Practice Final Exam for MAT 2377

Only faculty standard calculators are permitted. It is a closed book exam, but two sheets (double sided) are permitted. There are 3 short answer questions and 15 multiple choice questions.

Short Answer Questions

1. There is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. Suppose that X has the following probability mass function.

$$P(X = x) = \begin{cases} 0, & 0.66 \\ 1, & 0.29 \\ 2, & 0.04 \\ 3, & 0.009 \\ 4, & 0.001 \end{cases}$$

(a) What is the probability that there is at most 1 bit in error in the next four transmitted bits?
(b) What is the expected number of bits in error in the next four bits transmitted?
(c) Compute the standard deviation of the number of bits in error in the next four bits transmitted.
(d) Compute $P\left(-1.96 < \frac{X-E[X]}{\sqrt{V[X]}} < 1.96\right)$.

2. A manufacturer wants to test the lifetime of a small motor it builds; the mean lifetime is supposed to be at least 4 years. Assume that the distribution of lifetimes is approximately normal.
(a) The manufacturing unit will do a thorough investigation of the manufacturing process if it has reason to believe that the lifetime is too short. Formulate a null and alternative hypothesis.
(b) The CEO has called and said that selling substandard motors is unacceptable. Now an investigation will be done unless there is evidence that the mean lifetime meets requirements. Formulate a null and alternative hypothesis.
(c) A random sample of 15 motors is made. Under the situation of **part (b)**, and with a significance of 0.05, formulate the test procedure.
(d) Suppose that the sample mean is 4.13 years and the sample standard variation is 0.35 years. What does the test procedure in part (c) say to do?

3. Let X be the number of soldering defects for a device. It has the following mean $\mu = 1.2$ defects and standard deviation $\sigma = 1.6$ defects. Furthermore, the probability that a device will have more than 2 defects is 0.2.

   (a) Suppose that we ship 20 devices what is the probability that at most one device will have more than 2 defects?

(b) If we ship the devices one by one, what is the probability that the sixth device is the first device with more than 2 defects?

(c) Each soldering defect takes about 10 minutes to fix. If we ship 50 devices to the customer, approximate the probability a repair man would take more than 13.75 hours to fix all of the defects.

Multiple choice questions

1. Printed circuit cards are placed in a functional test after being populated with semiconductor chips. A lot contains 140 cards, and 20 are selected without replacement for functional testing. If 6 of the 140 cards are defective, what is the probability that at most 1 defective card will appear in the sample.

   A) 0.4062 B) 0.7956 C) 0.01578 D) 0.9754 E) 0.2065

2. A wire manufacturer wants to ensure that the mean radius of one model is very close to its listed value. Suppose that the standard deviation of the radius is known to be 10 μm. The population mean will be approximated by the average radius from a random sample of manufactured wires. How many wires need to be sampled to have 95% confidence that the error is at most 3 μm?

   A) 30 B) 31 C) 42 D) 43 E) 53

3. Calls to the help line of a large computer distributor occur according to a Poisson process with a rate of 20 calls per minute. What is the probability that three or more calls occur within 15 seconds?

   A) 0.1246 B) 0.8753 C) 0.7350 D) 0.2650 E) 0.4405

4. Suppose that the joint probability mass function of the random variables X and Y is given by the following table:

x	y	$p_{XY}(x,y)$
0	0	1/12
0	2	3/12
1	1	1/12
1	2	4/12
2	0	2/12
2	2	1/12

   Compute $P(X+Y \geq 2)$.

   A) 2/12 B) 7/12 C) 6/12 D) 9/12 E) 11/12

5. Suppose the cummulative distribution function of a continuous random variable X is

$$F(x) = \begin{cases} 0, & x \leq 0 \\ 0.25\,(x-2)+0.5, & 0 \leq x \leq 4 \\ 1, & x \geq 4 \end{cases}$$

   Compute the following expectation $E[X]$.

   A) 1.4142 B) 0.3333 C) 1.5333 D) 2 E) 1.3333

6. The strength of a steel alloy is normally distributed with a mean of 20 gigapascals (GPa) and a standard deviation of 3.54 GPa. What is the probability that a specimen of this alloy will have a strength at least 24 GPa?

A) 0.1151 B) 0.1292 C) 0.3206 D) 0.3409 E) 0.8849

7. Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 16 samples, exactly two contain the pollutant.

A) 0.3245 B) < 0.0001 C) 0.2745 D) 0.0065 E) 0.7892

8. A sample of 17 randomly selected boxes of a particular brand of cereal is taken, and each box is weighed. The sample mean is calculated to be 460 g, and the sample standard variation is calculated to be 23 g. Find the number x so that with probability 90%, the true population mean is at most x. Assume that the weights are normally distributed.

A) 1.333 B) 452.564 C) 467.140 D)467.436 E) 467.458

9. An engineering firm has a Facebook page. The visits to the page occur according to a Poisson process with a rate of 4 visits per day. What is the probability that we need to wait more than one day to observe 4 visits?

A) 0.0915 B) 0.4335 C) 0.6288 D) 0.0183 E) 0.2381

10. Let $X_1, \ldots, X_{20}$ be a random sample from a normal population with mean $\mu = 16$ and standard deviation $\sigma = 3$. Write $\bar{X}$ for the sample mean. Find the number c so that $P(\bar{X} < c) = 0.95$.
A) 16.556 B) 16.859 C) 17.100 D) 17.160 E) 20.920

11. Engineers of a fleet must continually check for corrosion inside the pipes that are part of the cooling systems. The inside condition of the pipes cannot be observed directly but a nondestructive test can give an indication of possible corrosion. The test is not perfect. The test has a probability of 0.7 of detecting internal corrosion when internal corrosion is present but it also has a probability of 0.1 of detecting internal corrosion, when there is no internal corrosion. Suppose the probability that any section of pipe has internal corrosion is 0.15. Given that the test detects internal corrosion, what is the probability that there truly is internal corrosion?

