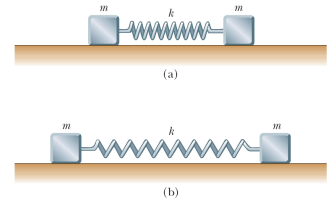


STUDENT LAST NAME: _____ STUDENT NUMBER: _____

- 1 Two identical metallic blocks resting on a frictionless horizontal surface are connected by a light metallic spring having the spring constant 100 N/m and an unstretched length of 0.300 m as shown in Figure P19.57a. A total charge of Q is slowly placed on the system, causing the spring to stretch to an equilibrium length of 0.400 m as shown. Determine the value of Q , assuming that all the charge resides on the blocks and modeling the blocks as point charges.



Charge $\frac{Q}{2}$ resides on each block, which repel as point charges: $F = \frac{k_e (Q/2)(Q/2)}{L^2} = k(L - L_i)$. Solving for

$$Q, Q = \sqrt{2L \frac{k(L - L_i)}{k_e}}$$

- 2 Two small spheres of mass m are suspended from strings of length ℓ that are connected at a common point. One sphere has charge Q , and the other has charge $2Q$. The strings make angles θ_1 and θ_2 with the vertical.

(a) How are θ_1 and θ_2 related?

(b) Assume that θ_1 and θ_2 are small. Show that the distance r between the spheres is given by

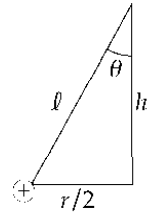
$$r \approx \left(\frac{4k_e Q^2 \ell}{mg} \right)^{1/3}$$

(a) From the $2Q$ charge we have $F_e - T_2 \sin \theta_2 = 0$ and $mg - T_2 \cos \theta_2 = 0$.

Combining these we find $\frac{F_e}{mg} = \frac{T_2 \sin \theta_2}{T_2 \cos \theta_2} = \tan \theta_2$.

From the Q charge we have $F_e = T_1 \sin \theta_1 = 0$ and $mg - T_1 \cos \theta_1 = 0$.

Combining these we find $\frac{F_e}{mg} = \frac{T_1 \sin \theta_1}{T_1 \cos \theta_1} = \tan \theta_1$ or $\theta_2 = \theta_1$.



(b) $F_e = \frac{k_e 2QQ}{r^2} = \frac{2k_e Q^2}{r^2}$

If we assume θ is small then $\tan \theta \approx \frac{r/2}{h}$. Substitute expressions for F_e and $\tan \theta$ into either equation found in part (a) and solve for r .

$$\frac{F_e}{mg} = \tan \theta \text{ then } \frac{2k_e Q^2}{r^2} \left(\frac{1}{mg} \right) \approx \frac{r}{2} \text{ and solving for } r \text{ we find } r \approx \left(\frac{4k_e Q^2}{mg} \right)^{1/3}$$

- 3 Find the electric field at the point 20 cm above the center of the square made of 4 charged rods of $L=20\text{cm}$ and $Q=1\text{mC}$ each. (use the opposite page to present your solution with full diagram)

Bases on geometry the field at this point is equal to 4 times the normal components of the $|E|$ from single rod at distance $\sqrt{500}\text{cm}$.

$$E = 4 \frac{20}{\sqrt{500}} E_1 = \frac{8}{\sqrt{5}} 2k_e \lambda l \left(\frac{1}{y \sqrt{l^2 + y^2}} \right) = \frac{16 \cdot 8.99 \cdot 10^9 \cdot 10^{-3}}{\sqrt{5} \cdot 0.2 \sqrt{0.04 + 0.04}} = 1.14 \cdot 10^9 \text{ N/C}$$

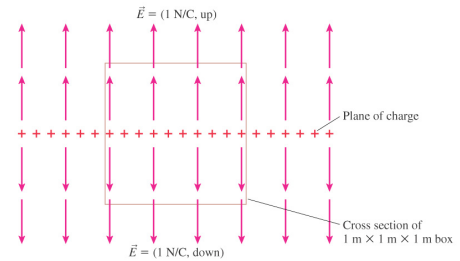
- 4 A vertical electric field of magnitude $2.00 \times 10^4 \text{ N/C}$ exists above the Earth's surface on a day when a thunderstorm is brewing. A car covers a rectangle measuring 6.00 m by 3.00 m on the roadway below it, which is built on dry fill. The roadway slopes downward at 10.0° . Determine the electric flux through the bottom of the car.

$$\Phi_E = EA \cos \theta = (2.00 \times 10^4 \text{ N/C})(18.0 \text{ m}^2) \cos 10.0^\circ = 355 \text{ kN} \cdot \text{m}^2/\text{C}$$

- 5 At some moment of time the following charges are placed (fixed) at given coordinates
 $Q(1,1,1)$; $-2Q(1,2,3)$; $-Q(0,0,2)$; $+2Q(1,0,2)$.
 Find the total flux through the surface of the sphere centered at $(0,0,0)$ and having radius of
- a) 1 ANS: TOTAL Flux=0 (0 charge inside)
 b) $3^{1/3}$ ANS:TOTAL Flux=0 (0 charge inside)
 c) $4^{1/3}$ ANS: TOTAL Flux=0 (0charge inside)
 d) 4 ANS: TOTAL Flux =0 (all charges inside \rightarrow sum of charges =0)

