Assignment 1

Assigned : Jan 13 Due Jan 27: 6:00 PM

Total Points: 50 (4 points for each correctly solved problem+10 points for presentation)
For full marks the solution should be clear and neat with clear diagram if needed
In the case of longer problems the main arguments should be presented in the space provided, while detailed calculations should be presented on the back of the page.

1 By algebraic manipulation of the first two kinematic equations for one-dimensional motion:

$$1) v_f = v_i + at \qquad 2) x_f = x_i + v_i t + \tfrac{1}{2} a t^2$$

Obtain the other two kinematic equations: $3) v_f^2 - v_i^2 = 2a\Delta x \qquad 4) x_f = x_i + \tfrac{1}{2}(v_i + v_f)t$

SOLUTION:

$$1) v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2) x_f = x_i + v_i \frac{(v_f - v_i)}{a} + \tfrac{1}{2} a \frac{(v_f - v_i)^2}{a^2} \Rightarrow x_f - x_i = v_i \frac{(v_f - v_i)}{a} + \tfrac{1}{2a}\left(v_f^2 - 2v_i v_f + v_i^2\right) \Rightarrow$$

$$\Rightarrow 2a(x_f - x_i) = 2v_i(v_f - v_i) + \left(v_f^2 - 2v_i v_f + v_i^2\right) \Rightarrow 2a(x_f - x_i) = 2v_i v_f - 2v_i v_i + v_f^2 - 2v_i v_f + v_i^2 \Rightarrow 2a(x_f - x_i) = v_f^2 - v_i^2$$

$$1) v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2) x_f = x_i + v_i t + \tfrac{1}{2} a \frac{(v_f - v_i)}{a} t \Rightarrow x_f = x_i + v_i t + \tfrac{1}{2}(v_f - v_i)t \Rightarrow x_f = x_i + v_i t + \tfrac{1}{2} v_f t - \tfrac{1}{2} v_i t \Rightarrow x_f = x_i + \tfrac{1}{2}(v_f + v_i)t$$

2. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of −5.60 m/s2 for 4.20 s, making straight skid marks 62.4 m long ending at the tree. With what speed does the car then strike the tree?

SOLUTION:

In the simultaneous equations:

$$\left\{\begin{array}{c} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{array}\right\} \text{ we have } \left\{\begin{array}{c} v_{xf} = v_{xi} - \left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}) \\ 62.4\ \text{m} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)(4.20\ \text{s}) \end{array}\right\}.$$

So substituting for v_{xi} gives $62.4\ \text{m} = \frac{1}{2}\left[v_{xf} + \left(56.0\ \text{m/s}^2\right)(4.20\ \text{s}) + v_{xf}\right](4.20\ \text{s})$

$14.9\ \text{m/s} = v_{xf} + \frac{1}{2}\left(5.60\ \text{m/s}^2\right)(4.20\ \text{s})$. Thus $v_{xf} = \boxed{3.10\ \text{m/s}}$.

3 The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

SOLUTION: $y = 3.00t^3$: At $t = 2.00\ \text{s}$, $y = 3.00(2.00)^3 = 24.0\ \text{m}$ and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0\ \text{m/s} \uparrow.$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2} g t^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting $y_b = 0$, $0 = 24.0 + 36.0t - 4.90t^2$.

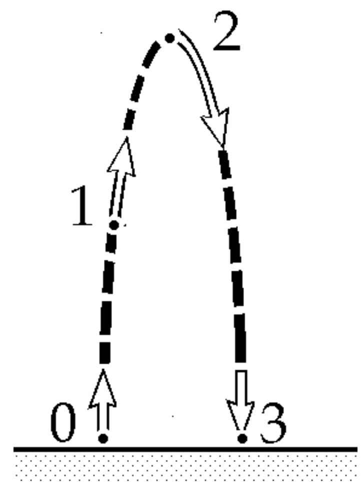
Solving for t, (only positive values of t count), $\boxed{t = 7.96\ \text{s}}$.

4 A rocket is fired vertically upward from a well. A catapult gives it initial velocity 80.0 m/s at ground level. Its engines then fire and it accelerates upward at 4.00 m/s^2 until it reaches an altitude of 1 000 m. At that point its

engines fail and the rocket goes into free fall, with an acceleration of –9.80 m/s^2. (a) How long is the rocket in motion above the ground? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (You will need to consider the motion while the engine is operating separate from the free-fall motion

2 SOLUTION:

Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact.
Below are the data found for each phase of the rocket's motion.



