University of Ottawa PHY1124B
Dr Czajkowski Winter 2017
Assignment 2

STUDENT NAME:____________________

STUDENT NUMBER:__________________

Newton's Dynamics, Work Energy Assigned : Jan 27 Due Feb 8: 6:00 PM sharp

1 A crate of weight $\mathbf{F}_g$ is pushed by a force $\mathbf{P}$ on a horizontal floor. (a) If the coefficient of static friction is μ_s and $\mathbf{P}$ is directed at angle θ below the horizontal, show that the minimum value of P that will move the crate is given by

$$P = \frac{\mu_s F_g \sec\theta}{1-\mu_s \tan\theta}$$

(b) Find the minimum value of P that can produce motion when $\mu_s = 0.400$, $F_g = 100$ N, and $\theta = 0°, 15.0°, 30.0°, 45.0°$, and $60.0°$.

The crate is in equilibrium, just before it starts to move.
Let the normal force acting on it be n and the friction force f_s.

Resolving vertically: $n = F_g + P\sin\theta$

Resolving Horizontally: $P\cos\theta = f_s$

But, $f_s \le \mu_s n$

i.e., $P\cos\theta \le \mu_s\left(F_g + P\sin\theta\right)$

or $P\left(\cos\theta - \mu_s\sin\theta\right) \le \mu_s F_g$.

Divide by $\cos\theta$: $P\left(1-\mu_s\tan\theta\right) \le \mu_s F_g \sec\theta$.

Then $\boxed{P_{\text{minimum}} = \dfrac{\mu_s F_g \sec\theta}{1-\mu_s\tan\theta}}$.

(b) $P = \dfrac{0.400(100\text{ N})\sec\theta}{1-0.400\tan\theta}$

θ(deg)	0.00	15.0	30.0	45.0	60.0
$P(N)$	40.0	46.4	60.1	94.3	260

If the angle were 68.2° or more, the expression for P would go to infinity and motion would become impossible.

2 A 1.30-kg toaster is not plugged in. The coefficient of static friction between the toaster and a horizontal countertop is 0.350. To make the toaster start moving, you carelessly pull on its electric cord. (a) For the cord tension to be as small as possible, you should pull at what angle above the horizontal? (b) With this angle, howlarge must the tension be?

SOLUTION:

With motion impending, $n + T\sin\theta - mg = 0$ *and* $f = \mu_s\left(mg - T\sin\theta\right)$

And $T\cos\theta - \mu_s mg + \mu_s T\sin\theta = 0$ So that: $T = \dfrac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}$.

To minimize T, we maximize $\cos\theta + \mu_s\sin\theta$ $\quad \dfrac{d}{d\theta}\left(\cos\theta + \mu_s\sin\theta\right) = 0 = -\sin\theta + \mu_s\cos\theta$.

(a) $\theta = \tan^{-1}\mu_s = \tan^{-1}0.350 = \boxed{19.3°}$

(b) $T = \dfrac{0.350(1.30\text{ kg})\left(9.80\text{ m/s}^2\right)}{\cos 19.3° + 0.350\sin 19.3°} = \boxed{4.21\text{ N}}$

3 A coin placed 30.0 cm from the center of a rotating, horizontal turntable slips when its speed is 50.0 cm/s. (a) What force causes the centripetal acceleration when the coin is stationary relative to the turntable? (b) What is the coefficient of static friction between coin and turntable?

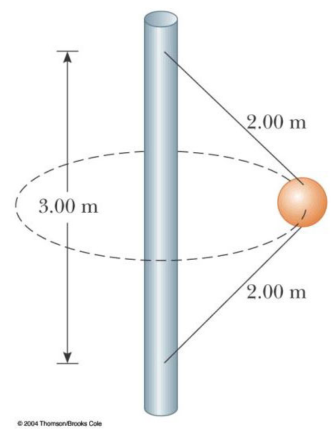
(a) static friction

(b) $ma\hat{\mathbf{r}} = f\hat{\mathbf{r}} + n\hat{\mathbf{j}} + mg\left(-\hat{\mathbf{j}}\right)$ $\quad \sum F_y = 0 = n - mg$

thus $n = mg$ and $\sum F_r = m\frac{v^2}{r} = f = \mu n = \mu mg$.