- 6 i) The total electric flux through this box is:

$$2 \text{ N/C m}^2$$



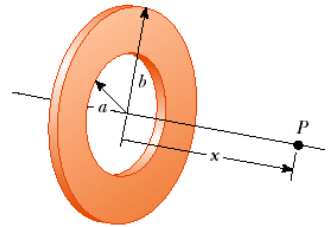
- ii) Find the flux through the circular disk of radius 5 cm which is placed at distance 5 m from the single point charge $Q=2\text{mC}$. The point charge is on the axis perpendicular to the disk and passing through its centre.

$$\Phi = \frac{\text{disc } q}{\text{full sphere } \epsilon_0} = \frac{\pi r^2 q}{4\pi R^2 \epsilon_0} = \frac{1}{40000} \frac{2 \cdot 10^{-3} \text{C}}{8.85 \cdot 10^{-12}} = 5649.7 = 5650 \frac{\text{N}}{\text{C}} \text{m}^2$$

- 7 Calculate the electric potential at point P on the axis of the annulus shown on the right which has a uniform charge density σ .

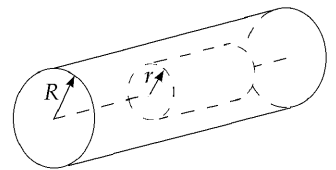
$$dV = \frac{k_e dq}{\sqrt{r^2 + x^2}} \text{ where } dq = \sigma dA = \sigma 2\pi r dr$$

$$V = 2\pi\sigma k_e \int_a^b \frac{r dr}{\sqrt{r^2 + x^2}} = \boxed{2\pi k_e \sigma \left[\sqrt{x^2 + b^2} - \sqrt{x^2 + a^2} \right]}$$



- 8 Obtain the expression for the electric potential at distance r from the centre of the charged cylindrical conductor of radius R and infinite length.

If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r , contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho \pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps; $\mathbf{E} \cdot d\mathbf{A} = E dA \cos 90.0^\circ = 0$. The curved surface has $\mathbf{E} \cdot d\mathbf{A} = E dA \cos 0^\circ$, and E must be the same strength everywhere over the curved surface.



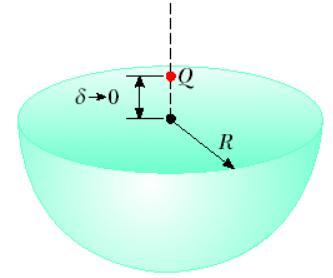
Gauss's law, $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$, becomes $E \int_{\text{Curved Surface}} dA = \frac{\rho \pi r^2 L}{\epsilon_0}$.

Now the lateral surface area of the cylinder is $2\pi r L$: $E(2\pi r)L = \frac{\rho \pi r^2 L}{\epsilon_0}$.

Thus, $\mathbf{E} = \boxed{\frac{\rho r}{2 \epsilon_0}}$ radially away from the cylinder axis.

- 8B) Use the result of the Part A to get the expression for the infinite cylindrical capacitor of inner radius a and out radius b

- 9 A point charge Q is located just above the center of the flat face of a hemisphere of radius R as shown in Figure P24.15. What is the electric flux
(a) through the curved surface and (b) through the flat face?



- (a) With δ very small, all points on the hemisphere are nearly at a distance R from the charge, so the field everywhere on the curved surface is $\frac{k_e Q}{R^2}$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:

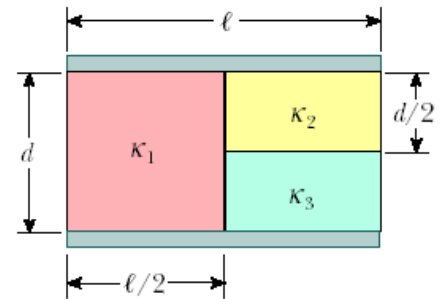
$$\Phi_{\text{curved}} = \int \mathbf{E} \cdot d\mathbf{A} = E_{\text{local}} A_{\text{hemisphere}}$$

$$\Phi_{\text{curved}} = \left(k_e \frac{Q}{R^2} \right) \left(\frac{1}{2} 4\pi R^2 \right) = \frac{1}{4\pi \epsilon_0} Q (2\pi) = \boxed{\frac{+Q}{2 \epsilon_0}}$$

- (b) The closed surface encloses zero charge so Gauss's law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \quad \text{or} \quad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = \boxed{\frac{-Q}{2 \epsilon_0}}$$

- 10 A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in Figure. You may assume that $\ell \gg d$. (a) Find an expression for the capacitance of the device in terms of the plate area A and d , κ_1 , κ_2 , and κ_3 . (b) Calculate the capacitance using the values $A = 1.00 \text{ cm}^2$, $d = 2.00 \text{ mm}$, $\kappa_1 = 4.90$, $\kappa_2 = 5.60$, and $\kappa_3 = 2.10$.