(0 to 1) $v_f^2 - (80.0)^2 = 2(4.00)(1\,000)$ so $v_f = 120$ m/s
$120 = 80.0 + (4.00)t$ giving $t = 10.0$ s

(1 to 2) $0 - (120)^2 = 2(-9.80)(x_f - x_i)$ giving $x_f - x_i = 735$ m
$0 - 120 = -9.80t$ giving $t = 12.2$ s
This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80)(-1\,735)$
$v_f = -184 = (-9.80)t$ giving $t = 18.8$ s

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0\text{ s}}$

(b) $(x_f - x_i)_{\text{total}} = \boxed{1.73\text{ km}}$

(c) $v_{\text{final}} = \boxed{-184\text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	-9.80
#3	Fall to Earth	41.0	0	-184	-9.80

5 A rock is dropped from rest into a well. (a) The sound of the splash is heard 2.40 s after the rock is released from rest. How far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (b) **What If?** If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated?

SOLUTION:

(a) $d = \frac{1}{2}(9.80)t_1^2$ $\qquad d = 336t_2$

$t_1 + t_2 = 2.40$ $\qquad 336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$ $\qquad t_2 = \dfrac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$

$t_2 = \dfrac{359.5 \pm 358.75}{9.80} = 0.076\,5$ s so $d = 336t_2 = \boxed{26.4\text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2$ m, an error of. $\boxed{6.82\%}$

6 Two railroad tracks intersect at right angles at station O. At 10AM the train A, moving west with constant speed of 50 km/h, leaves the station O. One hour later train B, moving south with the constant speed of 60 km/h, passes through the station O. Find minimum distance between these trains.

Train A moves along z axis and at time t it will have position: $x_A = V_A t = 50t$

Train B moves along the y axis and at time t it will have position: $y_B = 60 - V_B t = 60 - 60t$

The distance between the two trains is given by: $D = \sqrt{x_A^2 + y_B^2} = \sqrt{(50t)^2 + (60-60t)^2}$

The minimum distance is given by the contdition:

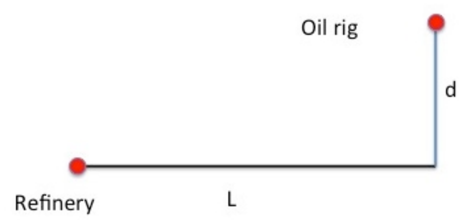
$$\frac{dD}{dt} = 0 \Rightarrow \frac{1}{2}\frac{1}{\sqrt{2500t^2 + (60-60t)^2}} \cdot [2(2500t) + 2(60-60t)(-60)] = 0$$

$$(2500t) + (60-60t)(-60) = 0 \Rightarrow (2500+3600)t = 3600 \Rightarrow t = \frac{3600}{6100} = \frac{36}{61}(hr) = 35.41\,\text{min} = 35\,\text{min}\,24.6\,\text{sec}$$

7 John who is member of certain NGO has missed the meeting of his protest group at the refinery, and now needs to get to the oil rig in the shortest time to join the demonstrators who are trying to disrupt the work of the petroleum company. John can run at 10km/h but can paddle only 3km/h.

a) How far from the Refinery should John enter the `water

b) What is the minimum time it will take to get to the oil rig L=12km, d=4km



$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L-x}{v_{land}} + \frac{\sqrt{x^2+d^2}}{v_{water}}$$

$$\frac{d(\Delta t)}{dx} = -\frac{1}{v_{land}} + \frac{1}{v_{water}}\frac{1}{2}(x^2+d^2)^{-1/2}(2x) \Rightarrow \Delta t = \min \quad when \quad \frac{d(\Delta t)}{dx} = 0$$

$$-\frac{1}{v_{land}} + \frac{1}{v_{water}}\frac{1}{2}(x^2+d^2)^{-1/2}(2x) = 0 \Rightarrow \frac{x}{\sqrt{x^2+d^2}} = \frac{v_{water}}{v_{land}} \Rightarrow \frac{x^2}{x^2+d^2} = \left(\frac{v_{water}}{v_{land}}\right)^2 \Rightarrow x^2[1-\left(\frac{v_{water}}{v_{land}}\right)^2] = d^2\left(\frac{v_{water}}{v_{land}}\right)^2$$

$$x = \sqrt{\frac{d^2\left(\frac{v_{water}}{v_{land}}\right)^2}{[1-\left(\frac{v_{water}}{v_{land}}\right)^2]}} = \sqrt{1.58}(km) = 1.26km$$