Then $\mu = \frac{v^2}{rg} = \frac{(50.0\ \text{cm/s})^2}{(30.0\ \text{cm})(980\ \text{cm/s}^2)} = \boxed{0.085\,0}$.

4 A 4.00-kg object is attached to a vertical rod by two strings, as in Figure P6.11. The object rotates in a horizontal circle at constant speed 6.00 m/s. Find the tension in (a) the upper string and (b) the lower string.



Solution:

$F_g = mg = (4\ \text{kg})(9.8\ \text{m/s}^2) = 39.2\ \text{N}$

$\sin\theta = \frac{1.5\ \text{m}}{2\ \text{m}}$

$\theta = 48.6°$

$r = (2\ \text{m})\cos 48.6° = 1.32\ \text{m}$

$\sum F_x = ma_x = \frac{mv^2}{r}$

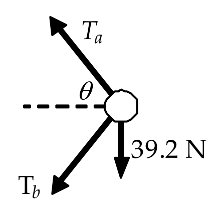
$T_a\cos 48.6° + T_b\cos 48.6° = \frac{(4\ \text{kg})(6\ \text{m/s})^2}{1.32\ \text{m}}$

$T_a + T_b = \frac{109\ \text{N}}{\cos 48.6°} = 165\ \text{N}$

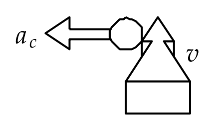
$\sum F_y = ma_y$

$+T_a\sin 48.6° - T_b\sin 48.6° - 39.2\ \text{N} = 0$

$T_a - T_b = \frac{39.2\ \text{N}}{\sin 48.6°} = 52.3\ \text{N}$



forces



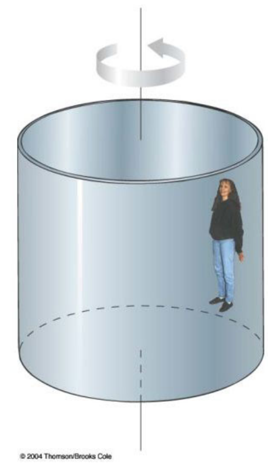
motion

(a) To solve simultaneously, we add the equations in T_a and T_b:

$$T_a + T_b + T_a - T_b = 165\ \text{N} + 52.3\ \text{N}$$

$$T_a = \frac{217\text{ N}}{2} = \boxed{108\text{ N}}$$

(b) $$T_b = 165\text{ N} - T_a = 165\text{ N} - 108\text{ N} = \boxed{56.2\text{ N}}$$



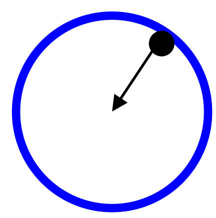
5 An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away (Fig. P6.65). The coefficient of static friction between person and wall is μ_s, and the radius of the cylinder is R. (a) Show that the maximum period of revolution necessary to keep the person from falling is $T = (4\pi^2 R\mu_s/g)^{1/2}$. (b) Obtain a numerical value for T if R = 4.00 m and μ_s = 0.400. How many revolutions per minute does the cylinder make?

(a) $$n = \frac{mv^2}{R} \qquad f - mg = 0$$

$$f = \mu_s n \qquad v = \frac{2\pi R}{T}$$

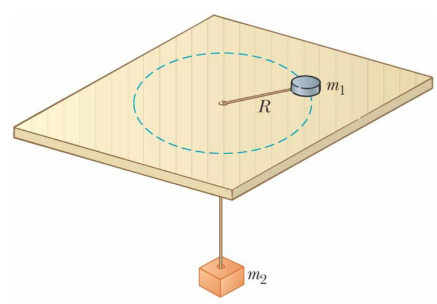
$$T = \boxed{\sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

(b) $$T = \boxed{2.54\text{ s}}$$



$$\#\frac{\text{rev}}{\text{min}} = \frac{1\text{ rev}}{2.54\text{ s}}\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{23.6\ \frac{\text{rev}}{\text{min}}}$$

6 An air puck of mass m_1 is tied to a string and allowed to revolve in a circle of radius R on a frictionless horizontal table. The other end of the string passes through a hole in the center of the table, and a counterweight of mass m_2 is tied to it (Fig. P6.58). The suspended object remains in equilibrium while the puck on the tabletop revolves. What is (a) the tension in the string? (b) the radial force acting on the puck? (c) the speed of the puck?