P26.61 (a) $C_1 = \frac{\kappa_1 \epsilon_0 A/2}{d}$; $C_2 = \frac{\kappa_2 \epsilon_0 A/2}{d/2}$; $C_3 = \frac{\kappa_3 \epsilon_0 A/2}{d/2}$

$$\left(\frac{1}{C_2} + \frac{1}{C_3} \right)^{-1} = \frac{C_2 C_3}{C_2 + C_3} = \frac{\epsilon_0 A}{d} \left(\frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3} \right)$$

$$C = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3} \right)^{-1} = \boxed{\frac{\epsilon_0 A}{d} \left(\frac{\kappa_1}{2} + \frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3} \right)}$$

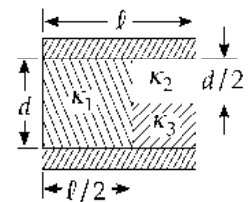
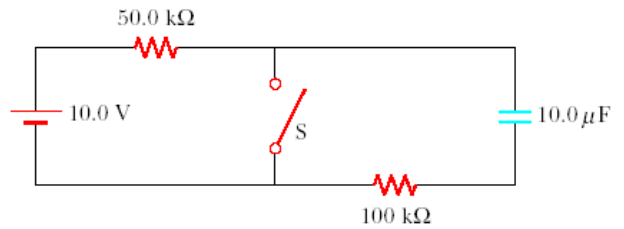


FIG. P26.61

- (b) Using the given values we find: $C_{\text{total}} = 1.76 \times 10^{-12} \text{ F} = \boxed{1.76 \text{ pF}}$.

11 In the circuit below, the switch S has been open for a long time. It is then suddenly closed. Determine the time constant (a) before the switch is closed and (b) after the switch is closed. (c) Let the switch be closed at $t = 0$. Determine the current in the switch as a function of time.



(a) $\tau = RC = (1.50 \times 10^5 \Omega)(10.0 \times 10^{-6} \text{ F}) = \boxed{1.50 \text{ s}}$

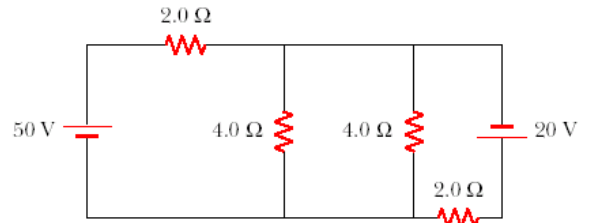
(b) $\tau = (1.00 \times 10^5 \Omega)(10.0 \times 10^{-6} \text{ F}) = \boxed{1.00 \text{ s}}$

(c) The battery carries current $\frac{10.0 \text{ V}}{50.0 \times 10^3 \Omega} = 200 \mu\text{A}$.

The $100 \text{ k}\Omega$ carries current of magnitude $I = I_0 e^{-t/RC} = \left(\frac{10.0 \text{ V}}{100 \times 10^3 \Omega} \right) e^{-t/1.00 \text{ s}}$.

So the switch carries downward current $\boxed{200 \mu\text{A} + (100 \mu\text{A})e^{-t/1.00 \text{ s}}}$.

12 Calculate the power delivered to each resistor shown below

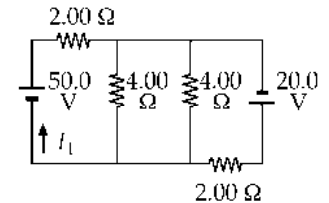


P28.30 We apply Kirchoff's rules to the second diagram.

$$50.0 - 2.00I_1 - 2.00I_2 = 0 \quad (1)$$

$$20.0 - 2.00I_3 + 2.00I_2 = 0 \quad (2)$$

$$I_1 = I_2 + I_3 \quad (3)$$



Substitute (3) into (1), and solve for I_1 , I_2 , and I_3

$$I_1 = 20.0 \text{ A}; \quad I_2 = 5.00 \text{ A}; \quad I_3 = 15.0 \text{ A}.$$

Then apply $P = I^2 R$ to each resistor:

$(2.00 \Omega)_1$:
 $P = I_1^2 (2.00 \Omega) = (20.0 \text{ A})^2 (2.00 \Omega) = \boxed{800 \text{ W}}$

(4.00Ω) : $P = \left(\frac{5.00}{2} \text{ A} \right)^2 (4.00 \Omega) = \boxed{25.0 \text{ W}}$

(Half of I_2 goes through each)

$(2.00 \Omega)_3$:
 $P = I_3^2 (2.00 \Omega) = (15.0 \text{ A})^2 (2.00 \Omega) = \boxed{450 \text{ W}}$.

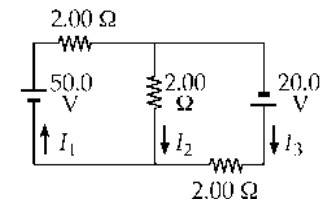


FIG. P28.30