Université d'Ottawa · University of Ottawa
Faculté des sciences Mathématiques et de statistique
Faculty of Science Mathematics and Statistics

Midterm for MAT 2377 A (Winter 2017)
Probability and statistics for engineers
Version 1

Duration: 80 minutes **Professor: Rachid Bentoumi**

Name: ____________________ Student Number: ____________________

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This is a closed book examination. However, one (double sided) sheet is permitted. Only basic calculators are allowed. There are two short answer questions and 6 multiple choice questions. The exam is marked on a total of 14 points.

Record your answer to each of the multiple choice questions in the table below.

Question	Answer	Question	Answer
1	C	4	A
2	E	5	A
3	E	6	C

Short Answer Questions

1. A chemical supply company ships a certain solvent in 10-gallon drums. Let X represent the number of drums ordered by a randomly chosen customer. Assume that X has the following probability mass function:

$$p_X(x) = \begin{cases} 0.4, & x = 1 \\ 0.2, & x = 2 \\ 0.2, & x = 3 \\ 0.1, & x = 4 \\ 0.1, & x = 5 \end{cases}$$

[1] (a) Compute the probability that more than 3 drums are ordered, i.e. compute $P(X > 3)$.

[1] (b) Give the expected number of drums ordered.

[1] (c) Compute the standard deviation of the number of drums ordered.

[2] (d) Suppose that the cost to ship an order is $C = 100\,X + 25$ (in dollars). It is \$100 per drum plus a \$25 processing fee for the order. Compute the mean and the standard deviation for the cost to ship an order.

(a) $P(X > 3) = P(X = 4) + P(X = 5) = 0.2.$

(b) We want

$$E[X] = \sum_{x \in R_X} x\, p_X(x) = 1(0.4) + 2(0.2) + \ldots + 5(0.1) = 2.3.$$

(c) Let's compute

$$E[X^2] = \sum_{x \in R_X} x^2\, p_X(x) = 1^2(0.4) + 2^2(0.2) + \ldots + 5^2(0.1) = 7.1.$$

So

$$\sigma_X = \sqrt{E[X^2] - \mu_X^2} = \sqrt{7.1 - (2.3)^2} = 1.3454.$$

(d) We want

$$\mu_C = E[100\,X + 25] = 100\,E[X] + 25 = 100(2.3) + 25 = 255$$

and

$$\sigma_C = \sqrt{V[C]} = \sqrt{V[100\,X + 25]} = \sqrt{100^2\,V[X]} = 100\,\sigma_X = 134.54.$$

(Question 1 cont.)

2. Customers who purchase a certain make of car can order an engine in any of three sizes. Of all cars sold, 45% have the smallest engine, 35% have the medium-size one, and 20% have the largest. Of cars with the smallest engine, 10% fail an emissions test within two years of purchase, while 12% of those with the medium size and 15% of those with the largest engine fail.

[**1**] (a) What is the probability that a randomly chosen car fails the emissions test within two years and it has the smallest engine?

[**1**] (b) What is the probability that a randomly chosen car fails the emissions test within two years?

[**1**] (c) A randomly chosen car fails the emissions test within two years. What is the probability that it has the smallest engine?

Let E be the event that the car fails the emissions test within two years. Let A, B, C be the event that the car has the smallest engine, has the medium-size engine, and has the largest size engine, respectively. We have $P(A) = 0.45$, $P(B) = 0.35$, $P(C) = 0.20$, $P(E|A) = 0.1$, $P(E|B) = 0.12$, and $P(E|C) = 0.15$.

(a) $P(E \cap A) = P(E|A)P(A) = (0.1)(0.45) = 0.045$.

(b)

$$\begin{aligned} P(E) &= P(E \cap A) + P(E \cap B) + P(E \cap C) \\ &= P(E|A)P(A) + P(E|B)P(B) + P(E|C)P(C) \\ &= (0.1)(0.45) + (0.12)(0.35) + (0.15)(0.20) = 0.117 \end{aligned}$$

(c) $P(A|E) = P(A \cap E)/P(E) = 0.045/0.117 = 0.385$

(Question 2 cont.)

Multiple Choice Questions

Please enter your answers to the multiple choice question in the table provided on the first page.

[**1**] 1. The probability that a submarine will have a bad propeller is 0.3, the probability that it will have a bad periscope is 0.4, and the probability that it will have both defects is 0.1

What is the probability that the submarine will have a bad propeller and a good periscope?

A) 0.1 B) 0.12 C) 0.2 D) 0.4 D) 0.5 E) 0.3

Answer: C
Let E be the event that the propeller is bad and let F be the event that the periscope is bad.

Then $P(E \cap F^c) = P(E) - P(E \cap F) = 0.3 - 0.1 = 0.2$.

