + 3}$$

5. **(3 points)** Rewrite in the trigonometric form $z = |z|(\cos\phi + i\sin\phi)$ the complex number

$$z = \frac{\sqrt{2} - i\sqrt{2}}{\sqrt{3} + i}$$

6. **(4 points)** Use de Moivre's theorem to write

$$\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^7$$

in the form $a + bi$.

Math 202 Midterm Exam Oct 2012

1\. (i)

$$\begin{array}{r|rrrrr} \frac{1}{2} & 6 & -23 & 28 & -13 & 2 \\ & & 3 & -10 & 9 & -2 \\ \hline & 6 & -20 & 18 & -4 & \boxed{0} \end{array}$$

Since Remainder $= 0$ $\Rightarrow P(\frac{1}{2}) = 0 \Rightarrow \frac{1}{2}$ is a solution

(ii)

$$\begin{array}{r|rrrrr} \frac{1}{3} & 6 & -23 & 28 & -13 & 2 \\ & & 2 & -7 & 7 & -2 \\ \hline & 6 & -21 & 21 & -6 & \boxed{0} \end{array}$$

$\Rightarrow P(\frac{1}{3}) = 0 \Rightarrow \frac{1}{3}$ is a solution

2\. Step 1 From i) $\dfrac{6x^4 - 23x^3 + 28x^2 - 13x + 2}{x - \frac{1}{2}} = 6x^3 - 20x^2 + 18x - 4$

$\Rightarrow 6x^4 - 23x^3 + 28x^2 - 13x + 2 = (6x^3 - 20x^2 + 18x - 4)(x - \frac{1}{2})$

Step 2 Now use $x = \frac{1}{3}$

$$\begin{array}{r|rrrr} \frac{1}{3} & 6 & -20 & 18 & -4 \\ & & 2 & -6 & 4 \\ \hline & 6 & -18 & 12 & \boxed{0} \end{array}$$

we have $\dfrac{6x^3 - 20x^2 + 18x - 4}{x - \frac{1}{3}} = (6x^2 - 18x + 12)$

Step 3 $\Rightarrow 6x^3 - 20x^2 + 18x - 4 = (6x^2 - 18x + 12)(x - \frac{1}{3})$

putting Results of step 2 into result of step 1

$6x^4 - 23x^3 + 28x^2 - 13x + 2 = (6x^2 - 18x + 12)(x - \frac{1}{3})(x - \frac{1}{2})$

$= 6(x^2 - 3x + 2)(x - \frac{1}{3})(x - \frac{1}{2})$

$= 6(x-1)(x-2)(x - \frac{1}{3})(x - \frac{1}{2})$

3\. Step 1 No Asymptotes

Step 2 x intercepts: (y = 0) let $(1 - 9x^2)(x^2 - 9) = 0$

$(1 - 3x)(1 + 3x)(x - 3)(x + 3) = 0$

$x = \frac{1}{3}, x = -\frac{1}{3}, x = 3, x = -3$

y intercept (x = 0) $y = (1 - 9[0]^2)([0]^2 - 9)$

$y = (1)(-9)$

$y = -9$

Step 3 Note:

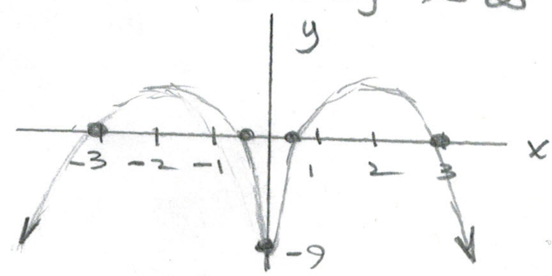
$-y = (9x^2 - 1)(x^2 - 9)$

if $x \to +\infty \Rightarrow -y \to +\infty \Rightarrow y \to -\infty$

if $x \to -\infty \Rightarrow -y \Rightarrow -\infty \Rightarrow y \to -\infty$

Step 4 if $x = -2$, $y > 0$

if $x = 2$, $y > 0$



4\. Step 1 Asymptotes

VA

$x + 3 = 0$

$x = -3$

when $x = -3$

Numerator $\neq 0$

$\Rightarrow x = -3$ is a VA

HA

$\lim\limits_{x \to \pm\infty} \dfrac{1 - x}{x + 3} = \dfrac{-\infty}{\infty}$

$= \lim\limits_{x \to \pm\infty} \dfrac{\frac{1}{x} - \frac{x}{x}}{\frac{x}{x} + \frac{3}{x}}$