SOLUTIONS:

Since the object of mass m_2 is in equilibrium, $$\sum F_y = T - m_2 g = 0$$

or $$T = \boxed{m_2 g}.$$

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus, $$F_c = T = \boxed{m_2 g}.$$

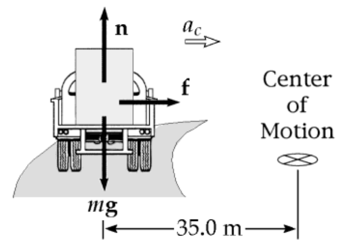
(c) From $$F_c = \frac{m_1 v^2}{R}$$

we have $$v = \sqrt{\frac{RF_c}{m_1}} = \boxed{\sqrt{\left(\frac{m_2}{m_1}\right) gR}}.$$

7 A crate of eggs is located in the middle of the flat bed of a pickup truck as the truck negotiates an unbanked curve in the road. The curve may be regarded as an arc of a circle of radius 35.0 m. If the coefficient of static friction between crate and truck is 0.600, how fast can the truck be moving without the crate sliding?

$n = mg$ since $a_y = 0$

The force causing the centripetal acceleration is the frictional force *f*.



From Newton's second law $f = ma_c = \frac{mv^2}{r}$.

But the friction condition is $f \le \mu_s n$

i.e., $\frac{mv^2}{r} \le \mu_s mg$

$v \le \sqrt{\mu_s r g} = \sqrt{0.600(35.0\text{ m})(9.80\text{ m/s}^2)}$ $\quad v \le \boxed{14.3\text{ m/s}}$

8. A skier of mass 70.0 kg is pulled up a slope by a motor-driven cable. (a) How much work is required to pull him a distance of 60.0 m up a 30.0° slope (assumed frictionless) at a constant speed of 2.00 m/s? (b) A motor of what power is required to perform this task?

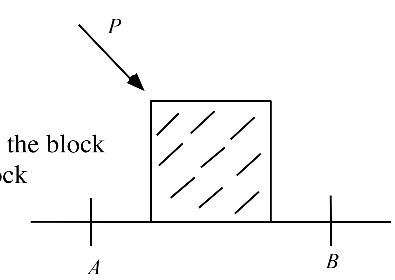
(a) $\sum W = \Delta K$, but $\Delta K = 0$ because he moves at constant speed. The skier rises a vertical distance of $(60.0\text{ m})\sin 30.0° = 30.0\text{ m}$. Thus,

$$W_{in} = -W_g = (70.0\text{ kg})(9.8\text{ m/s}^2)(30.0\text{ m}) = \boxed{2.06\times10^4\text{ J}} = \boxed{20.6\text{ kJ}}.$$

(b) The time to travel 60.0 m at a constant speed of 2.00 m/s is 30.0 s. Thus,

$$P_{input} = \frac{W}{\Delta t} = \frac{2.06\times10^4\text{ J}}{30.0\text{ s}} = \boxed{686\text{ W}} = 0.919\text{ hp}$$

9 A block is pushed across a rough horizontal surface from point A to point B by a force (magnitude *P* = 5.4 N) as shown in the figure. The magnitude of the force of friction acting on the block between A and B is 1.2 N and points A and B are 0.5 m apart. If the kinetic energies of the block at A and B are 4.0 J and 5.6 J, respectively, how much work in J is done on the block by the force *P* between A and B?

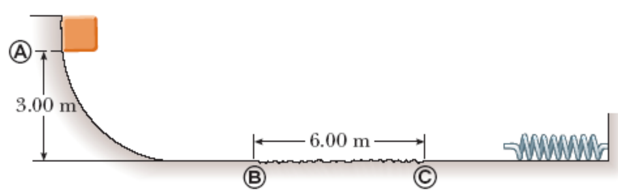


$\Delta K = W_P - W_f = W_P - 1.2(0.5)J \qquad 1.6\text{J} = W_P - 0.6J \qquad W_P = 2.2J$