8 An artillery shell is fired with an initial velocity of 300 m/s at 53.0° above the horizontal. It explodes on a mountainside 42.0 s after firing. What are the horizontal and vertical, coordinates (x and y) where the shell explodes, relative to its firing point.

$$x = v_{xi}t = v_i \cos\theta_i t$$

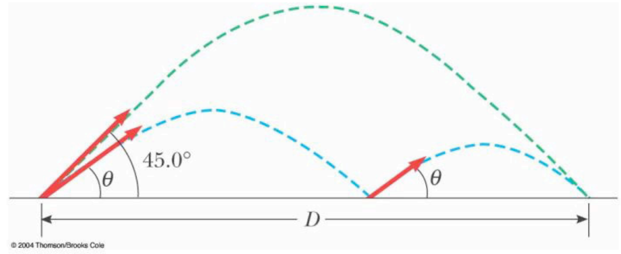
$$x = (300\ \text{m/s})(\cos 55.0°)(42.0\ \text{s})$$

$$x = \boxed{7.23 \times 10^3\ \text{m}}$$

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

$$y = (300\ \text{m/s})(\sin 55.0°)(42.0\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(42.0\ \text{s})^2 = \boxed{1.68 \times 10^3\ \text{m}}$$

9 When baseball players throw the ball in from the outfield, they usually allow it to take one bounce before it reaches the infield, on the theory that the ball arrives sooner that way. Suppose that the angle at which a bounced ball leaves the ground is the same as the angle at which the outfielder threw it, as in Figure P4.55, but that the ball's speed after the bounce is one half of what it was before the bounce. (a) Assuming the ball is always thrown with the same initial speed, at what angle θ should the fielder throw the ball to make it go the same distance D with one bounce (blue path) as a ball thrown upward at 45.0° with no bounce (green path)? (b) Determine the ratio of the times for the one-bounce and no-bounce throws.



SOLUTION:

The special conditions allowing use of the horizontal range equation applies.

For the ball thrown at 45°, $$D = R_{45} = \frac{v_i^2 \sin 90}{g}.$$

For the bouncing ball, $$D = R_1 + R_2 = \frac{v_i^2 \sin 2\theta}{g} + \frac{\left(\frac{v_i}{2}\right)^2 \sin 2\theta}{g}$$

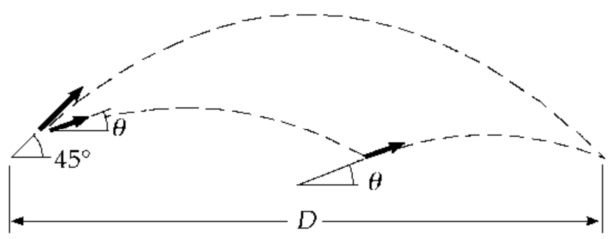
where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$$\sin 2\theta = \frac{4}{5}$$

$$\boxed{\theta = 26.6°}$$



(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi} t - \frac{1}{2} g t^2$$

$$0 = v_i \sin\theta_i t - \frac{1}{2} g t^2.$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing.

So for the ball thrown at 45.0° $t_{45} = \frac{2v_i \sin 45.0°}{g}$.

For the bouncing ball, $$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2\left(\frac{v_i}{2}\right)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}.$$

The ratio of this time to that for no bounce is $$\frac{\frac{3v_i \sin 26.6°}{g}}{\frac{2v_i \sin 45.0°}{g}} = \frac{1.34}{1.41} = \boxed{0.949}.$$

10 One strategy in a snowball fight is to throw a snowball at a high angle over level ground. While your opponent is watching the first one, a second snowball is thrown at a low angle timed to arrive before or at the same time as the first one. Assume both snowballs are thrown with a speed of 25.0 m/s. The first one is thrown at an angle of 70.0° with respect to the horizontal. (a) At what angle should the second snowball be thrown to arrive at the same point as the first? (b) How many seconds later should the second snowball be thrown after the first to arrive at the same time?