$= \lim\limits_{x \to \pm\infty} \dfrac{\frac{1}{x} - 1}{1 + \frac{3}{x}}$

$= \dfrac{0 - 1}{1 + 0}$

$= -1$

$\Rightarrow$ HA $y = -1$

Step 2 x int:

$0 = \dfrac{1 - x}{x + 3}$

$1 - x = 0$

$x = 1$

y int

$y = \dfrac{1 - 0}{0 + 3}$

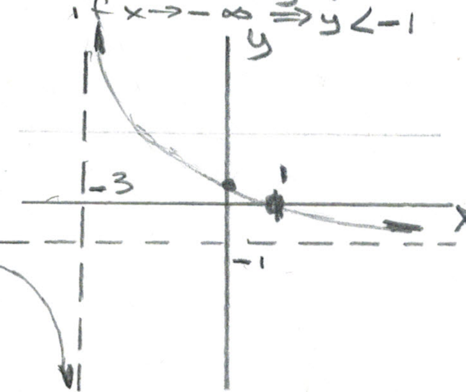
$y = \frac{1}{3}$

if $x \to -3^+ \Rightarrow y$ is $+$

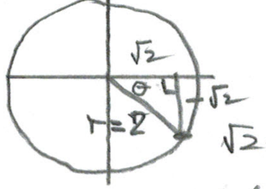
if $x \to -3^- \Rightarrow y$ is $-$

if $x \to +\infty \Rightarrow y > -1$

if $x \to -\infty \Rightarrow y < -1$



5. Step 1 Convert $(\sqrt{2}-\sqrt{2}i)$ to polar (Trig Form)

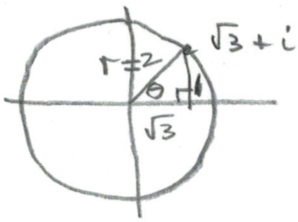


$r=\sqrt{a^2+b^2}=\sqrt{(\sqrt{2})^2+(-\sqrt{2})^2}=2$

$\Rightarrow \theta=-45^\circ$ or $\theta=-\frac{\pi}{4}R$

$\triangle:\ (\frac{2}{2},\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$ or $(1,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

Step 2 Convert $(\sqrt{3}+i)$ to polar



$r=\sqrt{a^2+b^2}=\sqrt{(\sqrt{3})^2+(1)^2}=2$

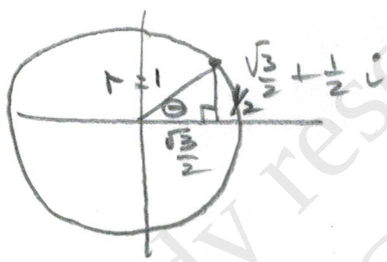
$\theta=30^\circ$ or $\theta=\frac{\pi}{6}R$

$\triangle:\ (\frac{2}{2},\frac{1}{2},\frac{\sqrt{3}}{2})$

Step 3 $z=\dfrac{\sqrt{2}-\sqrt{2}i}{\sqrt{3}+i}=\dfrac{2\ cis\ (-45^\circ)}{2\ cis\ 30^\circ}=\frac{2}{2}cis(-45^\circ-30^\circ)$

$z=\dfrac{\sqrt{2}-\sqrt{2}i}{\sqrt{3}+i}=1\ cis(-75^\circ)$

6. Step 1 Convert $(\frac{\sqrt{3}}{2}+\frac{1}{2}i)$ to polar Form



$r=\sqrt{a^2+b^2}=\sqrt{(\frac{\sqrt{3}}{2})^2+(\frac{1}{2})^2}$

$=\sqrt{\frac{3}{4}+\frac{1}{4}}=1$

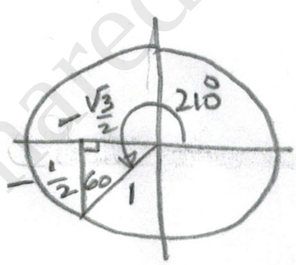
$\theta=30^\circ$ or $\frac{\pi}{6}R$

$\triangle:\ (1,\frac{\sqrt{3}}{2},\frac{1}{2})$

Step 2 $(\frac{\sqrt{3}}{2}+\frac{1}{2}i)^7=(1\ cis\ 30^\circ)^7$

$=1^7\ cis\ (30*7)$

$=1\ cis\ 210^\circ$



Step 3 Convert to Rectangular form

$1\ cis\ 210^\circ=1(\cos 210^\circ+i\sin 210^\circ)$

$=1\left(-\frac{\sqrt{3}}{2}+i\left[-\frac{1}{2}\right]\right)$

$=-\frac{\sqrt{3}}{2}-\frac{1}{2}i$

$210^\circ-180^\circ=30^\circ$

As Reference Angle