MAT 2377A (Winter 2017)

Assignment 1 - Solution

Remark : *Please only grade questions 2, 4 and 6.*

1. Consider the following events :
   M : a new car will require repairs on the engine.
   T : a new car will require repairs on the drive train.
   We have that

$$P(M) = 0.85; \quad P(T) = 0.37; \quad P(M \cap T) = 0.25$$

   (a) We want to find $P(M \cup T) = P(M) + P(T) - P(M \cap T) = 0.97$
   (b) We want to find $P(M') = 1 - P(M) = 1 - 0.85 = 0.15$
   (c) We want to find $P(M' \cup T') = 1 - P(M \cap T) = 0.75$

2. Consider the following events :
   C : the strand has a high conductivity.
   R : the strand has a high strength.

[1] a) We have to find $P(C \cap R) = 73/98 = 0.7448$

[1] b) We have to find $P(C' \cup R') = 1 - P((C' \cup R')') = 1 - P(C \cap R) = 1 - 0.7448 = 0.2551$

[1] c) We have to find $P(R'|C') = \frac{P(R' \cap C')}{P(C')} = \frac{4/98}{20/98} = \frac{4}{20} = 0.2$

[1] d) Since $R' \cap C' \neq \emptyset$, R' and C' are not mutually exclusive.

[1] e) We have $P(R') = 9/98 = 0.0918$, but $P(R'|C') = 0.2$. It follows that, $P(R') \neq P(R'|C')$. Therefore, the data suggests that R' and C' are NOT independent. **Note : Depending on which events you use for part (e), the answer might not be as obvious from the computed value. Please grade part (e) only based on whether one of the right formulas was used and whether the right numbers were plugged into the formula. Please do not grade based whether the student concluded that the events are not independent.**

3. Let A_i be the event such that among the five, we have i strands with hight resistance.
   (a)We want to find

$$P(A_5) = \frac{\binom{89}{5}}{\binom{98}{5}} = 0.6112.$$

   (b) We want to find

$$P(A_0) + P(A_1) = \frac{\binom{89}{0}\binom{9}{5}}{\binom{98}{5}} + \frac{\binom{89}{1}\binom{9}{4}}{\binom{98}{5}} = 0.00017.$$

4. Consider the following events :
   A : having a room at Ramada Inn.
   B : having a room at Sheraton.
   C : having a room at Lakeview Motor Lodge.
   D : the plumbing is defective.
   First, we have

$$P(A) = 0.21; P(B) = 0.49; P(C) = 0.3; P(D|A) = 0.04; P(D|B) = 0.035; P(D|C) = 0.075.$$

[1] To identify the probabilities in the statement.

[2] (a) We want to find

$$P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C) = 0.04805.$$

[2] (b) We want to find

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{P(D|C)P(C)}{P(D)} = \frac{(0.075)(0.3)}{0.04805} = 0.46826.$$

5. Let A_i be the event that the component i works. Consider the events $C = A_1 \cap A_2 \cap A_3$ and $D = A_4 \cup A_5 \cup A_6$. We compute

$$P(C) = P(A_1)P(A_2)P(A_3) = (0.9)^3 = 0.729$$

$$P(D) = 1 - P(A_4' \cap A_5' \cap A_6') = 1 - P(A_4')P(A_5')P(A_6') = 1 - (0.05)^3 = 0.999875.$$

Consider the event $E = D \cap A_7$. We have $P(E) = P(D \cap A_7) = P(D)P(A_7) = (0.999875)(0.9) = 0.8999$. The probability that the circuit will operate is

$$P(E \cup C) = 1 - P(E' \cap C') = 1 - P(E')P(C') = 1 - (1 - 0.8999)(1 - 0.729) = 0.973.$$

6. Here are two enumeration questions.

[2] (a) There are 60 choices for the first number, 57 choices for the second and 57 choices for the third. Then there are $60 \times 57 \times 57 = 194,940$ possible combinations.

[2] (b) For the first flavor, 4 of the 24 students are selected, then for the second flavor, 4 students are selected from the 20 students who remain and so on. So, the number of different ways to distribute the 6 flavors of candy to 24 children is

$$\binom{24}{4}\binom{20}{4}\binom{16}{4}\binom{12}{4}\binom{8}{4}\binom{4}{4} = \frac{24!}{(4!)^6} = 3.24 \times 10^{15}.$$

[/14]