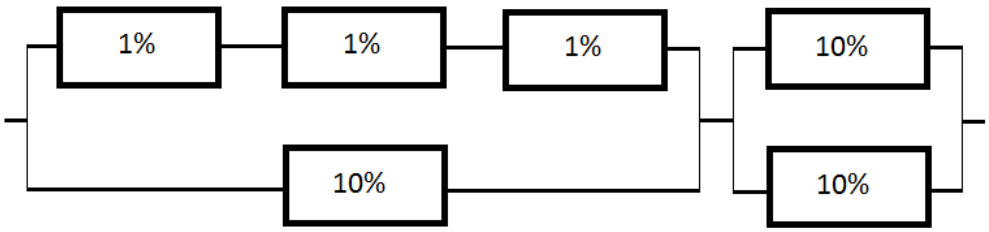
A) 0.2916 B) 0.3600 C) 0.1500 D) 0.3425 E) 0.1926

12. Consider a random sample of size $n = 22$ from a normal population with a mean of 15. Let $\overline{X}$ and S be the sample mean and the sample standard deviation, respectively. Find a such that

$$P\left(\frac{\overline{X} - 5}{S/\sqrt{22}} \leq a\right) = 0.01.$$

A) 2.518 B) -2.518 C) -2.831 D) 2.508 E) -2.508

13. The following circuit operates if and only if there is a path of functional devices from left to right. Assume devices fail independently and that the probability of *failure* of each device is as shown. What is the probability that the circuit operates?



A) 0.9871 B) 0.9975 C) 0.9725 D) 0.0211 E) 0.7885

14. Let X equal the grade of a student in a statistics course. Suppose that X has a normal distribution. Using traditional method of teaching, the average student grade was $\mu = 60$. We want to test if the the students' grades in this course have increased on average due to a new teaching method. In order to test $H_0 : \mu = 60$ versus $H_1 : \mu > 60$, we will use a random sample of $n = 26$ students. We computed $\overline{x} = 62.5$ and $s = 10$ for the 26 students. Find the p-value for this test.

A) p-value > 0.10
B) $0.05 < p$-value < 0.10
C) $0.025 < p$-value < 0.05
D) $0.01 < p$-value < 0.025
E) p-value < 0.01

15. The determination of the shear strength of spot welds is relatively difficult, whereas measuring the weld diameter of spot welds is relatively simple. We will describe the shear strength (in psi) as a linear function of the weld diameter (in 0.0001 inch). The data are as follows:

i	Shear Strength y_i (psi)	Weld diameter x_i (0.0001 inch)
1	370	400
2	780	800
3	1210	1250
4	1560	1600
5	1980	2000
6	2450	2500
7	3070	3100
8	3550	3600
9	3940	4000
10	3950	4000
total	22,860.0	23,250.0

From these data, we computed the following quadratic forms:

$$\sum_{i=1}^{10}(x_i - \overline{x})^2 = 15,686,250.0; \quad \sum_{i=1}^{10}(y_i - \overline{y})^2 = 15,461,440.0;$$

$$\sum_{i=1}^{10}(x_i - \overline{x})(y_i - \overline{y}) = 15,573,000.0.$$

Assuming that is is reasonable to use a simple linear regression model to describe the shear strength as a function of the weld diameter, predict the shear strength in (psi) of a spot weld when the weld diameter is 0.375 inch.

A) 3701 B) 3850 C) 3576 D) 3695 E) 3687

Cumulative distribution function for $N(0,1)$: $\Phi(z) = P(Z \leq z)$

0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	z
.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	−3.8
.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	−3.7
.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	−3.6
.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	−3.5
.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	−3.4
.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005	−3.3
.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007	−3.2
.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010	−3.1
.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013	−3.0
.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019	−2.9
.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026	−2.8
.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035	−2.7
.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047	−2.6
.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062	−2.5
.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082	−2.4
.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107	−2.3
.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139	−2.2
.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179	−2.1
.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228	−2.0
.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287	−1.9
.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359	−1.8
.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446	−1.7
.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548	−1.6
.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668	−1.5
.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808	−1.4
.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968	−1.3
.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151	−1.2
.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357	−1.1
.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587	−1.0
.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841	−0.9
.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119	−0.8
.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.242	−0.7
.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743	−0.6
.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085	−0.5
.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446	−0.4
.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821	−0.3
.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207	−0.2
.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602	−0.1
.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000	−0.0

Cumulative distribution function for $N(0,1)$: $\Phi(z) = P(Z \leq z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

The T distribution with ν degrees of freedom

	$F_T(t) = P(T \leq t)$								
	0.6	.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999
ν	$t_{0.40,\nu}$	$t_{0.25,\nu}$	$t_{0.10,\nu}$	$t_{0.05,\nu}$	$t_{0.025,\nu}$	$t_{0.01,\nu}$	$t_{0.005,\nu}$	$t_{0.0025,\nu}$	$t_{0.001,\nu}$
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852
14	0.258	0.692	1.345	1.761	2.145	2.624	2.997	3.326	3.787
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408
29	0.256	0.683	1.311	1.699	2.045	2.464	2.756	3.038	3.396
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385
∞	0.253	0.674	1.282	1.645	1.96	2.326	2.576	2.807	3.090

Note: $z_\alpha = t_{\alpha,\infty}$