10 1 On Oct 31 2015massive asteroid TB145 nicknamed "Spooky" passed near the Earth vicinity. Given the measured diameter of the asteroid (450meters) and its speed relative to the Sun:125000km/h, find the total maximum and minimum energy released in the completely inelastic collision of this object with Earth. (NOTE you are not given the parameters of collision so they becem important part of the "worst case –best case scenario.") Treat the asteroid as spherical object with the density of between 3g/ cm^3 to 6g/cm^3. Earth orbit around the sun has radius =150 x 10^6 km.
State your answers in Jules and in mega-tones of TNT.(1 megaton of TNT= 4.184 10^{15})
34.722km/s vs 29.89km/s

WORST CASE SCENARIO:	BEST CASE SCENARIO:
Maximum	*Minimum*
density = 6g/cm^3	density 3g/ cm^3
mass 2.29 10^{12}kg	mass 1.15 10^{12}
relative speed 64.6 km/s	relative speed 4.83 km/s
Kinetic Energy 9.51 10^{21} J	Kinetic Energy 1.34 10^{19}
2 273 000 megatonnes of TNT.	3203 megatonnes of TNT.
Approx. 1000 of the world total stockpiled nuclear weapons!	(about 50% of the world stockpiled weapons)

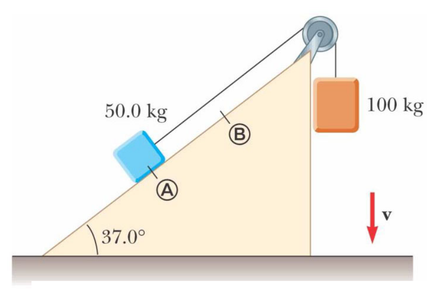
11 10.0-kg block is released from point A with speed of initial speed 1m/s. (Block is given a push). The track is frictionless except for the portion between points B and C which has a length of 6.00 m. The block travels down the track, hits a spring of force constant 2 000 N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between B and C.



$$E_C - E_A = W_{BC} \Rightarrow E_C - E_A = W_{BC} \Rightarrow \frac{1}{2}kx^2 - \left(mgh + \frac{1}{2}mv^2\right) = -\mu mgL \Rightarrow$$

$$\Rightarrow \mu = -\frac{1}{mgL}\left(\frac{1}{2}kx^2 - mgh - \frac{1}{2}mv^2\right) = \frac{209}{588}$$

12. A 50.0-kg block and 100-kg block are connected by a string as in Figure P8.36. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between the 50-kg block and incline is 0.250. Determine the change in the kinetic energy of the 50-kg block as it moves from A to B, a distance of 20.0 m.



$$\sum F_y = n - mg\cos 37.0° = 0$$

$$\therefore n = mg\cos 37.0° = 400 \text{ N}$$

$$f = \mu n = 0.250(400 \text{ N}) = 100 \text{ N}$$

$$-f\Delta x = \Delta E_{\text{mech}}$$

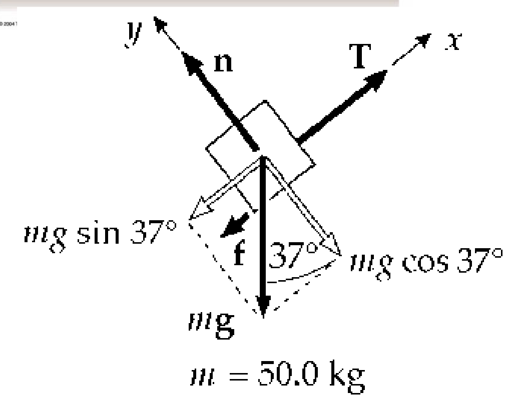
$$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$$

$$\Delta U_A = m_A g\left(h_f - h_i\right) = (50.0)(9.80)(20.0\sin 37.0°) = 5.90\times 10^3$$

$$\Delta U_B = m_B g\left(h_f - h_i\right) = (100)(9.80)(-20.0) = -1.96\times 10^4$$

$$\Delta K_A = \frac{1}{2}m_A\left(v_f^2 - v_i^2\right)$$

$$\Delta K_B = \frac{1}{2}m_B\left(v_f^2 - v_i^2\right) = \frac{m_B}{m_A}\Delta K_A = 2\Delta K_A$$



Adding and solving, $\Delta K_A = \boxed{3.92 \text{ kJ}}$.