[**1**] 2. We program a computer to generate a 4-digit pin at random, allowing repeated digits. The digits are chosen from the set $\{0,1,2,\ldots,9\}$. Assuming that all possible 4-digit pins are equally likely of being generated, what is the probability that the computer will generate a 4-digit pin with no repeated digits, i.e. we get 4 distinct digits.

A) $(6 \times 5 \times 4 \times 3 \times 2)/(10 \times 9 \times 8 \times 7)$
B) $\binom{6}{4}/\binom{10}{4}$
C) $\binom{10}{4}/(10 \times 9 \times 8 \times 7)$
D) $\binom{10}{4}/10^4$
E) $(10 \times 9 \times 8 \times 7)/10^4$

Answer: E
The number of 4-digit pins without repetition is $10 \times 9 \times 8 \times 7$ and with repetition it is 10^4. Using the equally likely model, the probability of getting four distinct digits is $10 \times 9 \times 8 \times 7/10^4$.

[**1**] 3. Let R be the event that an airplane's radar will be functioning, and let G be the event that the airplane's GPS will be functioning. We know that R and G are independent. We also know that $P(R) = 0.7$

and $P(G) = 0.9$. The airplane needs either a functioning radar or a functioning GPS to find its way at night (it is enough to have at least one of these two instruments functioning for the airplane to find its way). What is the probability that the airplane will be able to find its way at night?

A) 0.03 B) 0.75 C) 0.99 D) 0.85 E) 0.97

Answer: E
$P(R \cup G) = P(R) + P(G) - P(R \cap G) = P(R) + P(G) - P(R) \times P(G) = 0.7 + 0.9 - 0.7 \times 0.9 = 0.97.$

[**1**] 4. The concentration of a reactant is a random variable with the following probability density function

$$f_X(x) = \begin{cases} k(x+x^2), & \text{if } 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

What is the probability that the concentration is greater than 0.5? (Hint: You will need to first find the value of the constant k.)

A) 0.8 B) 0.75 C) 0.5 D) 0.25 E) 0.2

Answer: A
Solving

$$1 = \int_{-\infty}^{\infty} f_X(x)dx = k\int_0^1 x + x^2\,dx = k\,(1/2 + 1/3),$$

we get $k = 6/5$. We want

$$P(X > 0.5) = \int_{0.5}^{1} (6/5)(x+x^2)\,dx = (6/5)(x^2/2 + x^3/3)\big|_{0.5}^{1} = 0.8.$$

[**1**] 5. Consider the following joint probability mass function:

x	y	$p_{XY}(x,y)$
1	1	5/20
1	2	1/20
2	1	2/20
2	3	3/20
3	1	3/20
3	2	1/20
3	3	5/20

We can compute

$$E[X] = 2.15, \quad E[Y] = 1.9, \quad V[X] = 0.7275, \quad V[Y] = 0.89,$$

Determine $V[3X + 5Y]$, rounded to the nearest integer.

A) 40 B) 25 C) 82 D) 7 E) 24

Answer: A; We compute

$$\begin{aligned}
& E[XY] \\
= & 1 \times 1 \times \frac{5}{20} + 1 \times 2 \times \frac{1}{20} + 2 \times 1 \times \frac{2}{20} + 2 \times 3 \times \frac{3}{20} + 3 \times 1 \times \frac{3}{20} \\
& + 3 \times 2 \times \frac{1}{20} + 3 \times 3 \times \frac{5}{20} \\
= & \frac{89}{20} = 4.45
\end{aligned}$$

and so

$$\text{Cov}[X,Y] = E[XY] - E[X] \times E[Y] = 4.45 - (2.15)(1.9) = 0.365.$$

Thus,

$$\begin{aligned}
V[3X + 5Y] &= V[3\,X] + V[5\,Y] + 2\text{Cov}[3\,X, 5\,Y] \\
&= 3^2\,V[X] + 5^2\,V[Y] + 2(3)(5)\text{Cov}[X,Y] \\
&= 9\,(0.7275) + 25\,(0.89) + 30(0.365) = 39.7475 \approx 40
\end{aligned}$$

[1] 6. A large industrial firm allows a discount on any invoice that is paid within one month. Of all invoices, 10% receive the discount. In a company audit, 12 invoices are sampled at random. What is the probability that at most 1 of the 12 sampled invoices had received the discount?

A) 0.3766 B) 0.1667 C) 0.6590 D) 0.7665 E) 0.5795

Answer: C; We compute

Let X be the number invoices that received the discount among the 12 selected invoices. X has a binomial distribution with $n = 12$ and $p = 0.1$. We want

$$P(X \leq 1) = \binom{12}{0}(0.1)^0(0.9)^{12} + \binom{12}{1}(0.1)^1(0.9)^{11} = 0.6590.$$